

# Statistics | S

Statistics is designed to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. This course serves as a 4<sup>th</sup> year mathematics credit and it is aligned with the Statewide Dual Credit Probability and Statistics Course Learning Objectives. Schools intending to offer this course for SDC credit should follow the Statewide Dual Credit Guidelines [Statewide Dual Credit \(tn.gov\)](https://www.tn.gov/statewide-dual-credit)

## Statistics includes the following Learning Objectives:

- Topic 1: Sampling and Data
- Topic 2: Descriptive Statistics
- Topic 3: Probability
- Topic 4; Discrete Random Variables
- Topic 5: Continuous Random Variables and the Normal Distribution
- Topic 6: Central Limit Theorem
- Topic 7: Confidence Intervals
- Topic 8: Hypothesis Testing
- Topic 9: Regression Correlation

## Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

## Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
<ol style="list-style-type: none"><li>1. Make sense of problems and persevere in solving them.</li><li>2. Reason abstractly and quantitatively.</li><li>3. Construct viable arguments and critique the reasoning of others.</li><li>4. Model with mathematics.</li><li>5. Use appropriate tools strategically.</li><li>6. Attend to precision.</li><li>7. Look for and make use of structure.</li><li>8. Look for and express regularity in repeated reasoning.</li></ol>

## Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
<ol style="list-style-type: none"><li>1. Use multiple reading strategies.</li><li>2. Understand and use correct mathematical vocabulary.</li><li>3. Discuss and articulate mathematical ideas.</li><li>4. Write mathematical arguments.</li></ol>

## Learning Objectives

### Topic 1: Sampling and Data★

- 1a** Understand the investigative process of statistics and differentiate between descriptive and inferential statistics.
- 1b** Differentiate between a population and a sample.
- 1c** Construct a simple random sample.
- 1d** Understand the differences between stratified sampling, cluster sampling, systematic sampling, and convenience sampling.
- 1e** Determine when samples of convenience are acceptable and how sampling bias and error can occur.
- 1f** Identify and classify data as either qualitative or quantitative and classify quantitative data as either discrete or continuous data.
- 1g** Display and interpret qualitative data with graphs: pie graphs, bar graphs, and pareto charts.
- 1h** Differentiate between levels of measurement: nominal, ordinal, interval, and ratio.
- 1i** Create a frequency distribution from a list of quantitative and/or qualitative data.
- 1j** Calculate relative frequencies and cumulative frequencies using a frequency distribution table.
- 1k** Understand differences between a designed experiment and an observational study.
- 1l** Differentiate between the types of variables used in a designed experiment.
- 1m** Understand different methods used in an experiment to isolate effects of the explanatory variable.

### Topic 2: Descriptive Statistics★

- 2a** Display and interpret graphs using quantitative data including stem-and-leaf plots, line graphs, and box plots.
- 2b** Construct a histogram from a frequency distribution table.
- 2c** Interpret data using histograms and time series graphs.
- 2d** Analyze a frequency distribution table and determine the sample size, class width and class midpoints.
- 2e** Recognize, describe, and calculate the measures of locations of data: quartiles, median, five number summary, interquartile range outliers, upper and lower fences, and percentiles.
- 2f** Distinguish between a parameter and a statistic.
- 2g** Calculate and differentiate between different measures of center: mean, median, and mode.
- 2h** Calculate the mean of a frequency distribution: GPA and weighted grade.
- 2i** Interpret the shape of the distribution from a graph: normal/symmetric, skewed, or uniform.
- 2j** Calculate and differentiate between different measures of spread: range, variance, and standard deviation.
- 2k** Determine if a data value is unusual based on standard deviations,  $\mu \pm 2\sigma$ .

### Topic 3: Probability★

- 3a** Understand and use terminology and symbols of probability.
- 3b** List the elements of events and the sample space from an experiment.
- 3c** Understand the concept of randomness: flipping a coin, rolling a die, and drawing a card from a standard 52 card deck.
- 3d** Differentiate between and calculate different types of probabilities: empirical and theoretical.
- 3e** Explain the Law of Large Numbers.
- 3f** Calculate and interpret probabilities using the complement rule, addition rule, and multiplication rule.
- 3g** Differentiate between and calculate probabilities for different types of events: independent, dependent, with or without replacement, conditional, and mutually exclusive.
- 3h** Use Venn diagrams and lists to solve probability problems when appropriate.

#### Topic 4: Discrete Random Variables★

- 4a Identify the random variable in a probability experiment.
- 4b Recognize and understand discrete probability distribution functions.
- 4c Create a probability distribution for the values of a discrete random variable.
- 4d Use a probability function to determine probabilities associated with a discrete random variable.
- 4e Calculate and interpret the mean (expected value), variance, and standard deviation for discrete random variables and binomial probability distributions.
- 4f Determine when a probability distribution should be classified as a discrete binomial probability distribution, and calculate probabilities associated with such a distribution.

#### Topic 5: Continuous Random Variables and the Normal Distribution★

- 5a Recognize and understand continuous probability density functions.
- 5b Use a probability density curve to describe a population, including a normal population.
- 5c Calculate and interpret the area under a probability density curve.
- 5d Calculate and interpret a z-score, understanding the concept of "standardizing" data.
- 5e Calculate and interpret z-scores using the Empirical Rule, understanding the general properties of the normal distribution: 100% is the total area under the curve, exactly 50% is to the left and right of the mean, and it is perfectly symmetric about the mean.
- 5f Use technology to calculate the area under the curve for any normal distribution model: left, right, and between.
- 5g Use technology to calculate percentiles, quartiles, and other numerical values of  $X$  for a specified area under a normal curve, including unusual values ( $P(X) < 5\%$  and  $\mu \pm 2\sigma$ ).

#### Topic 6: Central Limit Theorem★

- 6a Recognize the characteristics of the mean of sample means taken from different types of populations: normal and non-normal.
- 6b Calculate the mean of sample means taken from different types of populations: normal and non-normal.
- 6c Describe how the means of samples calculated from a non-normal population might be distributed.
- 6d Apply the Central Limit Theorem to normal and non-normal populations and compute probabilities of a sample mean.
- 6e Determine whether the Central Limit Theorem can be used for a given situation.
- 6f Assess the impact of sample size on sampling variability.

#### Topic 7: Confidence Intervals★

- 7a Read and write confidence intervals using two different forms: point estimate plus/or minus margin of error (error bound) and interval notation.
- 7b Calculate and interpret confidence intervals for estimating a population mean and a population proportion.
- 7c Calculate the margin of error (error bound) using sample statistics.
- 7d Predict if a confidence interval will become wider or narrower given larger or smaller sample sizes as well as higher or lower confidence levels.
- 7e Find the point estimate and margin of error (error bound) when given a confidence interval.
- 7f Estimate the sample size necessary to estimate a population mean.
- 7g Recognize the difference between the sample mean,  $\bar{x}$ , and the population mean,  $\mu$ , as well as the difference between the sample standard deviation,  $s$ , and standard error of the mean,  $s/\sqrt{n}$ .
- 7h Find critical values for  $z_{\alpha/2}$  and  $t_{\alpha/2}$  given a value of  $\alpha$  and degrees of freedom.
- 7i Estimate the sample size necessary to estimate a population proportion.

## Topic 8: Hypothesis Testing★

- 8a** Determine the appropriate null and alternative hypotheses when presented with a problem.
- 8b** Differentiate between Type I and Type II errors.
- 8c** Understand and list the assumptions needed to conduct z-tests and t-tests.
- 8d** Determine whether to reject or fail to reject the null hypothesis using the p-value method.
- 8e** Determine if a test is left-tailed, right-tailed, or two-tailed.
- 8f** Differentiate between independent group and matched pair sampling.
- 8g** Calculate test statistics and p-values for hypotheses tests: single proportion, single mean, and difference between two means.
- 8h** Conduct hypotheses tests for a single proportion and a single mean.
- 8i** Test hypotheses regarding the difference of two independent means (assume the variances are not pooled).
- 8j** Draw conclusions and make inferences about claims based on hypotheses tests.

## Topic 9: Regression Correlation★

- 9a** Differentiate between the independent (explanatory variable,  $x$ ) and the dependent (response variable,  $y$ ) in a bivariate data set.
- 9b** Create a scatter plot and determine the type of relationship that exists between two variables: positive or negative correlation and weak or strong correlation.
- 9c** Calculate and interpret the correlation coefficient using technology.
- 9d** Calculate the line of best fit and interpret the coefficient of determination.
- 9e** Use the line of best fit to make conclusions about the relationship between two variables, understanding correlation does not imply causation.
- 9f** Calculate a residual using the line of best fit.
- 9g** Use the p-value to determine if a line of best fit is statistically significant.
- 9h** For a given value of  $x$ , find the appropriate estimated value of  $y$ .
- 9i** Distinguish between interpolated and extrapolated values and explain why interpolated values are more reliable.
- 9j** Perform a residual analysis to check assumptions of regression.