

# Basic Math for All Certifications

Course # 1014



Department of  
**Environment &  
Conservation**



**BASIC MATH FOR ALL  
CERTIFICATIONS  
APRIL 31-MARCH 3, 2020  
COURSE #1014**

**Monday**

8:30 am	Fractions, Decimals, Percents, Averages	Amanda Carter
10:00	Order of Operations	Amanda
11:00	Lunch	
12:15	Powers and Roots	Amanda
1:00	Solving for the Unknown Value	Amanda

**Tuesday**

8:30 am	Solving for the Unknown Value Cont'd	Amanda
9:30	Ratios and Proportions	Amanda
11:00	Lunch	
12:15 pm	Metric System and Temperature	Amanda
1:00	Dimensional Analysis (Conversions)	Amanda

**Wednesday**

8:30 am	Circumference, Area, and Volume	Amanda
12:00	Lunch	
1:00 pm	Velocity and Flow	Amanda

**Thursday**

8:30 am	End of Week Review	Amanda
11:00	Course Evaluation and Exam	





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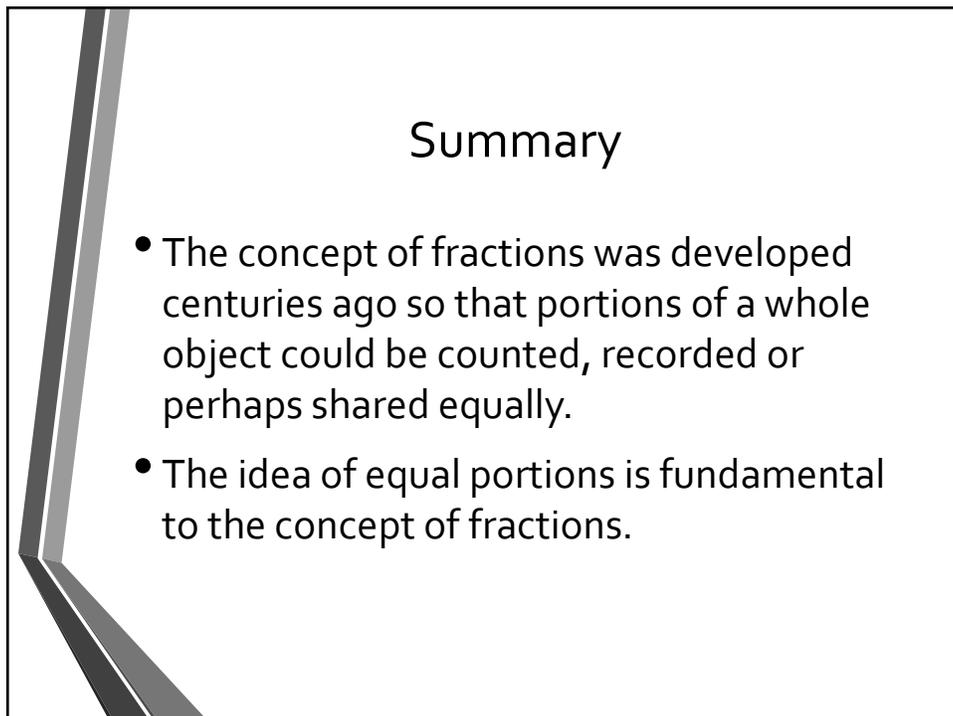
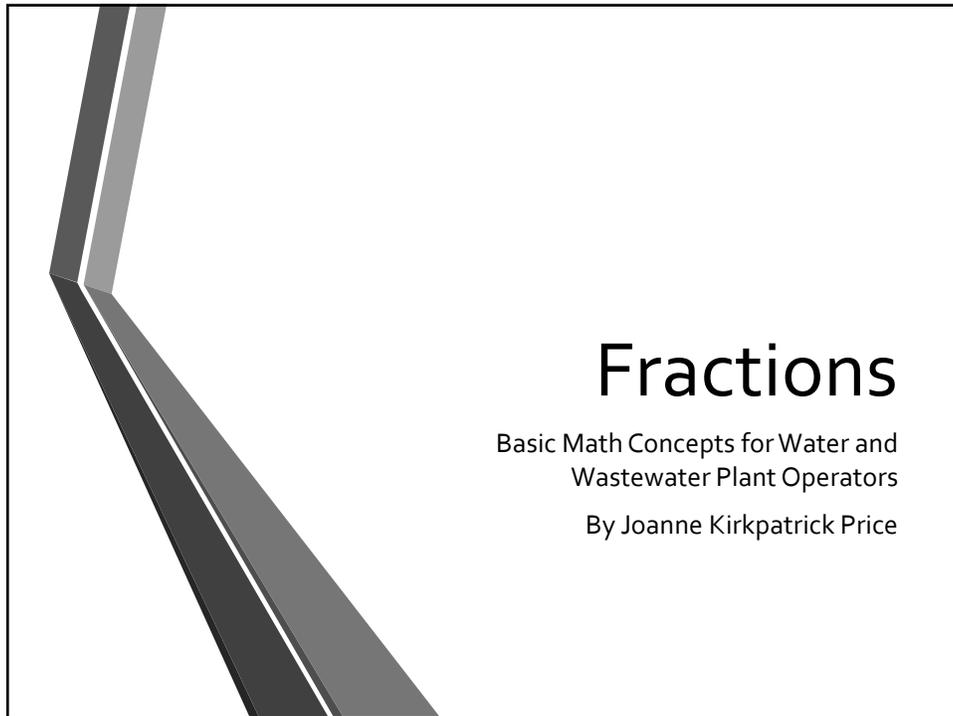
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## Section 1

### Fractions, Decimals, Percents & Averages





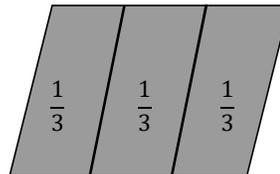
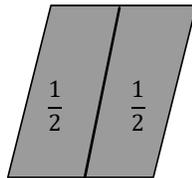
## Summary

- Numerator
  - Top portion of a fraction
  - Indicates how many parts are being considered
- Denominator
  - Bottom portion of fraction
  - Tells how many equal parts the whole has been divided into

$$\frac{\text{Numerator}}{\text{Denominator}}$$
  
Division line

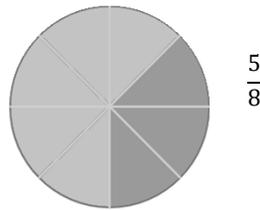
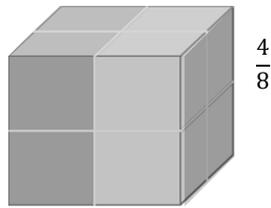
## Denominator

- Bottom of fraction
- Gives the name of the fraction
  - Halves, thirds, fourths, fifths, twentieths, etc
  - A denominator of two indicates that the whole has been divided into two equal parts



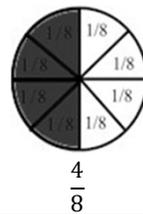
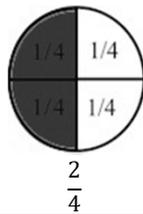
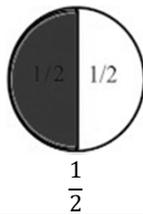
## Numerator

- Top of fraction
- Indicates number of equal parts



## Equivalent Fractions

- Fractions with different numerators and denominators that refer to the same portion
- Fractions that represent equal parts of the whole



## Finding Equivalent Fractions

- Multiply the numerator and denominator by the same number

$$\frac{4}{11} \times \frac{3}{3} = \frac{12}{33}$$

- $\frac{4}{11}$  and  $\frac{12}{33}$  are equivalent fractions

## Determining Equivalent Fractions

- Verify equivalent fractions by cross multiplying

$$\frac{1}{8} \neq \frac{5}{32}$$
$$1 \times 32 = 32 \qquad 8 \times 5 = 40$$
$$32 \neq 40$$

- Fractions are **not** equivalent

## Reducing Fractions

- Divide the numerator and denominator by the same number

$$\frac{156}{204} \div \frac{2}{2} = \frac{78}{102}$$

- Simplify fractions by dividing by a common dividend

$$\frac{78}{102} \div \frac{6}{6} = \frac{13}{17}$$

- Fraction is now in its lowest terms (cannot be reduced any further)

## Complex Fractions

- A fraction whose numerator and/or denominator contains a fraction
- To complete these
  - Simplify the numerator and denominator
  - Restate the original problem
  - Divide as needed

$$\frac{25}{\frac{6}{7}}$$

$$6 \div 7 = 0.8571$$

$$25 \div 0.8571 = 29.17$$

$\frac{1}{4} \div \frac{3}{8}$

## Solving Complex Fractions

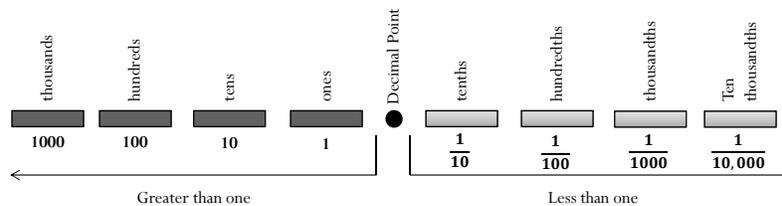
- Step 1 – Move bottom fraction to top by inverting (flip) and multiplying
$$\frac{1}{4} \times \frac{8}{3}$$
- Step 2 - Multiply fraction  $\frac{1 \times 8}{4 \times 3} = \frac{8}{12}$
- Step 3 – Reduce fraction  $\frac{8}{12} \div \frac{4}{4} = \frac{2}{3}$

## Decimals

Basic Math Concepts for Water and Wastewater  
Plant Operators  
By Joanne Kirkpatrick Price

## Decimal System

- The word decimal comes from the Latin word meaning *decem*, meaning ten.
- The decimal system is based on ten and multiples of ten.
- In a place value system the size of any number depends on two things:
  - Which digits are used and
  - Where these digits are placed in relation to the decimal point



## Percents and Decimals

- To convert from a decimal to a percent
  - Move the decimal point two places to the right

$$0.46 \rightarrow 46.0\%$$

- Multiply decimal by 100

$$0.46(100) = 46\%$$

- To convert from a percent to a decimal
  - Move the decimal two points to the left

$$79.5\% \rightarrow 0.795$$

- Divide percent by 100

$$\frac{79.5}{100} = 0.795$$

## Converting Decimals and Fractions

- To convert a decimal to a fraction
  - The entire number becomes the numerator, disregarding the decimal point
  - The denominator is determined by how many decimal places to the right the number goes
  - Reduce the fraction

0.53

- 53 becomes the numerator
- The number goes 2 places past the decimal, so we will put 2 zeros in the denominator

$$\frac{53}{100}$$

## Converting Decimals and Fractions

- To convert a fraction to a decimal
  - Simply divide the numerator by the denominator

$$\frac{1}{2} = 1 \div 2 = 0.5$$

$$\frac{10}{13} = 10 \div 13 = 0.7692$$

## Percents and Fractions

- To convert a fraction into a percent
  - Divide the fraction to obtain a decimal number. Then convert the decimal number to a percent.

$$\frac{4}{17}$$

$$4 \div 17 = 0.24$$

$$0.24 \times 100 = 24\%$$

## Percents and Fractions

- To convert a percent to a fraction
  - Simply write the number over 100
- If a percent has a decimal, the decimal must be taken out

$$33\% = \frac{33}{100}$$

$$12.5\% = \frac{12.5}{100}$$

$$\frac{12.5}{100} \times \frac{10}{10}$$

$$\frac{125}{1000}$$

- Reduce fraction to lowest terms

$$\frac{125}{1000} \div \frac{25}{25} = \frac{5}{40}$$

$$\frac{5}{40} \div \frac{5}{5} = \frac{1}{8}$$

## Key Words in Math

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### Key Words

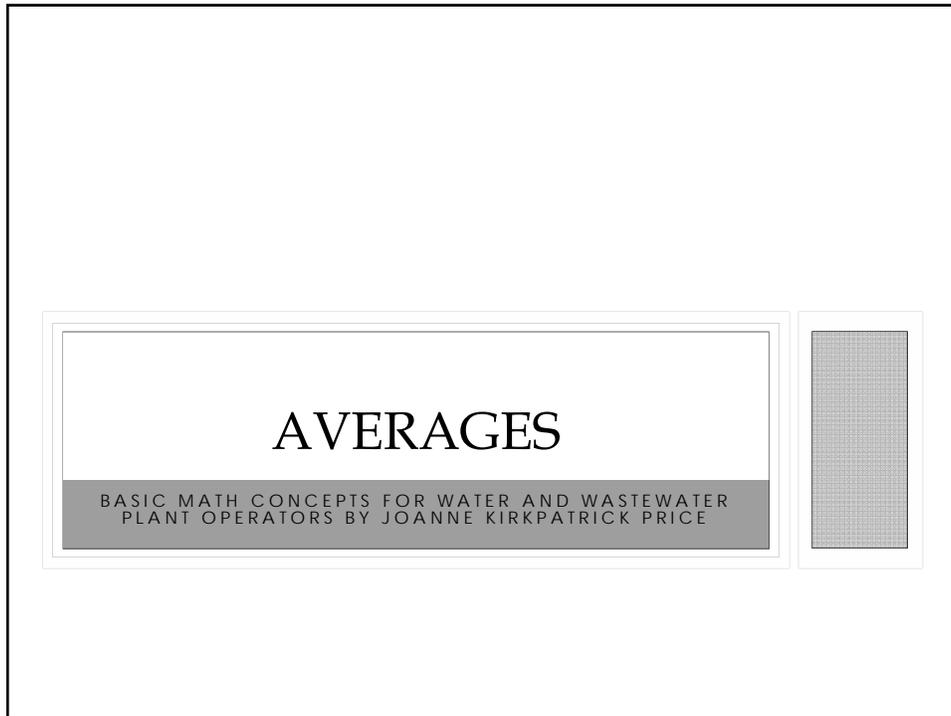
- Of  $\longrightarrow$  multiply

$$5\% \text{ of } 100 = 0.05 \times 100$$

- Per  $\longrightarrow$  divide

$$25 \text{ miles per gallon} = 25 \frac{\text{miles}}{\text{gallon}}$$

- Is  $\longrightarrow$  equals



A presentation slide with a large black border. At the top, the word "AVERAGES" is centered in a large, bold, serif font, enclosed in a white box with a double-line border. Below this, a bulleted list item states: "• By calculating averages, a group of data is represented by a single number". Underneath the list item is the formula for the mean: 
$$\text{Mean} = \frac{\text{Sum of all measurements}}{\text{Number of measurements used}}$$

## EXAMPLE

- What is the average temperature for a week given the following data:

72°F, 70°F, 79°F, 80°F, 77°F, 77°F, 73°F

$$\text{Mean} = \frac{\text{Sum of all measurements}}{\text{Number of measurements used}}$$

$$\text{Mean} = \frac{72 + 70 + 79 + 80 + 77 + 77 + 73}{7}$$

$$\text{Mean} = \frac{528}{7}$$

$$\text{Mean} = 75.4^\circ\text{F}$$

## Basic Math

### Fractions, Decimals, Percents and Averages

Determine if the following are equivalent fractions (1-10).

1.  $\frac{1}{15} = \frac{4}{60}$

6.  $\frac{28}{72} = \frac{7}{18}$

2.  $\frac{145}{175} = \frac{29}{45}$

7.  $\frac{44}{116} = \frac{11}{30}$

3.  $\frac{60}{66} = \frac{10}{13}$

8.  $\frac{12}{81} = \frac{4}{27}$

4.  $\frac{140}{180} = \frac{32}{36}$

9.  $\frac{115}{160} = \frac{32}{23}$

5.  $\frac{5}{24} = \frac{25}{110}$

10.  $\frac{4}{128} = \frac{6}{32}$

Reduce the following fractions to their simplest form (11-20).

11.  $\frac{45}{90} =$

14.  $\frac{14}{21} =$

12.  $\frac{45}{81} =$

15.  $\frac{18}{54} =$

13.  $\frac{14}{35} =$

16.  $\frac{5}{10} =$

17.  $\frac{4}{8} =$

19.  $\frac{16}{80} =$

18.  $\frac{20}{35} =$

20.  $\frac{48}{80} =$

Convert the following fractions to decimals (21-30).

21.  $\frac{3}{5} =$

26.  $\frac{17}{53} =$

22.  $\frac{9}{13} =$

27.  $\frac{2}{5} =$

23.  $\frac{7}{4} =$

28.  $\frac{13}{169} =$

24.  $\frac{1}{3} =$

29.  $\frac{22}{100} =$

25.  $\frac{5}{6} =$

30.  $\frac{33}{99} =$

Convert the following decimals to fractions in lowest terms (31-40).

31.  $0.98 =$

34.  $0.84 =$

32.  $0.516 =$

35.  $7.5 =$

33.  $1.23 =$

36.  $0.5833 =$

37.  $0.67 =$

39.  $0.75 =$

38.  $0.8333 =$

40.  $0.785 =$

Convert the following percents into fractions in lowest terms (41-50).

41.  $33\% =$

46.  $0.5\% =$

42.  $12\% =$

47.  $16.3\% =$

43.  $45\% =$

48.  $25\% =$

44.  $75\% =$

49.  $100\% =$

45.  $110\% =$

50.  $30.4\% =$

Convert the following percents into decimals (51-60).

51.  $16\% =$

56.  $88.7\% =$

52.  $75\% =$

57.  $0.5\% =$

53.  $20\% =$

58.  $112\% =$

54.  $0.07\% =$

59.  $12.5\% =$

55.  $120\% =$

60.  $57.94\% =$

Convert the following decimals into a percent (61-70).

61.  $0.531 =$

63.  $1.21 =$

62.  $0.66 =$

64.  $0.08 =$

65.  $19.5 =$

68.  $1.0 =$

66.  $0.406 =$

69.  $0.278 =$

67.  $11.0 =$

70.  $0.785 =$

Solve the following word problems (71-80).

71. What is 10% of 55?

72. What is 15% of 125?

73. 50% of 840 is what?

74. What is 7% of 1125?

75. 110% of 50 is what?

76. What is 5% of 10.7?

77. 68% of 2140 is how much?

78. 4% of 4175 is what number?

79. What is 78.5% of 150,000?

80. You need to disinfect a 300,000 gallon storage tank. The method you are using calls for you to dose 5% of the tank volume with 50 mg/L chlorine. What is 5% of 300,000 gallons?

Find the arithmetic mean (average) of the following sets of values.

81. What is the average high temperature of the week in °C? (Data for seven days : 21°C, 25.2°C, 19°C, 22°C, 20°C, 19.4°C, and 20.1°C)
82. What was the average chlorine residual measured in the distribution system? (0.2 mg/L, 0.7 mg/L, 0.5 mg/L, 0.8 mg/L, 1.2 mg/L)
83. What is the average weight of a 1 L volumetric flask? (700 g, 701 g, 698 g, 690 g, 704 g, 697 g, 705 g)
84. What was the average flow for the year in MGD through the Townsville Treatment Plant? (Jan = 1.32 MGD, Feb = 1.21 MGD, Mar = 1.5 MGD, Apr = 1.6 MGD, May = 1.95 MGD, June = 1.8 MGD, July = 1.7 MGD, Aug = 1.65 MGD, Sep = 1.5 MGD, Oct = 1.25 MGD, Nov = 1.6 MGD, Dec = 1.92 MGD)

## Basic Math

## Fractions, Decimals, Percent, and Averages Answers

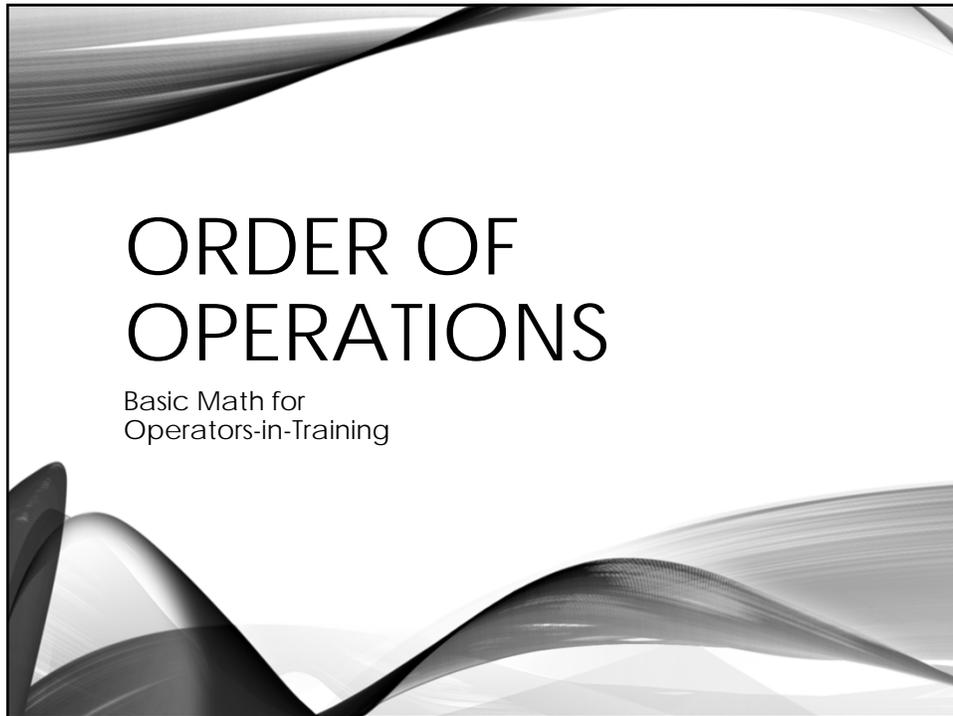
- |     |               |     |                      |     |                    |     |                     |
|-----|---------------|-----|----------------------|-----|--------------------|-----|---------------------|
| 1.  | yes           | 22. | 0.69                 | 42. | $\frac{3}{25}$     | 63. | 121%                |
| 2.  | no            | 23. | 1.75                 | 43. | $\frac{9}{20}$     | 64. | 8%                  |
| 3.  | no            | 24. | 0.33                 | 44. | $\frac{3}{4}$      | 65. | 1950%               |
| 4.  | no            | 25. | 0.83                 | 45. | $\frac{11}{10}$    | 66. | 40.6%               |
| 5.  | no            | 26. | 0.32                 | 46. | $\frac{1}{200}$    | 67. | 1100%               |
| 6.  | yes           | 27. | 0.4                  | 47. | $\frac{163}{1000}$ | 68. | 100%                |
| 7.  | no            | 28. | 0.08                 | 48. | $\frac{1}{4}$      | 69. | 27.8%               |
| 8.  | yes           | 29. | 0.22                 | 49. | 1.00               | 70. | 78.5%               |
| 9.  | no            | 30. | 0.33                 | 50. | $\frac{38}{125}$   | 71. | 5.5                 |
| 10. | no            | 31. | $\frac{49}{50}$      | 51. | 16                 | 72. | 18.75               |
| 11. | $\frac{1}{2}$ | 32. | $\frac{129}{250}$    | 52. | 0.75               | 73. | 420                 |
| 12. | $\frac{5}{9}$ | 33. | $\frac{123}{100}$    | 53. | 0.20               | 74. | 78.75               |
| 13. | $\frac{2}{5}$ | 34. | $\frac{21}{25}$      | 54. | 0.0007             | 75. | 55                  |
| 14. | $\frac{2}{3}$ | 35. | $\frac{15}{2}$       | 55. | 1.20               | 76. | 0.535               |
| 15. | $\frac{1}{3}$ | 36. | $\frac{5833}{10000}$ | 56. | 0.887              | 77. | 1455.2              |
| 16. | $\frac{1}{2}$ | 37. | $\frac{67}{100}$     | 57. | 0.0005             | 78. | 167                 |
| 17. | $\frac{1}{2}$ | 38. | $\frac{8333}{10000}$ | 58. | 1.12               | 79. | 117,750             |
| 18. | $\frac{4}{7}$ | 39. | $\frac{3}{4}$        | 59. | 0.125              | 80. | 15,000              |
| 19. | $\frac{1}{5}$ | 40. | $\frac{157}{200}$    | 60. | 0.5794             | 81. | 20.96°C             |
| 20. | $\frac{3}{5}$ | 41. | $\frac{1}{3}$        | 61. | 53.1%              | 82. | 0.68 $\frac{mg}{L}$ |
| 21. | 0.6           |     |                      | 62. | 66%                | 83. | 699.3 g             |
|     |               |     |                      |     |                    | 84. | 1.5 MGD             |



## Section 2

### Order of Operations

**P**ARENTHESSES  
**E**XPONENTS  
**M**ULTIPLY  
**D**IVIDE  
**A**DD  
**S**UBTRACT



WHAT IS ORDER OF OPERATIONS?

- A set way to solve an calculation  
 $8 + 16 \div 4$

Two empty rectangular boxes with horizontal lines, intended for students to write their solutions to the calculation.

- Which way is the correct way?

The slide has the same decorative background as the title slide. The title 'WHAT IS ORDER OF OPERATIONS?' is centered in a large, bold, sans-serif font. Below it, a bullet point describes the concept as 'A set way to solve an calculation' followed by the expression  $8 + 16 \div 4$ . Two empty rectangular boxes with horizontal lines are provided for student input. A second bullet point asks 'Which way is the correct way?'.

## PEMDAS

- Parenthesis
- Exponents
- Multiplication/Division
- Addition/Subtraction

$$\begin{array}{r} 8 + 16 \div 4 \\ 24 \div 4 \\ 6 \end{array}$$

$$\begin{array}{r} 8 + 16 \div 4 \\ 8 + 4 \\ 12 \end{array}$$

## EXAMPLE

Parenthesis  
Exponents  
Multiplication/Division  
Addition/Subtraction

$$(7 \times 3) \times 4 \div 2 - 5 \times 6$$

$$\underline{(7 \times 3)} \times 4 \div 2 - 5 \times 6$$

$$21 \times 4 \div 2 - 5 \times 6$$

Since Multiplication and Division are on the same "level," work left to right

$$\underline{21 \times 4} \div 2 - 5 \times 6$$

$$84 \div 2 - 5 \times 6$$

Skip the subtraction because it is on the next "level"

$$\underline{84 \div 2} - 5 \times 6$$

$$42 - 5 \times 6$$

$$42 - \underline{5 \times 6}$$

$$\underline{42 - 30}$$

$$12$$

## Basic Math for Operators-in-Training

### Order of Operations

1.  $(14 + 2) \times 8 - 4$

2.  $4 \times 3 + (3 + 6)$

3.  $(11 + 5) + 10 \times 5$

4.  $(8 + 27 - 5) \times 6$

5.  $(10 + 3) \times (7 - 5)$

6.  $(12 + 7) \times 9 + 2$

$$7. 2 \times 3 + (9 + 6)$$

$$8. (9 + 3) + 15 \times 5$$

$$9. (10 + 20 - 6) \times 6$$

$$10. (14 + 3) \times (12 + 5)$$

$$11. (14 + (15 - 3)) \times 7$$

$$12. 12 + ((17 + 4) + 2)$$

$$13. (7 + (18 - 3 + 2))$$

$$14. ((11 + 4) + 4) + 8$$

$$15. (10 + (18 - 3)) \times 7$$

$$16. 2 + ((13 + 5) + 6)$$

$$17. ((10 - 2) \times 5) - 10$$

$$18. 13 + (10 + (11 - 5))$$

$$19. 15 + (5 \times (17 - 6))$$

$$20. 8 + (14 - 7 - 6)$$

$$21. 18 + (5 \times (11 - 4)^2)$$

$$22. ((14 - 2) + 14 - 2)^2$$

$$23. 14 + (5 \times (4 + 3)^2)$$

$$24. 18 + ((10 + 3) + 2^2)$$

$$25. (4^2 + (10 - 2 + 4^2))$$

$$26. (6^2 + (20 - 5 + 3^2))$$

$$27. 18 + ((11 + 7) + 3^2)$$

$$28. ((5 + 4)^2 \times 2) + 2^2$$

$$29. ((18 + 2) + (20 - 4)^2)$$

$$30. ((10 - 4)^2 + 6) - 4^2$$

Answers:	19. 70
1. 124	20. 9
2. 21	21. 263
3. 66	22. 576
4. 180	23. 259
5. 26	24. 35
6. 173	25. 40
7. 21	26. 60
8. 87	27. 45
9. 144	28. 166
10. 289	29. 276
11. 182	30. 26
12.35	
13. 24	
14. 27	
15. 175	
16. 26	
17. 30	
18. 29	



## Section 3

### Powers and Roots

$$\left(\sqrt[5]{32}\right)^3$$

$$x^5$$

$$\sqrt[3]{27}$$

$$25^{1/2}$$

$$3^{-4} = \frac{1}{3^4}$$

$$y^3 \times y^6 = y^9$$

# POWERS & ROOTS

BASIC MATH CONCEPTS FOR WATER AND WASTEWATER  
PLANT OPERATORS BY JOANNE KIRKPATRICK PRICE

## TERMS

- Exponent – indicates how many times a number is to be multiplied together
- Base – the number that is being multiplied

$$\begin{array}{c} \text{base} \quad \text{exponent} \\ \swarrow \quad \nwarrow \\ 7^4 = (7)(7)(7)(7) \end{array}$$

- Power – entire expression (seven to the 4<sup>th</sup> power)
- Same rule applies to letters and measurement units

$$x^3 = (x)(x)(x) \text{ or } ft^2 = (ft)(ft)$$

- Any number that does not have an exponent is considered to have an exponent of 1

$$10 = 10^1$$

## EXAMPLE 1

- Write the following numbers in expanded form:

- $6^2$

\_\_\_\_\_

- $20^3$

\_\_\_\_\_

- $3^5$

\_\_\_\_\_

- $x^3$

\_\_\_\_\_

## EXAMPLE 2

- Write the factors using exponential notation:

- $(4)(4)(4)$

\_\_\_\_\_

- $(2)(2)(2)(2)$

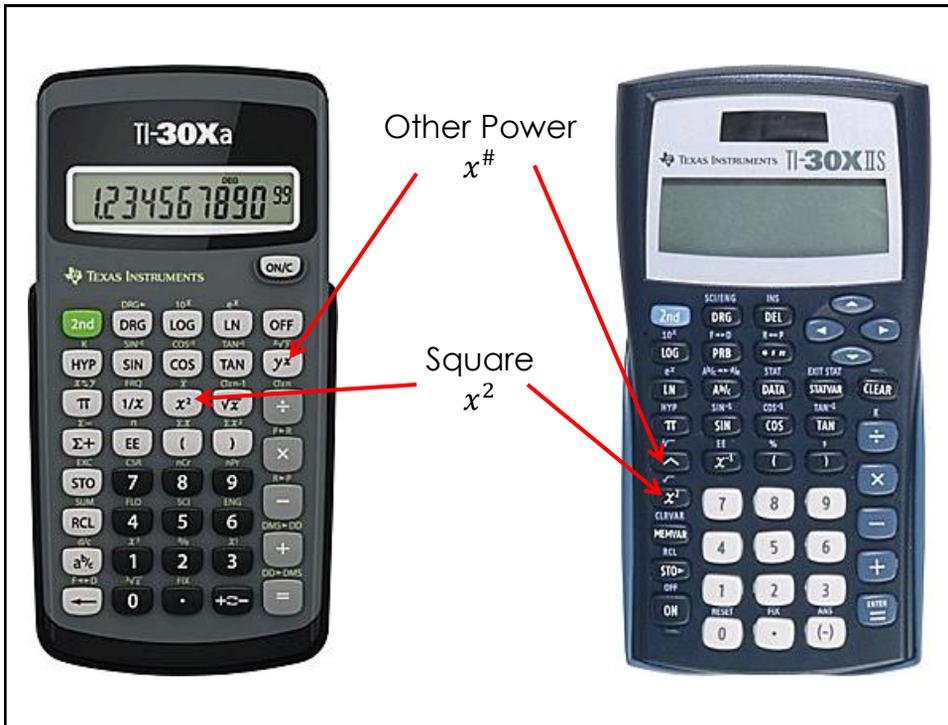
\_\_\_\_\_

- $(7)(7)(7)(7)(7)(7)(7)$

\_\_\_\_\_

- $(x)(x)$

\_\_\_\_\_



### EXAMPLE 3

- Complete the following calculations

- $27^2$

- $14^5$

- $5^{7.9}$

## NEGATIVE EXPONENTS

- A factor with a negative exponent can be inverted and written with a positive exponent

$$3^{-2} = \frac{1}{3^2}$$

- When a power is moved from the numerator to denominator (and vice versa) the sign of the exponent must be changed
- Any number that has an exponent of zero is equal to one.

$$7^0 = 1$$

$$x^0 = 1$$

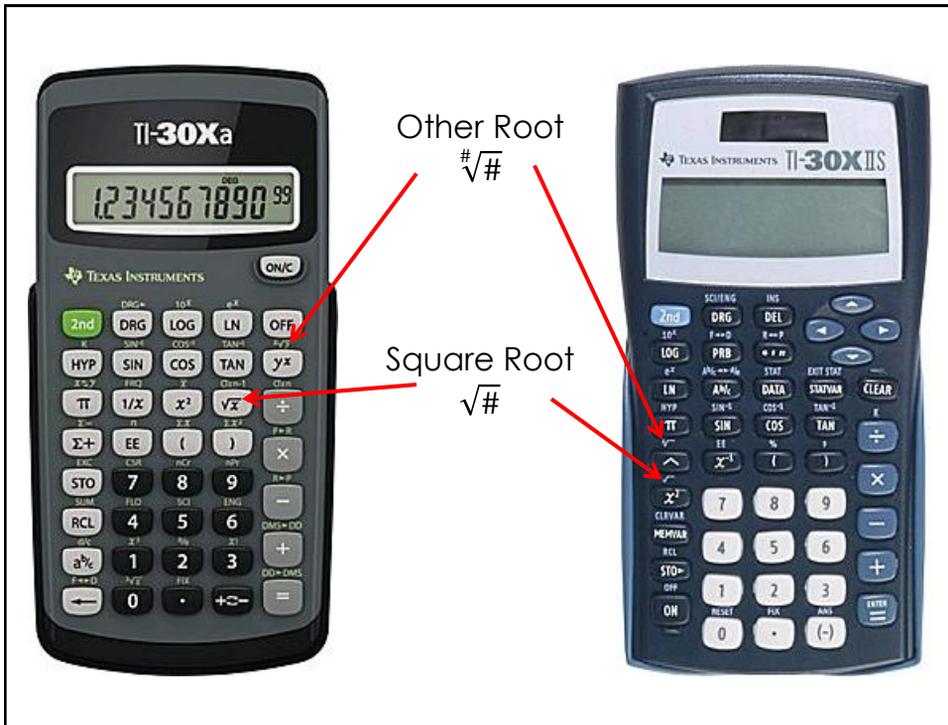
## ROOTS

- Root - a number which, when multiplied together two or more times, equals the original number
- Square root – a number in which, when multiplied together twice, equals the original number

$$\sqrt[2]{100} = \sqrt{100}$$

- Cube root – a number which, when multiplied together three times equals the original number

$$\sqrt[3]{8}$$



## ROOTS

- Complete the following calculations

- $\sqrt{6400}$

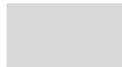
- $\sqrt[3]{912673}$

- $\sqrt[8]{390625}$

## FRACTIONAL EXPONENTS

- Fractional exponent
  - Numerator indicates power to raise base to
  - Denominator indicates root to be taken

Power to raise to  $\rightarrow$   $15^{2/3}$   $\leftarrow$  Root to be taken



## EXAMPLE 4

- Express the following numbers using radicals

- $4^{1/2}$



- $64^{1/3}$



- $90^{2/3}$



- $1.4^{5/2}$



### EXAMPLE 5

- Express the following numbers using fractional exponents

- $\sqrt{5^3}$

- $\sqrt[3]{x^2}$

- $\sqrt[5]{150}$

### EXAMPLE 6

- Complete the following calculations

- $484^{1/2}$

- $\sqrt[3]{27}$

- $4^{3/2}$

## MULTIPLYING POWERS

- When multiplying powers with the same base, simply add exponents

$$x^2 * x^3$$



$$x^3 \bullet x^4 = (x)(x)(x)(x)(x)(x)(x)$$



## EXAMPLE 7

- Simplify the following terms using the rule for multiplying powers:

- $3^2 \bullet 3^5$



- $x^3 \bullet x^3$



## DIVIDING POWERS

- When dividing powers with the same base, subtract the power of the denominator from the power of the numerator

$$\frac{x^5}{x^3}$$

## EXAMPLE 8

- Simplify the following terms using the rule for dividing powers

- $\frac{a^3}{a^2}$

\_\_\_\_\_

- $\frac{9^5}{9^3}$

\_\_\_\_\_

## Powers and Roots Practice Problems

Write the following numbers in expanded form as factors.

1.  $6^2$  \_\_\_\_\_

6.  $x^2$  \_\_\_\_\_

2.  $10^4$  \_\_\_\_\_

7.  $8^1$  \_\_\_\_\_

3.  $x^3$  \_\_\_\_\_

8.  $14^4$  \_\_\_\_\_

4.  $5^0$  \_\_\_\_\_

9.  $17^3$  \_\_\_\_\_

5.  $13^6$  \_\_\_\_\_

10.  $1^7$  \_\_\_\_\_

Write the following numbers using exponential notation.

11.  $(4)(4)(4)$  \_\_\_\_\_

12.  $(x)(x)(x)(x)$  \_\_\_\_\_

13.  $(9)(9)$  \_\_\_\_\_

14.  $(16)(16)(16)(7)(7)$  \_\_\_\_\_

15.  $\frac{1}{(2)(2)(2)(2)(2)}$  \_\_\_\_\_

16.  $(D)(D)(D)$  \_\_\_\_\_

17.  $\frac{1}{8}$  \_\_\_\_\_

18.  $(3)(3)(3)(3)$  \_\_\_\_\_

19.  $(12)(12)(12)$  \_\_\_\_\_

20.  $(0.5)(0.5)$  \_\_\_\_\_

Solve the following problems.

21.  $(19)(19)(19) =$  \_\_\_\_\_

22.  $(0.785)(0.3333)^2(150) =$  \_\_\_\_\_

23.  $(0.785)(4)^2 =$  \_\_\_\_\_

24.  $(2^2)(3^4) =$  \_\_\_\_\_

25.  $(36)(14)(2^3) =$  \_\_\_\_\_

26.  $(5^3)(2^3) =$  \_\_\_\_\_

27.  $(5^5)(5^5) =$  \_\_\_\_\_

28.  $(7x3)^2 =$  \_\_\_\_\_

29.  $(7.5^1)(2^2) =$  \_\_\_\_\_

30.  $(0.5^4)(2.2^{-2}) =$  \_\_\_\_\_

Write the following in radical form. (fractional exponents into  $\sqrt{x}$  )

31.  $144^{1/2} =$  \_\_\_\_\_

32.  $27^{1/3} =$  \_\_\_\_\_

33.  $15^{3/6} =$  \_\_\_\_\_

34.  $10^{2/3} =$  \_\_\_\_\_

35.  $54^{4/5} =$  \_\_\_\_\_

36.  $16^{2/1} =$  \_\_\_\_\_

37.  $98^{3/4} =$  \_\_\_\_\_

Write the following numbers in exponential form ( $\sqrt{x}$  into fractional exponents).

38.  $\sqrt{450} =$  \_\_\_\_\_

39.  $\sqrt[3]{27} =$  \_\_\_\_\_

40.  $\sqrt[4]{45^7} =$  \_\_\_\_\_

41.  $\sqrt[8]{12^5} =$  \_\_\_\_\_

42.  $\sqrt{21^3} =$  \_\_\_\_\_

43.  $\sqrt{16^3} =$  \_\_\_\_\_

44.  $\sqrt[3]{11^2} =$  \_\_\_\_\_

45.  $\sqrt[2]{87^3} =$  \_\_\_\_\_

46.  $\sqrt{49^3} =$  \_\_\_\_\_

$$47. \quad \sqrt[3]{14^{-1}} = \underline{\hspace{2cm}}$$

Complete the following problems.

$$48. \quad 144^{1/2} = \underline{\hspace{2cm}}$$

$$49. \quad \sqrt{6400} = \underline{\hspace{2cm}}$$

$$50. \quad \sqrt[3]{1000} = \underline{\hspace{2cm}}$$

$$51. \quad \sqrt{4^3} = \underline{\hspace{2cm}}$$

$$52. \quad 64^{1/3} = \underline{\hspace{2cm}}$$

$$53. \quad (2)(3)(\sqrt{81}) = \underline{\hspace{2cm}}$$

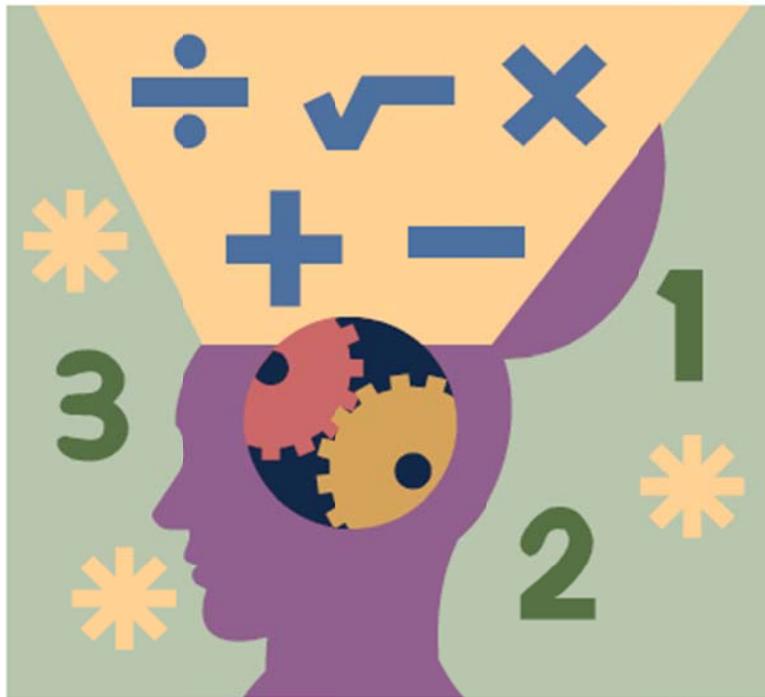
## Powers and Roots Practice Problems Answers

1.  $(6)(6)$
2.  $(10)(10)(10)(10)$
3.  $(x)(x)$
4. 1
5.  $(13)(13)(13)(13)(13)(13)$
6.  $(x)(x)$
7. (8)
8.  $(14)(14)(14)(14)$
9.  $\left(\frac{1}{17}\right)\left(\frac{1}{17}\right)\left(\frac{1}{17}\right)$
10.  $(1)(1)(1)(1)(1)(1)(1)$
11.  $4^3$
12.  $x^4$
13.  $9^2$
14.  $(16^3)(7^2)$
15.  $2^{-5}$
16.  $D^3$
17.  $8^{-1}$
18.  $3^4$
19.  $12^3$
20.  $0.5^2$
21. 6859
22. 13.08
23. 12.56
24. 324
25. 4032
26. 1000
27. 9765625
28. 441
29. 30
30. 0.0129
31.  $\sqrt{144}$
32.  $\sqrt[3]{27}$
33.  $\sqrt[6]{15^3}$
34.  $\sqrt[3]{10^2}$
35.  $\sqrt[5]{54^4}$
36.  $16^2$
37.  $\sqrt[4]{98^3}$
38.  $450^{\frac{1}{2}}$
39.  $27^{\frac{1}{3}}$
40.  $45^{\frac{7}{4}}$
41.  $12^{\frac{5}{8}}$
42.  $21^{\frac{3}{2}}$
43.  $16^{\frac{3}{2}}$
44.  $11^{\frac{2}{3}}$
45.  $87^{\frac{3}{2}}$
46.  $49^{\frac{3}{2}}$
47.  $14^{\frac{-1}{3}}$
48. 12
49. 80
50. 10
51. 8
52. 4
53. 54



## Section 4

### Solving for an Unknown



## Basic Math Concepts

For Water and Wastewater Plant  
Operators  
by Joanne Kirkpatrick Price

## Difficulties in Math

- ◎ **A Poor Foundation**
  - Mathematics is sequential – concepts build upon concepts
- ◎ **No Linking or Steps Missing**
  - Link new concepts to what you already know
- ◎ **The “Big Picture” is Missing**
  - The skeleton on which all the details can be hung
- ◎ **“Use It or Lose It” Syndrome**
  - The more you practice and use math calculations, the easier they become

## Setting Up and Solving Math Problems

- ◎ **Theoretical Math** – concepts such as fractions, decimals, percents, areas, volumes, etc.
  - “Tools” of math - more tools you have, the easier the applied math problems will be
- ◎ **Applied Math** – basic math concepts applied in solving practical problems
  - Applied math calculations have a strategy – a way of approaching every problem that leads them methodically to the answer

## Suggested Strategy

- ◎ Disregarding all numbers, what type of problem is it?
- ◎ What diagram, if any, is associated with the concept identified?
- ◎ What information is required to solve the problem and how is it expressed in the problem?
- ◎ What is the final answer?
- ◎ Does the answer make sense?

## Solving for the Unknown Value (X)

---

### Solving for X

---

◎ Solve for X

$$(4)(1.5)(x) = 1100$$

- X must be by itself on one side of equal sign
- 4 and 1.5 must be moved away from X

$$x = \frac{1100}{(4)(1.5)}$$

$$x = 183.3$$

- How was this accomplished?

## Movement of Terms

---

- ◎ To understand how we move the numbers, we will need to consider more closely the math concepts associated with moving the terms.
- ◎ An equation is a mathematical statement in which the terms or calculation on one side equals the terms or calculation on the other side.

## Movement of Terms

---

- ◎ To preserve this equality, anything done to one side of the equation must be done to the other side as well.

$$(3)(x) = 14$$

- ◎ Since X is multiplied by 3, you can get rid of the 3 by using the opposite process: division.

## Movement of Terms

- ◎ To preserve the equation, you must divide the other side of the equation as well.

$$\frac{\cancel{3}(x)}{\cancel{3}} = \frac{14}{3}$$

$$(1)(x) = \frac{14}{3}$$

$$x = 4.67$$

- ◎ Since both sides of the equation are divided by the same number, the value of the equation remains unchanged.

## Example 1

$$730 = \frac{x}{3847}$$

What you do to one side of the equation, must be done to the other side.

$$730 = \frac{x}{3847} \times \frac{3847}{1}$$

$$\frac{3847}{1} \times 730 = \frac{x}{\cancel{3847}} \times \frac{\cancel{3847}}{1}$$

$$3847 \times 730 = x$$

$$2,808,310 = x$$

## Example 2

Simplify

What you do to one side of the equation, must be done to the other side.

$$0.5 = \frac{(165)(3)(8.34)}{x}$$

$$0.5 = \frac{4128.3}{x}$$

$$0.5 = \frac{4128.3}{x} \times \frac{x}{1}$$

$$\frac{x}{1} \times 0.5 = \frac{4128.3}{\cancel{x}} \times \frac{\cancel{x}}{1}$$

$$(x)(0.5) = 4128.3$$

$$\frac{(x)(\cancel{0.5})}{\cancel{0.5}} = \frac{4128.3}{0.5}$$

$$x = \frac{4128.3}{0.5}$$

$$x = 8256.6$$

## Solving for X when squared

- ⦿ Follow same procedure as solving for X
- ⦿ Then take the square root

$$x^2 = 15,625$$

$$\sqrt{x^2} = \sqrt{15,625}$$

$$x = 125$$

### Example 3

$$(0.785)(x^2) = 2826$$

$$\frac{\cancel{(0.785)}(x^2)}{\cancel{0.785}} = \frac{2826}{0.785}$$

$$x^2 = \frac{2826}{0.785}$$

$$x^2 = 3600$$

$$\sqrt{x^2} = \sqrt{3600}$$

$$x = 60$$

## Solving for X

- ⦿ When solving for x involving addition and subtraction, the balance of the equation must still remain.
  - What you do to one side you must do to the other

## Example 4

---

$$115 + 105 + 80 + x = 386$$

Step 1. Simplify

$$\cancel{300} + x = 386$$

$$\cancel{-300} \quad - 300$$

$$x = 386 - 300$$

$$x = 86$$

## Example 5

---

$$17 + 23 + 7 - x = 38$$

Step 1. Simplify

$$47 - \cancel{x} = 38$$

Step 2. Make x  
positive

$$+\cancel{x} \quad + x$$

$$47 = \cancel{38} + x$$

$$-38 \quad - \cancel{38}$$

$$47 - 38 = x$$

$$9 = x$$

## Basic Math

### Solving for the Unknown

1.  $7 + 10 + x + 7 + 9 = 41$

2.  $9.5 + x = 8.7$

3.  $x - 93 = 165$

4.  $10.1 = 9.5 + x$

5.  $x + 15 = 19 + 22$

6.  $16 = (2)(x)$

7.  $8.1 = (3)(x)(1.5)$

$$8. (0.785)(0.33)(0.33)(x) = 0.49$$

$$9. \frac{10}{x} = 50$$

$$10. \frac{233}{x} = 44$$

$$11. 56.5 = \frac{3800}{(x)(8.34)}$$

$$12. 10 = \frac{x}{4}$$

$$13. 940 = \frac{x}{(0.785)(90)(90)}$$

$$14. \quad x = \frac{(165)(3)(8.34)}{0.5}$$

$$15. \quad 114 = \frac{(230)(1.15)(8.34)}{(0.785)(70)(70)}$$

$$16. \quad 2 = \frac{x}{180}$$

$$17. \quad 46 = \frac{(105)(x)(8.34)}{(0.785)(100)(100)(4)}$$

$$18. \quad 2.4 = \frac{(0.785)(5)(5)(4)(7.48)}{x}$$

$$19. \quad 19747 = (20)(12)(x)(7.48)$$

$$20. \quad \frac{(15)(12)(1.25)(7.48)}{x} = 337$$

$$21. \quad \frac{x}{(4.5)(8.34)} = 213$$

$$22. \quad \frac{x}{246} = 2.4$$

$$23. \quad 6 = \frac{(x)(0.18)(8.34)}{(65)(1.3)(8.34)}$$

$$24. \quad \frac{(3000)(3.6)(8.34)}{(0.785)(x)} = 23.4$$

$$25. \quad 109 = \frac{x}{(0.785)(80)(80)}$$

$$26. \quad (x)(3.7)(8.34) = 3620$$

$$27. \quad 2.5 = \frac{1,270,000}{x}$$

$$28. \quad 0.59 = \frac{(170)(2.42)(8.34)}{(1980)(x)(8.34)}$$

$$29. \quad 142 = (2)(x) + 13$$

$$30. \quad (3.5)(x) - 62 = 560$$

Solve for the unknown value.

31.  $x^2 = 100$

32.  $(2)(x^2) = 288$

33.  $942 = (0.785)(x^2)(12)$

34.  $6358.5 = (0.785)(x^2)$

35.  $835 = \frac{4,200,000}{(0.785)(x^2)}$

36.  $920 = \frac{3,312,000}{x^2}$

$$37. \quad 23.9 = \frac{(3650)(3.95)(8.34)}{(0.785)(x^2)}$$

$$38. \quad (0.785)(D^2) = 5024$$

$$39. \quad (x^2)(10)(7.48) = 10,771.2$$

$$40. \quad 51 = \frac{64,000}{(0.785)(x^2)}$$

$$41. \quad (0.785)(D^2) = 0.54$$

$$42. \quad 2.1 = \frac{(0.785)(D^2)(15)(7.48)}{(0.785)(80)(80)}$$

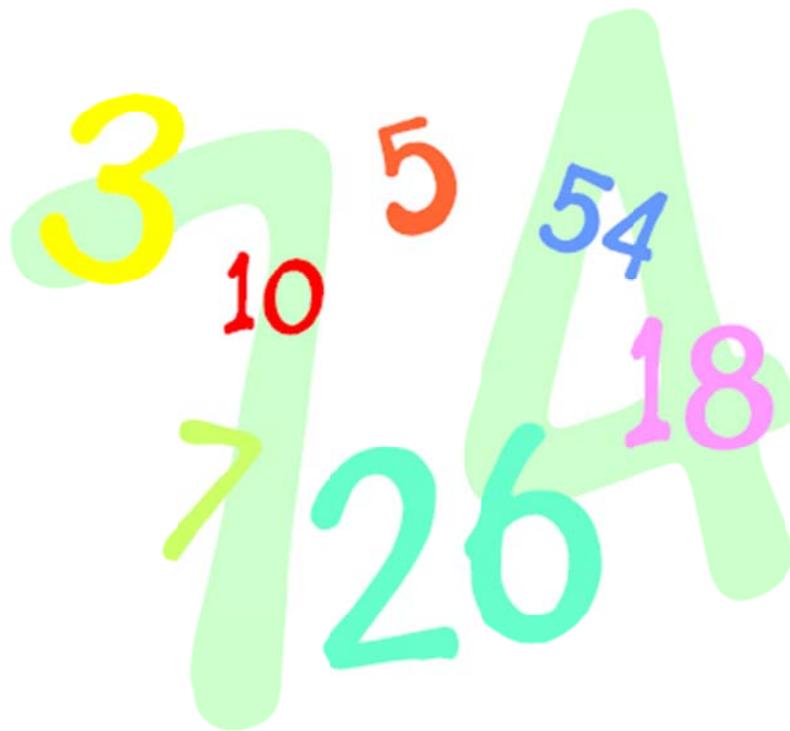
**Answers**

- |               |             |
|---------------|-------------|
| 1. 8          | 22. 590.4   |
| 2. 0.8        | 23. 2816.67 |
| 3. 72         | 24. 4903.48 |
| 4. 0.6        | 25. 547,616 |
| 5. 26         | 26. 117.31  |
| 6. 8          | 27. 508,000 |
| 7. 1.8        | 28. 0.35    |
| 8. 5.73       | 29. 64.5    |
| 9. 2          | 30. 177.71  |
| 10. 5.3       | 31. 10      |
| 11. 8.06      | 32. 12      |
| 12. 40        | 33. 10      |
| 13. 5,976,990 | 34. 90      |
| 14. 8256.6    | 35. 80      |
| 15. 0.005     | 36. 60      |
| 16. 360       | 37. 80      |
| 17. 1649.42   | 38. 80      |
| 18. 244.66    | 39. 12      |
| 19. 10.99     | 40. 39.98   |
| 20. 4.99      | 41. 0.83    |
| 21. 7993.89   | 42. 10.94   |



## Section 5

### Ratios and Proportions



# RATIOS AND PROPORTIONS

For Water and Wastewater Plant Operators

by Joanne Kirkpatrick Price



Department of  
**Environment &  
Conservation**

## WHAT ARE RATIOS & PROPORTIONS?

- ▶ A **ratio** is the established relationship between two numbers
  - ▶ i.e. 3 feet to every yard is a 3:1 ratio
- ▶ A **proportion** exists when the value of one ratio is equal to the value of a second ratio
  - ▶  $3:1 = 15:5$

## CROSS MULTIPLYING

- ▶ If the proportion is written using fractions, cross-multiplied terms will be equal

$$\frac{2}{3} = \frac{6}{9}$$

## EXAMPLE 1

- ▶ Determine if the following ratio is proportionate using cross multiplying

$$\frac{4}{5} = \frac{72}{95}$$

## SOLVING A PROPORTION



## SOLVING PROPORTIONS

- ▶ To solve a proportion problem, use the same steps as solving for the unknown value
- ▶ There are four terms in every proportion
- ▶ In a proportion problem, three of the terms are known and one is unknown (X)

## EXAMPLE 2

- ▶ Solve for the unknown value X in the problem given below

$$\frac{3.2}{2} = \frac{6}{x}$$

- ▶ First, cross multiply terms
- ▶ Now solve for the unknown

## EXAMPLE 3

- ▶ Given the proportion  $5:9 = x:72$ , solve for the unknown value
- ▶ First, rewrite the proportion as a fraction
- ▶ Then, cross multiply and solve for the unknown

## SETTING UP A PROPORTION PROBLEM



## DIRECT PROPORTIONS

- ▶ As one unit increases, the other increases as well

As one unit increases  
the other unit increases

As one unit decreases  
the other unit decreases

- ▶ To set up and solve a direct proportion
  1. Write the two fractions, take care that the location of the units is the same for each fraction
  2. Fill in the given values for both fractions
  3. Solve for the unknown value

## EXAMPLE 1

- ▶ If 1 foot is 12 inches, how many feet is 78 inches?

*1 ft: 12 inches*

*x ft: 78 inches*

- ▶ First, write the two fractions

- ▶ Cross Multiply

## EXAMPLE 1

- ▶ If a 0.5 lb of chlorine is dissolved in 45 gallons of water, how many pounds of chlorine would have to be dissolved in 100 gallons of water to have the same concentration?

- ▶ First, write the two fractions

- ▶ Next, fill in the given information

- ▶ Now, cross multiply and solve for the unknown

## Basic Math for All Certifications Proportions

### Solving a Proportion Problem

1.  $2 : 3 = 6 : X$

6.  $15 : 3 = X : 4$

2.  $25 : X = 10 : 2$

7.  $X : 30 = 8 : 12$

3.  $\frac{9}{3} = \frac{X}{8}$

8.  $\frac{3}{8} = \frac{21}{X}$

4.  $\frac{X}{27} = \frac{3}{9}$

9.  $\frac{4}{X} = \frac{196}{1225}$

5.  $1 : 144 = X : 1296$

10.  $\frac{X}{8} = \frac{49}{56}$

## Setting Up a Proportion

11. One gallon is equivalent to 3.785 liters. How many gallons are equivalent to 75 liters?
  
  
  
  
  
  
  
  
  
  
12. On the average one bag of chemical is used up in 3.5 days. At this rate, how many bags of chemical will be required during a 120-day period?
  
  
  
  
  
  
  
  
  
  
13. Suppose you wish to maintain a weir overflow rate of 12,000 gpd/ft (this is 12,000 gpd flow for each one-foot of weir length). If the weir length is 180 ft, what gpd flow will result in the desired weir overflow rate?
  
  
  
  
  
  
  
  
  
  
14. A total of 5.4 lbs of hypochlorite are dissolved in 80 gallons of water. For a solution with the same concentration, how many lbs of hypochlorite must be dissolved in 30 gallons of water?
  
  
  
  
  
  
  
  
  
  
15. A treatment pond is designed for a population loading of 300 persons per acre-ft of pond. If the population to be served is 1240 people, how many acre-ft of treatment pond will be required?
  
  
  
  
  
  
  
  
  
  
16. Dina prepared 4 kilograms of dough after working 2 hours. How much dough did Dina prepare if she worked for 9 hours? Assume the relationship is directly proportional.

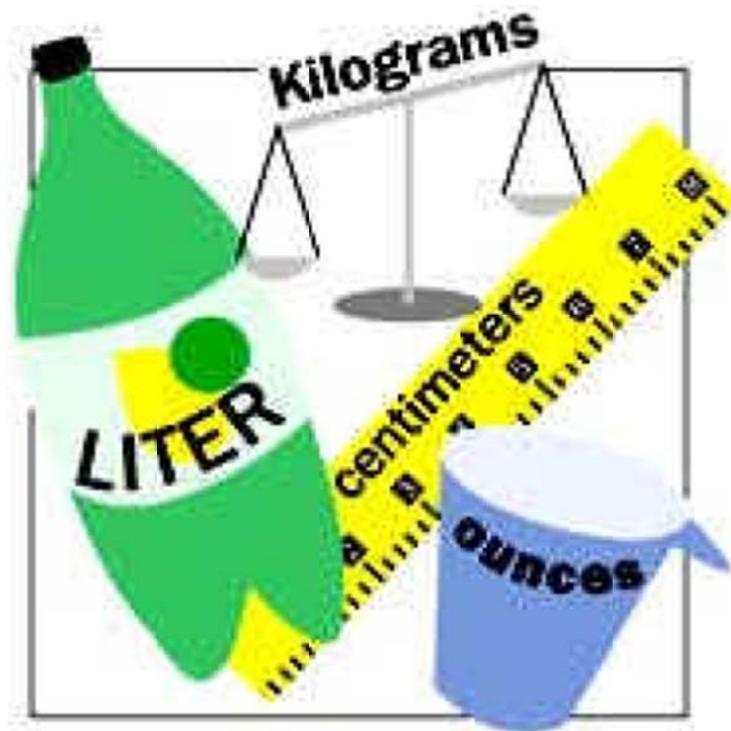
17. Four bags of chemical cost \$130. At the same unit price, how much would 11 bags of chemical cost?
18. If 3 gallons of paint cover 950 sq ft, how many gallons of paint will be required to paint 2400 sq ft?
19. On the average, one barrel of chemical is used up in 17 days. At this rate, how many barrels will be required during a 90 day period?
20. An average of 3 cubic feet of screenings are removed per million gallons of wastewater treated. At this rate, how much cubic feet of screenings would be expected to be removed from a flow of 4.6 MGD?

### Answers

- |       |                          |
|-------|--------------------------|
| 1. 9  | 11. 19.82 gal            |
| 2. 5  | 12. 34.29 bags           |
| 3. 24 | 13. 2,160,000 gpd        |
| 4. 9  | 14. 2.03 lbs             |
| 5. 9  | 15. 4.13 ac-ft           |
| 6. 20 | 16. 18 kg                |
| 7. 20 | 17. \$357.5              |
| 8. 56 | 18. 7.58 gal             |
| 9. 25 | 19. 5.29 barrels         |
| 10. 7 | 20. 13.8 ft <sup>3</sup> |

## Section 6

### Metric System and Temperature



## Metric System & Temperature

For Water and Wastewater  
Plant Operators  
by Joanne Kirkpatrick Price

## Metric Units

mega	....	kilo	hecto	deka	no	deci	centi	milli	....	micro
(M)		(k)	(h)	(da)	prefix	(d)	(c)	(m)		(μ)
1,000,000		1,000	100	10	1	1/10	1/100	1/1,000		1/1,000,000

↓

meter – linear measurement  
liter – volume measurement  
gram – weight measurement

King Henry Died By Drinking Chocolate Milk

## Metric Units

Kilo	Hecto	Deca	Basic Unit	Deci	Centi	Milli
King	Henry	Died	By	Drinking	Chocolate	Milk
1000X larger	100X larger	10X larger	Meter Liter Gram <b>1 unit</b>	10X smaller	100X smaller	1000X smaller

**MULTIPLY** numbers by 10 if you are getting smaller

**DIVIDE** number by 10 if you are getting bigger

## Problem 1

- Convert 2500 milliliters to liters
 

Primary Unit

kilo	hecto	deka	no	deci	centi	milli
(k)	(h)	(da)	prefix	(d)	(c)	(m)
- Converting milliliters to liters requires a move of three place values to the left
- Therefore, move the decimal point 3 places to the left

2 5 0 0 . =

3	2	1		

## Problem 2

- Convert 0.75 km into cm

Primary Unit						
kilo	hecto	deka	no	deci	centi	milli
(k)	(h)	(da)	prefix	(d)	(c)	(m)
1,000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$

- From kilometers to centimeters there is a move of 5 value places to the right



## Examples

- Convert 1.34 Liters to mL.
- Convert 76,897 m into km.
- Convert 34,597 cg into kg.

9

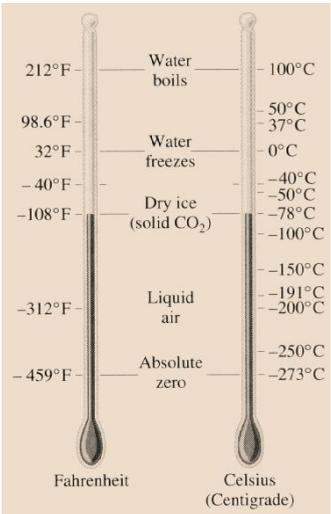
# Temperature




9

# Temperature

The **Fahrenheit** scale is named for the 18th-century German physicist Daniel Fahrenheit. His scale is based on 32 for the freezing point of water and 212 for the boiling point of water, the interval between the two being divided into 180 parts. The scale was in common use in English speaking countries until the 1970's when Europe and Canada adopted the centigrade (Celsius) scale. The U.S is the only country that still uses the Fahrenheit scale.

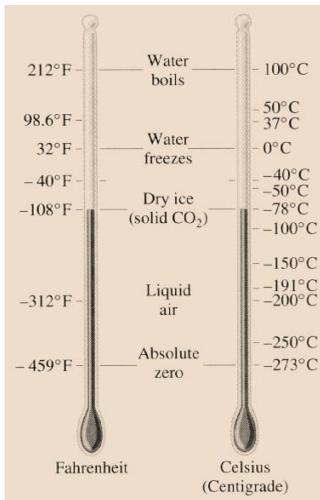


The **Celsius** temperature scale is named for the Swedish astronomer Anders Celsius who invented the scale in 1742. The scale is based on 0 for the freezing point of water and 100 for the boiling point of water. It is sometimes called the centigrade scale because of the 100-degree interval between the defined points.

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## Temperature Scales

The conversion formula for a temperature that is expressed on the Celsius (°C) scale to its Fahrenheit (°F) representation is:  
 $F^{\circ} = (C^{\circ})(1.8) + 32.$



The following formula can be used to convert a temperature from its representation on the Fahrenheit (°F) scale to the Celsius (°C) value:  
 $C^{\circ} = \frac{(F^{\circ} - 32)}{1.8}$

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## Temperature Conversions



You are going on a vacation in the U.K. The BBC news weather report says the temperature in London is 22°C, so should you pack shorts or sweaters?



12

## Temperature Conversions

You are calculating the Langelier Index which is a measure of a water's corrosiveness. The formula requires that you know your water temperature in °C . Your thermometer only reads °F.

The temperature of the water is 50°F.

$$^{\circ}\text{C} = \frac{^{\circ}\text{F} - 32}{1.8}$$



TDEC - Fleming Training Center

## Metric System and Temperature Conversion Practice Problems

Convert the following.

1. 23 g into \_\_\_\_\_ mg
2. 12,456 m into \_\_\_\_\_ km
3. 4235 mL into \_\_\_\_\_ L
4. 200 mg into \_\_\_\_\_ kg
5. 1000 watts into \_\_\_\_\_ kw
6. 0.05 g into \_\_\_\_\_  $\mu$ g
7. 20 deciliters into \_\_\_\_\_ mL
8. 140 kg into \_\_\_\_\_ g
9. 9.5 cm into \_\_\_\_\_ mm
10. 100 milliseconds into \_\_\_\_\_ seconds

Convert the following.

1. 12 C° into \_\_\_\_\_ °F
2. 80 F° into \_\_\_\_\_ °C
3. 150 F° into \_\_\_\_\_ °C
4. 100 C° into \_\_\_\_\_ °F
5. 32 F° into \_\_\_\_\_ °C

**Answers**

1. 23,000 mg
2. 12.456 km
3. 4.235 L
4. 0.0002 kg
5. 1 kw
6. 50,000  $\mu\text{g}$
7. 2000 mL
8. 140,000 g
9. 95 mm
10. 0.1 seconds

**Part 2**

1. 53.6°F
2. 26.67°C
3. 65.6°C
4. 212°F
5. 0°C

**Basic Math****Metric Systems Conversion Practice Problems**

- 1) 317.22 m = \_\_\_\_\_ cm
- 2) 265,400 cm = \_\_\_\_\_ m
- 3) 3.11 m = \_\_\_\_\_ mm
- 4) 911,200 mm = \_\_\_\_\_ m
- 5) 24.01 km = \_\_\_\_\_ m
- 6) 31,570 m = \_\_\_\_\_ km
- 7) 3.82 km = \_\_\_\_\_ cm
- 8) 7,154,900 cm = \_\_\_\_\_ km
- 9) 14.72 L = \_\_\_\_\_ mL
- 10) 5,618 mL = \_\_\_\_\_ L
- 11) 4.13 g = \_\_\_\_\_ mg
- 12) 2,089 mg = \_\_\_\_\_ g
- 13) 91.42 kg = \_\_\_\_\_ mg
- 14) 24,020 mg = \_\_\_\_\_ kg
- 15) 69.98 kg = \_\_\_\_\_ g
- 16) 742,400 g = \_\_\_\_\_ kg
- 17) 42.07 m = \_\_\_\_\_ cm
- 18) 261,600 cm = \_\_\_\_\_ m
- 19) 21.67 m = \_\_\_\_\_ mm
- 20) 803,500 mm = \_\_\_\_\_ m
- 21) 2.45 m = \_\_\_\_\_ cm
- 22) 93,200 cm = \_\_\_\_\_ m
- 23) 129.9 m = \_\_\_\_\_ mm
- 24) 30,940 mm = \_\_\_\_\_ m
- 25) 126.35 km = \_\_\_\_\_ m
- 26) 573,100 m = \_\_\_\_\_ km
- 27) 62.83 km = \_\_\_\_\_ cm
- 28) 91,470 cm = \_\_\_\_\_ km
- 29) 123.78 L = \_\_\_\_\_ mL
- 30) 45,050 mL = \_\_\_\_\_ L
- 31) 315.07 g = \_\_\_\_\_ mg
- 32) 360 mg = \_\_\_\_\_ g
- 33) 319.77 kg = \_\_\_\_\_ mg
- 34) 345,600 mg = \_\_\_\_\_ kg
- 35) 19.2 kg = \_\_\_\_\_ g
- 36) 3,849 g = \_\_\_\_\_ kg
- 37) 6.47 m = \_\_\_\_\_ cm
- 38) 933,800 cm = \_\_\_\_\_ m
- 39) 9.64 m = \_\_\_\_\_ mm
- 40) 9,840 mm = \_\_\_\_\_ m

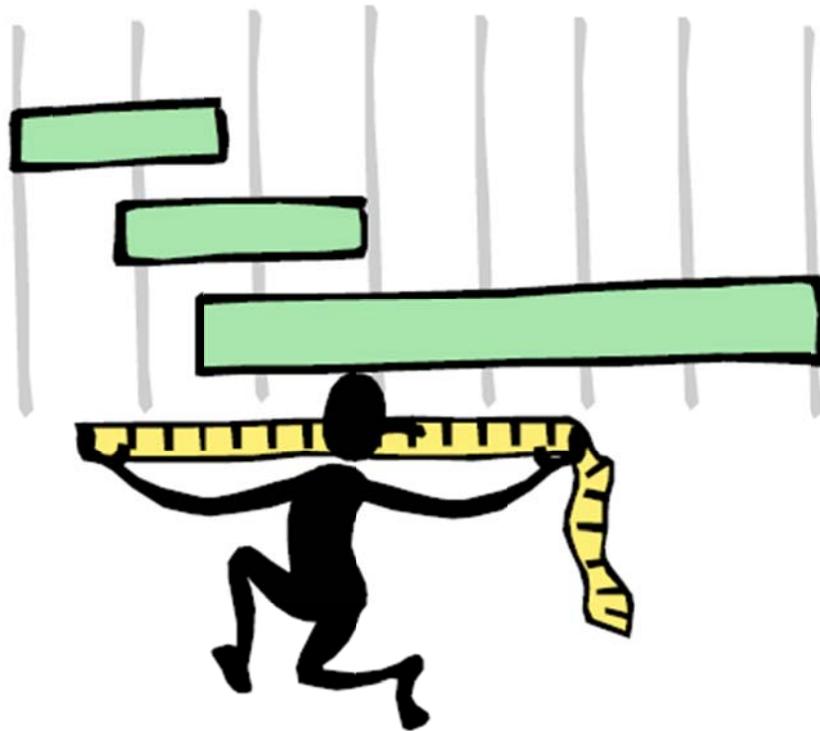
**ANSWERS**

- |                   |                    |
|-------------------|--------------------|
| 1) 31,722 cm      | 21) 245 cm         |
| 2) 2,654 m        | 22) 932 m          |
| 3) 3,110 mm       | 23) 129,900 mm     |
| 4) 911.2 m        | 24) 30.94 m        |
| 5) 24,010 m       | 25) 126,350 m      |
| 6) 31.57 km       | 26) 573.1 km       |
| 7) 382,000 cm     | 27) 6,283,000 cm   |
| 8) 71.549 km      | 28) 0.9147 km      |
| 9) 14,720 mL      | 29) 123,780 mL     |
| 10) 5.618 L       | 30) 45.05 L        |
| 11) 4,130 mg      | 31) 315,070 mg     |
| 12) 2.089 g       | 22) 0.36 g         |
| 13) 91,420,000 mg | 33) 319,770,000 mg |
| 14) 0.02402 kg    | 34) 0.3456 kg      |
| 15) 69,980 g      | 35) 19,200 g       |
| 16) 742.4 kg      | 36) 3.849 kg       |
| 17) 4,207 cm      | 37) 647 cm         |
| 18) 2,616 m       | 38) 9,338 m        |
| 19) 21,670 mm     | 39) 9,640 mm       |
| 20) 803.5 m       | 40) 9.84 m         |



## Section 7

# Unit Conversions and Dimensional Analysis



# DIMENSIONAL ANALYSIS

**MATHEMATICS MANUAL FOR WATER AND  
WASTEWATER TREATMENT PLANT OPERATORS  
BY FRANK R. SPELLMAN**

## DIMENSIONAL ANALYSIS

Used to check if a problem is set up correctly

Work with the units of measure, not the numbers

**Step 1:**

- Express fraction in a vertical format

$$gal/ft^3 \text{ to } \frac{gal}{ft^3}$$

**Step 2:**

- Be able to divide a fraction

$$\frac{\frac{lb}{day}}{\frac{min}{day}} \text{ becomes } \frac{lb}{day} \times \frac{day}{min}$$

## DIMENSIONAL ANALYSIS

### Step 3:

- Know how to divide terms in the numerator and denominator
- Like terms can cancel each other out
  - For every term that is canceled in the numerator, a similar term must be canceled in the denominator

$$\frac{lb}{\cancel{day}} \times \frac{\cancel{day}}{min} = \frac{lb}{min}$$

Units with exponents should be written in expanded form

$$ft^3 = (ft)(ft)(ft)$$

## EXAMPLE 1

Convert 1800 ft<sup>3</sup> into gallons.

We need the conversion factor that connects the two units

$$1 \text{ cubic foot of water} = 7.48 \text{ gal}$$

This is a ratio, so it can be written two different ways

$$\frac{1 \text{ ft}^3}{7.48 \text{ gal}} \quad \text{OR} \quad \frac{7.48 \text{ gal}}{1 \text{ ft}^3}$$

We want to use the version that allows us to cancel out units

$$\frac{1 \text{ ft}^3}{7.48 \text{ gal}} \quad \text{OR} \quad \frac{7.48 \text{ gal}}{1 \text{ ft}^3}$$

## EXAMPLE 1

$$\left( \frac{1800 \text{ ft}^3}{1} \right)$$

Will anything cancel out?

NO

Let's try the other version

Will anything cancel out?

YES

## EXAMPLE 2

Determine the square feet given  $70 \text{ ft}^3/\text{sec}$  and  $4.5 \text{ ft}/\text{sec}$

Use units to determine set up

- Two ways to write the number

$$\frac{4.5 \text{ ft}}{\text{sec}} \quad \text{OR} \quad \frac{\text{sec}}{4.5 \text{ ft}}$$

- Which way is the right way?

- Will anything cancel?

## EXAMPLE 2 CONT'D

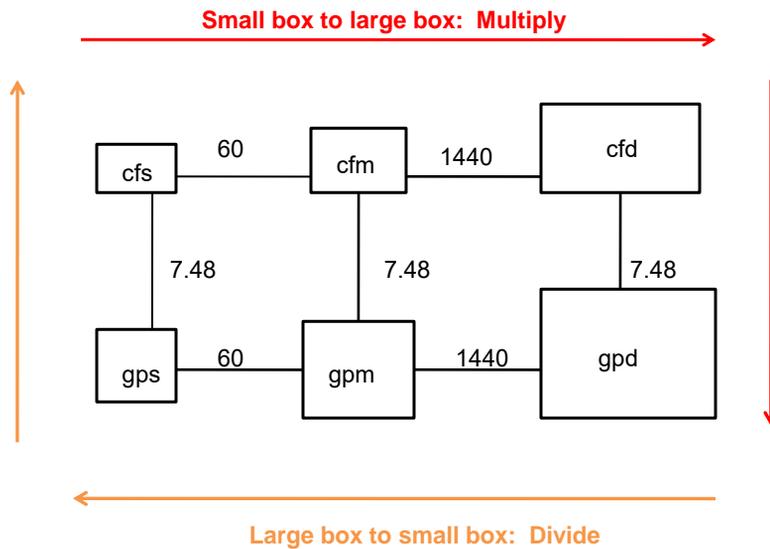
Remember, units function the same as numbers.

$$ft^3 = (ft)(ft)(ft)$$

Therefore

Will anything cancel out?

## FLOW CONVERSIONS – BOX METHOD



TDEC - Fleming Training Center

## EXAMPLE 3

Convert 3.8 ft<sup>3</sup>/sec to MGD.

- Little to big – multiply

$$(3.8 \text{ cfs})(60)(1440)(7.48) = 2455833.6 \text{ gpd}$$

- Big to little – divide

$$2455833.6 \text{ gpd} \div 1,000,000 = 2.46 \text{ MGD}$$

## Basic Math for Operators Conversions (1)

### Linear Measurement

1.  $\frac{1}{4}$  mile = feet
2. 4200 feet = miles
3. 17 feet = yds
4. 122 inches = feet
5. 30 yds = inches
6. 0.6 feet = inches
7. 492 inches = feet
8. The total weir length for a sedimentation tank is 142 feet 7 inches. Express this length in terms of feet only.
9. A one-eighth mile section of pipeline is to be replaced. How many feet of pipeline is this?
10. 2.7 miles of pipe is how many inches?

**Metric/English Conversions**

11.20 feet = meters

12.50 L = gal

13.70 cm = inches

14.35 yds = feet

15.600 mL = gal

16.1 lb = g

17.2.7 gal = L

**Area Measurement**

18.1017 in<sup>2</sup> = ft<sup>2</sup>

19.500 yd<sup>2</sup> = ft<sup>2</sup>

20.4 acres = ft<sup>2</sup>

21.1 yd<sup>2</sup> = in<sup>2</sup>

22.9.5 ft<sup>2</sup> = in<sup>2</sup>

23.78.5 in<sup>2</sup> = ft<sup>2</sup>

24.25,000 ft<sup>2</sup> = acres

$$25. 0.9 \text{ acre} = \quad \text{ft}^2$$

26. For solids treatment, a total of 60,000  $\text{ft}^2$  will be required. How many acres is this?

27. A pipe has a cross-sectional area of 452  $\text{in}^2$ . How many  $\text{ft}^2$  is this?

**Volume Measurement**

$$28. 325 \text{ ft}^3 = \quad \text{yd}^3$$

$$29. 2512 \text{ in}^3 = \quad \text{ft}^3$$

$$30. 25 \text{ yd}^3 = \quad \text{ft}^3$$

$$31. 1500 \text{ in}^3 = \quad \text{ft}^3$$

$$32. 2.2 \text{ ac-ft} = \quad \text{yd}^3$$

$$33. 21 \text{ ft}^3 = \quad \text{yd}^3$$

$$34. 92,600 \text{ ft}^3 = \quad \text{ac-ft}$$

$$35. 17,260 \text{ ft}^3 = \quad \text{yd}^3$$

$$36. 0.6 \text{ yd}^3 = \quad \text{ft}^3$$

$$37. 3 \text{ ft}^3 = \quad \text{in}^3$$

38. A screening pit must have a capacity of 400  $\text{ft}^3$ . How many  $\text{yd}^3$  is this?

39. A reservoir contains 50 ac-ft of water. How many  $\text{ft}^3$  of water does it contain?

**Flow Conversions**

40. 3.6 cfs = gpm

41. 1820 gpm = gpd

42. 45 gps = cfs

43. 8.6 MGD = gpm

44. 2.92 MGD = gpm

45. 385 cfm = gpd

46. 1,662,000 gpd = gpm

47. 3.77 cfs = MGD

48. The flow through a pipeline is 8.4 cfs. What is the flow in gpd?

49. A treatment plant receives a flow of 6.31 MGD. What is the flow in gpm?

## Basic Math for Operators

### Conversions (1) Answers

1. 1320 ft.
2. 0.80 miles
3. 5.67 yds.
4. 10.17 ft.
5. 1080 in.
6. 7.2 in.
7. 41 ft.
8. 142.58 ft.
9. 660 ft.
10. 171,072 in.
11. 3.42 m.
12. 13.21 gal.
13. 27.56 in.
14. 105 ft.
15. 0.16 gal
16. 454 g
17. 10.22 L
18.  $7.06 \text{ ft}^2$
19.  $4500 \text{ ft}^2$
20.  $174,240 \text{ ft}^2$
21.  $1296 \text{ in}^2$
22.  $1368 \text{ in}^2$
23.  $0.55 \text{ ft}^2$
24. 0.57 ac
25.  $39,204 \text{ ft}^2$
26. 1.38 ac
27.  $3.14 \text{ ft}^2$
28.  $12.04 \text{ yd}^3$
29.  $1.45 \text{ ft}^3$
30.  $675 \text{ ft}^3$
31.  $0.87 \text{ ft}^3$
32.  $3549.33 \text{ yd}^3$
33.  $0.78 \text{ yd}^3$
34. 2.13 ac-ft
35.  $639.26 \text{ yd}^3$
36.  $16.2 \text{ ft}^3$
37.  $5184 \text{ in}^3$
38.  $14.81 \text{ yd}^3$
39.  $2,177,995 \text{ ft}^3$
40. 1615.68 gpm
41. 2,620,800 gpd
42. 6.02 cfs
43. 5972.22 gpm
44. 2027.78 gpm
45. 41,469.12 gpd
46. 1154.17 gpm
47. 2.44 MGD
48. 5,428,684.8 gpd
49. 4381.94 gpm

## Basic Math for Operators Conversions (2)

1. How many pounds are there in 1 ft<sup>3</sup> of water?
2. How much (lb) does exactly 100 gal of water weigh?
3. Convert 8.2 ft<sup>3</sup>/sec to gallons per minute.
4. How many gallons are there in 82 ft<sup>3</sup>?
5. Convert 2,445 gal to cubic feet.
6. How much does 725 gal of water weigh in pounds?
7. Convert 5.1 MGD to cfs.
8. Convert 15.0 acre-ft to cubic feet.

9. 4,078,611 ft<sup>3</sup> to acre-feet.
  
10. Convert 11.9 MGD to cubic feet per second.
  
11. Convert 5.6 ft<sup>3</sup>/sec to gallons per minute.
  
12. Convert 3.2 ft<sup>3</sup>/sec to millions of gallons per day.
  
13. How many million gallons are there in 22 ac-ft?
  
14. How many million gallons are there in 43,000 ac-ft?
  
15. Convert 23 lb/million gallons to milligrams per liter.
  
16. How many gallons are there in 8,492 ft<sup>3</sup>?
  
17. Convert a solution that has 52,600 ppm to percent.

18. Convert 45 lb/MG to mg/L.
19. The distance between your plant and the nearest customer is 1.535 kilometers. What is this distance in yards?
20. The minimum area required for building a 1 MG storage facility is 900 ft<sup>2</sup>. What is this size in acres?
21. Sunny Slope water system daily maximum demand is 556,000 gallons. What is this system demand in ft<sup>3</sup>?
22. Based on the dimensions of your storage reservoir, you calculate the total volume to be 210,000ft<sup>3</sup>. How many million gallons (MG) of water can you store at this reservoir?
23. During a fire flow test, the gauge shows a flow of 79.3 L/sec. What is the flow at this location in ac-ft/day?
24. Based on your measurements, the maximum flow out of a ¼ inch pipe at your chemical pump effluent is 24.5 gpd. What is the flow rate in mL/min?

25. The length of the pipe between Main St. and Beach Ave. as measured on a map is 0.224 miles. The purchasing department requires you to submit all distances in feet when you place an order for new pipes. What is the distance in feet?

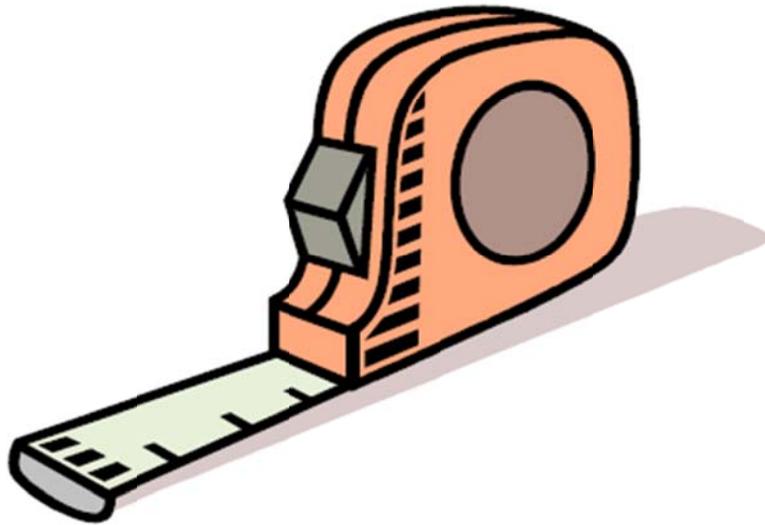
**Answers**

1. 62.4 lbs
2. 834 lbs
3. 3680.16 gal/min
4. 613.36 gal
5. 326.87 ft<sup>3</sup>
6. 6046.5 lbs
7. 7.89 cfs
8. 653,743.32 ft<sup>3</sup>
9. 93.58 ac-ft
10. 18.41 cfs
11. 2513.28 gal/min
12. 2.07 MGD
13. 7.17 MG
14. 14,018 MG
15. 2.76 mg/L
16. 65,520.16 gal
17. 5.26 %
18. 5.4 mg/L
19. 1678.01 yds
20. 0.02 ac
21. 74331.55 ft<sup>3</sup>
22. 1.57 MG
23. 5.56 ac-ft/day
24. 64.4 mL/min
25. 1182.72 ft



## Section 8

### Circumference, Area and Volume



## LINEAR MEASUREMENT CIRCUMFERENCE & PERIMETER

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Basic Math Concepts for Water and  
Wastewater Plant Operators  
by Joanne Kirkpatrick Price

### Suggested Strategy to Solving Word Problems

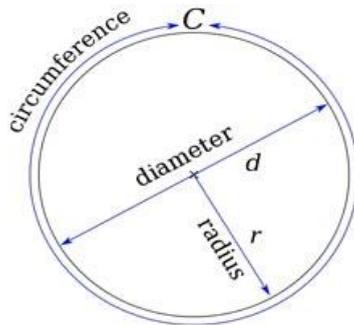
- Disregarding all numbers, what type of problem is it?
- What diagram is associated with the concept identified? If there is not one provided draw one yourself.
- What information is required to solve the problem and how is it expressed in the problem?
- What is the final answer?
- Does the answer make sense?

## Linear Measurement

- Linear measurement is simply the measurement along a line and expressed in units of length (ft, m, in, yd, mi, km)
- Many collections system calculations require tank or channel dimensions, pipe lengths and diameters, traffic control zone measurements
- One particular type of length measurement is the distance around the outside edge of an area or object – the perimeter and circumference

## Parts of a Circle

- Diameter is the distance across the center of circle
- Radius is the distance from circle's center to the edge and is half of the diameter
- Circumference is the distance around a circle or a circular object



## Circumference & Perimeter

- Circumference of a Circle

$$\text{Circumference} = (3.14)(\text{Diameter})$$

- Perimeter is obtained by adding the lengths of the four sides of a square or rectangle

$$\text{Perimeter} = 2(\text{length}) + 2(\text{width})$$



### Example 1

- Find the circumference in inches of a 6 inch diameter pipe.

$$\text{Circumference} = (3.14)(\text{diameter})$$

$$C = (3.14)(6 \text{ inches})$$

$$C = 18.85 \text{ inches}$$

- Find the perimeter of a trench that is 15 ft by 22 ft.

$$\text{Perimeter} = 2(\text{length}) + 2(\text{width})$$

$$P = 2(15 \text{ ft}) + 2(22 \text{ ft})$$

$$P = 30 \text{ ft} + 44 \text{ ft}$$

$$P = 74 \text{ ft}$$

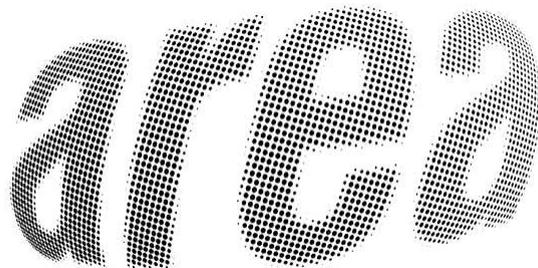
To add the units must be the same

# AREA

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## Area

- Area is the measurement of the amount of space on the surface of an object
- Two dimensional measurement
- Measured in:  $\text{in}^2$ ,  $\text{ft}^2$ ,  $\text{m}^2$ , acres, etc.

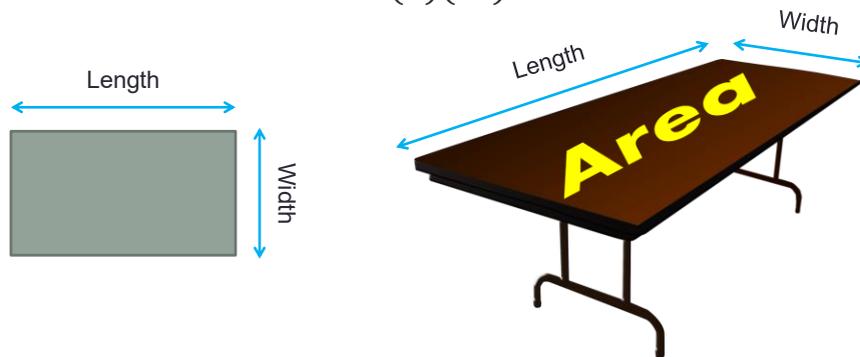
The word "area" is displayed in a large, stylized font where each letter is composed of a grid of small black dots. The letters have a slight 3D effect, appearing to float above the surface.

## Area

- Area of Rectangle

$$\text{Area} = (\text{length})(\text{width})$$

$$A = (L)(W)$$



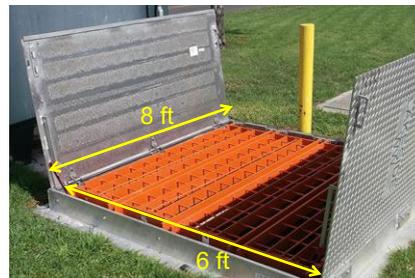
## Example 1

- Find the area in  $\text{ft}^2$  of a rectangular access hatch door that is 8 feet long and 6 feet wide.

$$A = (L)(W)$$

$$A = (8\text{ft})(6\text{ft})$$

$$A = 48\text{ft}^2$$

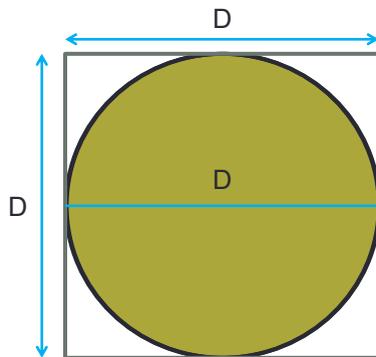


## Area

- Area of Circle

$$\text{Area} = (0.785) (\text{Diameter})^2$$

$$A = (0.785)(D)^2$$



The circle takes up  
78.5% of a square

## Example 2

- Find the area of the cross section of a pipe in  $\text{ft}^2$  that has a diameter of 2 feet.

$$\text{Area} = (0.785)(D)^2$$

$$A = (0.785)(2\text{ft})(2\text{ft})$$

$$A = 3.14 \text{ ft}^2$$

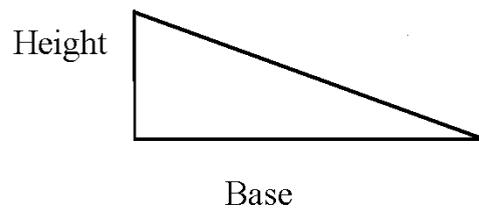


## Area

- Area of Right Triangle

$$Area = \frac{(base)(height)}{2}$$

$$A = \frac{(b)(h)}{2}$$



## Example 3

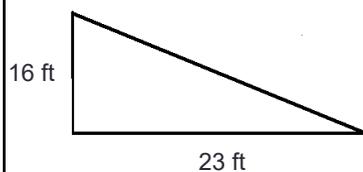
- Determine the area in  $ft^2$  of a right triangle where the base is 23 feet long with a height of 16 feet.

$$A = \frac{(b)(h)}{2}$$

$$A = \frac{(23ft)(16ft)}{2}$$

$$A = \frac{368ft^2}{2}$$

$$A = 184ft^2$$



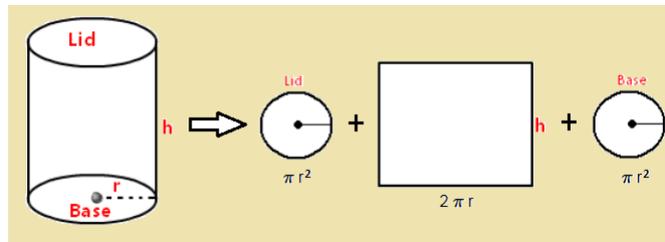
## Area

- Area of Cylinder (total exterior surface area)

$$\text{Area} = [\text{End \#1 SA}] + [\text{End \#2 SA}] + [(3.14)(D)(h)]$$

Where SA = surface area

$$A = A_1 + A_2 + [(3.14)(D)(h)]$$



## Example 4

- Find the total surface area in  $\text{ft}^2$  of a drum that is 2 ft in diameter and 4 ft tall.



$$A = A_1 + A_2 + [(3.14)(D)(h)]$$

$$A_1 = (0.785)(D)^2$$

$$A_1 = (0.785)(2\text{ft})(2\text{ft})$$

$$A_1 = 3.14\text{ft}^2$$

$$A_1 = A_2$$

$$A = 3.14\text{ft}^2 + 3.14\text{ft}^2 + [(3.14)(2\text{ft})(4\text{ft})]$$

$$A = 3.14\text{ft}^2 + 3.14\text{ft}^2 + 25.12\text{ft}^2$$

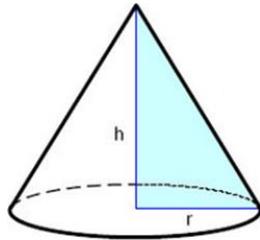
$$A = 31.40\text{ft}^2$$

## Area

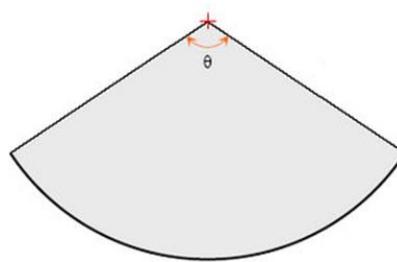
- Area of Cone (lateral area)

$$\text{Area} = (3.14)(\text{radius})\sqrt{\text{radius}^2 + \text{height}^2}$$

$$A = (3.14)(r)\sqrt{r^2 + h^2}$$



Right Circular Cone



Unrolled Lateral Area

## Example 5

- Find the lateral area ( $\text{in}^2$ ) of a conical funnel that is 7 inches tall and has a radius of 9 inches.

$$A = (3.14)(r)\sqrt{r^2 + h^2}$$

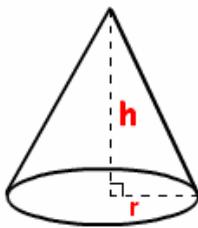
$$A = (3.14)(9\text{in})\sqrt{(9\text{in})(9\text{in}) + (7\text{in})(7\text{in})}$$

$$A = (3.14)(9\text{in})\sqrt{81\text{in}^2 + 49\text{in}^2}$$

$$A = (3.14)(9\text{in})\sqrt{130\text{in}^2}$$

$$A = (3.14)(9\text{in})(11.4018\text{in})$$

$$A = 322.21\text{in}^2$$

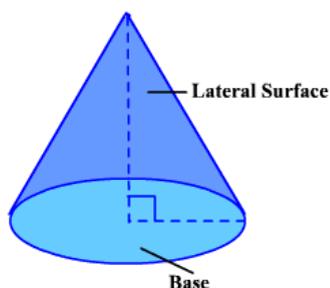


## Area

- Area of Cone (total surface area)

$$\text{Area} = (3.14)(\text{radius})(\text{radius} + \sqrt{\text{radius}^2 + \text{height}^2})$$

$$A = (3.14)(r)(r + \sqrt{r^2 + h^2})$$



## Example 6

- Find the total surface area in  $\text{ft}^2$  of a cone that is 4.5 feet deep with a diameter of 6 feet.

$$A = (3.14)(r)(r + \sqrt{r^2 + h^2})$$

$$A = (3.14)(3\text{ft})(3\text{ft} + \sqrt{(3\text{ft})(3\text{ft}) + (4.5\text{ft})(4.5\text{ft})})$$

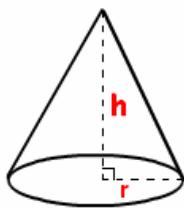
$$A = (3.14)(3\text{ft})(3\text{ft} + \sqrt{9\text{ft}^2 + 20.25\text{ft}^2})$$

$$A = (3.14)(3\text{ft})(3\text{ft} + \sqrt{29.25\text{ft}^2})$$

$$A = (3.14)(3\text{ft})(3\text{ft} + 5.4083\text{ft})$$

$$A = (3.14)(3\text{ft})(8.4083\text{ft})$$

$$A = 79.21\text{ft}^2$$



$$\begin{aligned} \text{Radius} &= (1/2)D \\ r &= (1/2)(6\text{ft}) \\ r &= 3\text{ft} \end{aligned}$$

# VOLUME

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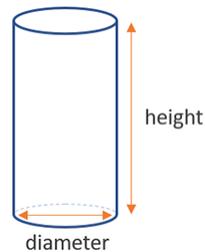
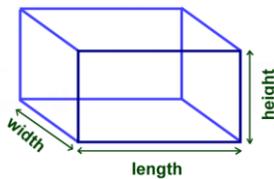
## Volume

- Volume is the capacity of a unit or how much it will hold
- General types of volume calculations are:
  - Tank or Basin Volume
  - Channel or Pipeline Volume
  - Pond Volume



## Volume

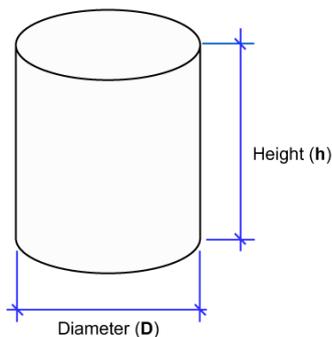
- Volume calculations are measured in:
  - cubic units (ft<sup>3</sup>, m<sup>3</sup>, yd<sup>3</sup>)
  - liquid volume units (gallons, liters, MG)
- Calculated volumes will always be in cubic units and must be converted to liquid measurement units if they are desired



## Volume of a Cylinder

$$\text{Volume} = (0.785)(\text{Diameter}^2)(\text{height})$$

$$\text{Vol} = (0.785)(D^2)(h)$$



## Example 1

- Determine the volume in  $\text{ft}^3$  for a tank that is 20 feet long with a diameter of 7.5 ft.

$$\text{Vol} = (0.785)(D)^2(h)$$

$$\text{Vol} = (0.785)(7.5\text{ft})(7.5\text{ft})(20\text{ft})$$

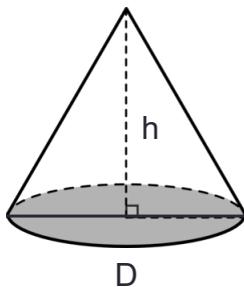
$$\text{Vol} = 883.13 \text{ ft}^3$$



## Volume of a Cone

$$\text{Volume} = \left(\frac{1}{3}\right)(0.785)(\text{Diameter}^2)(\text{height})$$

$$\text{Vol} = \left(\frac{1}{3}\right)(0.785)(D^2)(h)$$



## Example 2

- Determine the volume in gallons of a conical tank that is 8 feet wide and 15 feet tall.

$$Vol = \left(\frac{1}{3}\right)(0.785)(D^2)(h)$$

$$Vol = \left(\frac{1}{3}\right)(0.785)(8ft)(8ft)(15ft)$$

$$Vol = (0.3333)(753.6 ft^3)$$

$$Vol = 251.1749 ft^3 \leftarrow \text{Not done yet!}$$

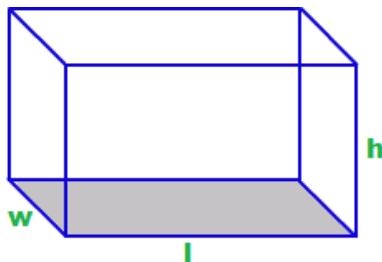
$$Vol, gal = (251.1749 \cancel{ft^3})(7.48 \frac{gal}{\cancel{ft^3}})$$

$$Vol, gal = 1878.79 \text{ gallons}$$

## Volume of a Rectangle

$$Volume = (length)(width)(height)$$

$$Vol = (l)(w)(h)$$



### Example 3

- Determine the volume in  $m^3$  for a tank that measures 12 meters by 8 meters by 5 meters.

$$Vol = (l)(w)(h)$$

$$Vol = (12m)(8m)(5m)$$

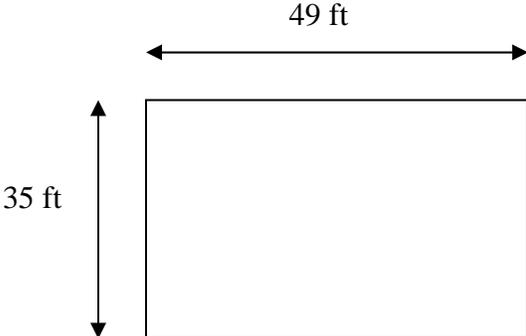
$$Vol = 480m^3$$

## Math Problem Strategies

Strategy for solving word problems:

- 1) Read the problem, disregard the numbers (What type of problem is it? What am I asked to find?)
- 2) Refer to the diagram, if provided. If there isn't one, draw your own.
- 3) What information do I need to solve the problem, and how is it given in the statement of the problem?
- 4) Work it out.
- 5) Does it make sense?

It might be helpful to write out everything that is known in one column and the unknown (what am I asked to find?) in another column. Identify the correct formula and write it in the middle, plug in the numbers and solve.

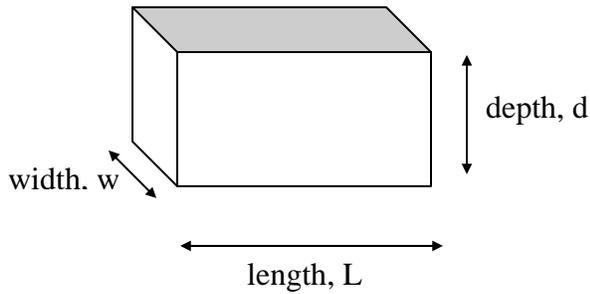
<u>Known</u>		<u>Unknown</u>
Length = 35 ft Width = 49 ft	$A = (l)(w)$  $A = (35 \text{ ft})(49 \text{ ft})$ $A = 1715 \text{ ft}^2$	Area = ?  <div style="text-align: center;">  </div>

***\*\*Remember: make sure measurements agree; if diameter of pipe is in inches then change to feet; if flow is in MGD and you need feet or feet/sec then change to ft<sup>3</sup>/sec before you plug values into formula.***

	..								..	
mega (M)		kilo (k)	hecto (h)	deka (da)	no prefix	deci (d)	centi (c)	milli (m)		micro (μ)
1,000,000		1,000	100	10	1	1/10	1/100	1/1,000		1/1,000,000

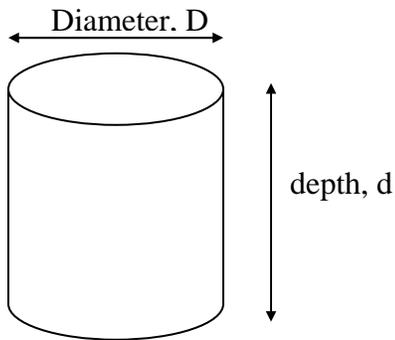
Tank Volume Calculations: Most tank volumes calculations are for tanks that are either rectangular or cylindrical in shape.

**Rectangular Tank**



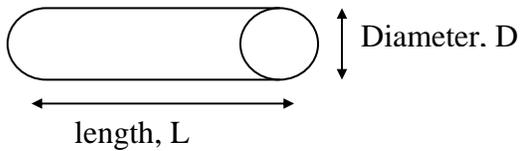
$$\text{Volume} = (L)(W)(d)$$

**Cylindrical Tank**



$$\text{Volume} = (0.785)(D)^2(d)$$

**Portion of a Pipeline**



$$\text{Volume} = (0.785)(D)^2(L)$$

**Basic Math**  
**Circumference, Area, and Volume**  
**Examples**

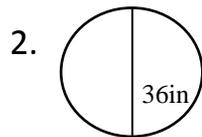
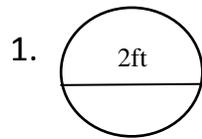
1. A chemical holding tank has a diameter of 24 feet. What is the circumference of the tank in feet?
  
  
  
  
  
  
  
  
  
  
2. A basin has a length of 45 feet and a width of 12 feet. Calculate the area in  $\text{ft}^2$ .
  
  
  
  
  
  
  
  
  
  
3. Calculate the surface area (in  $\text{ft}^2$ ) of the top of a basin which is 90 feet long, 25 feet wide, and 10 feet deep.
  
  
  
  
  
  
  
  
  
  
4. Calculate the cross-sectional area (in  $\text{ft}^2$ ) for a 24 inch diameter main that has just been laid.
  
  
  
  
  
  
  
  
  
  
5. A triangular portion of concrete has just been poured. Calculate the surface area in square feet if the base concrete is 10 feet with a height of 15 ft.



10. Calculate the volume of water in a rectangular tank (in gallons), which measures 12 feet long, 6 feet wide, 5 feet deep, and contains 8 inches of water.
11. A new water main needs to be disinfected. The main is 30" in diameter and has a length of 1320 ft. How many gallons of water will it hold?
12. Calculate the volume (in  $\text{ft}^3$ ) of a cone shaped chemical hopper with a diameter of 12 feet and a depth of 18 feet.

**Basic Math for All Certifications  
Circumference, Area, and Volume**

Circumference



3. What is the area (in  $\text{ft}^2$ ) of a rectangle 5 ft by 4 ft?
4. A rectangle has a length of 5 feet and a width of 3 feet. What is the area (in  $\text{ft}^2$ ) of the rectangle?
5. The diameter of a circle is 5 feet. What is its area (in  $\text{ft}^2$ )?
6. What is the cross-sectional area (in  $\text{ft}^2$ ) of a pipe with a diameter of 7 inches?

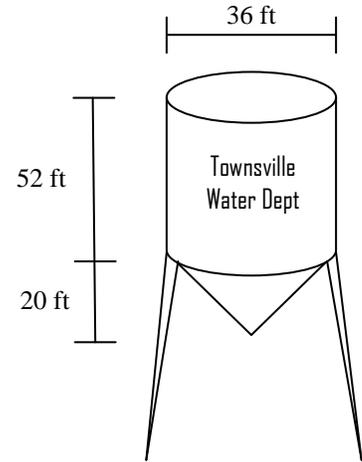
7. What is the lateral surface area (in  $\text{ft}^2$ ) of a cone with a radius of 12.5 ft and a height of 18 ft?
  
  
  
  
  
  
  
  
  
  
8. Calculate the total surface area (in  $\text{ft}^2$ ) of a cone that has a diameter of 15 feet and a height of 7 feet.
  
  
  
  
  
  
  
  
  
  
9. The dimensions of a tank are 60 feet wide, 10 feet deep and 15 feet long. Calculate the volume of the tank in cubic feet.
  
  
  
  
  
  
  
  
  
  
10. A square tank is 25 ft wide, 75 ft long and can hold water to a depth of 10 ft. What is the volume of the tank, in gallons?
  
  
  
  
  
  
  
  
  
  
11. The diameter of a tank is 60 ft. When the water depth is 25 ft, what is the volume of the water in the tank, in  $\text{ft}^3$ ?

12. Calculate the volume (in gal) of a water tank that is 19 feet in diameter with a height of 25 feet.
13. Calculate the volume of water (in  $\text{ft}^3$ ) in a section of rectangular channel that is 4 feet deep, 5 feet wide, and 50 feet long.
14. A tank is 12 ft wide, 20 ft long and 15 deep. If the depth of the water is 11 ft, how many gallons of water are in the tank?
15. A new section of 12-inch diameter pipe is to be disinfected before it is put into service. If the length of the pipe is 2000 ft, how many gallons of water will be needed to fill the pipeline?
16. A section of 6 inch diameter pipeline is to be filled with chlorinated water for disinfection. If  $\frac{1}{4}$  mile of pipeline is to be disinfected, how many gallons of water will be required?

17. A circular clarifier has a diameter of 40 ft. What is the surface area (in  $\text{ft}^2$ ) of the clarifier?
18. The surface area of a tank is  $2000 \text{ ft}^2$ . If the width of the tank is 25 feet, what is the length of the tank?
19. What is the cubic yard volume of a trench 500 ft long, 2.25 ft wide and 4 ft deep? ( $1 \text{ yd}^3 = 27 \text{ ft}^3$ )
20. What is the diameter of a pipe (in feet) that is 750 feet long and holds  $1324 \text{ ft}^3$  of water?
21. The top of a tank has a surface area of  $3150 \text{ ft}^2$ . If the width of the tank is 35 ft, what is the length of the tank?

22. Calculate the volume of water in gallons in a 6 foot deep channel holding 4 feet of water. The channel is 5 feet wide and 120 feet long.
23. A tank is 12 ft wide and 20 feet long. If the depth of water is 11 feet, what is the volume of water in the tank in gallons?
24. Determine the amount of water, in gallons, to be disinfected in a new 36 inch water main that is 2 miles long.
25. A tank will hold 75,000 gallons. What is the volume of water in the tank, in gallons, if the depth is 12.5 feet, width is 20 ft, and length is 25 ft?
26. A 55 gallon steel drum with a diameter of 24 inches and a height of 42 inches needs to be painted. Calculate how many square feet of paint you would need to paint the entire outside of the barrel.

27. A circular water tower that is tapered at the bottom has a diameter of 36 feet and a height of 52 feet from the top to the beginning of the taper. The cone created by the taper has a height of 20 feet. Calculate the total exterior surface area of the water tower.



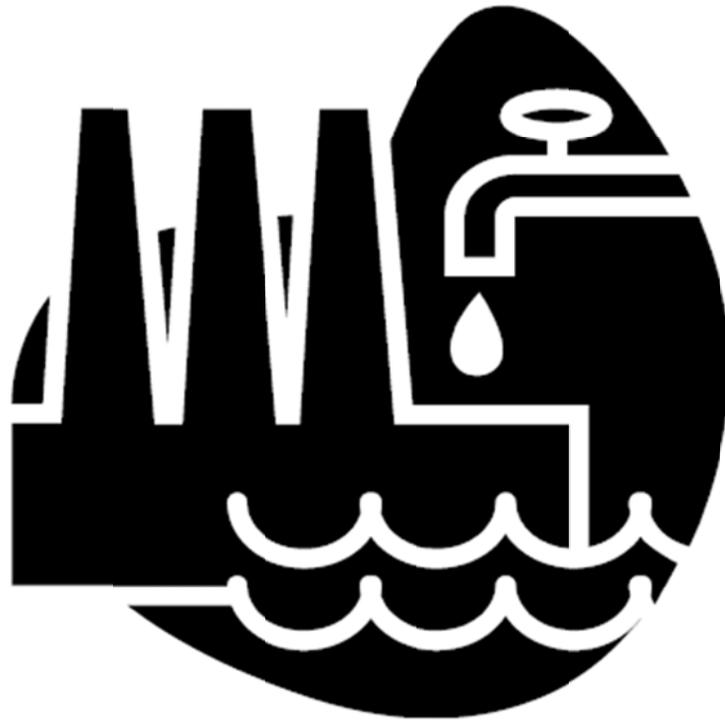
28. Refer back to the water tower in #29. Calculate the total volume (in gallons) when the tower is full.

Answers

1. 6.28 ft
2. 9.42 ft
3.  $20 \text{ ft}^2$
4.  $15 \text{ ft}^2$
5.  $19.63 \text{ ft}^2$
6.  $0.27 \text{ ft}^2$
7.  $215.04 \text{ ft}^2$
8.  $104.56 \text{ ft}^2$
9.  $9,000 \text{ ft}^3$
10. 140,250 gal
11.  $70,560 \text{ ft}^3$
12. 52,992.99 gal
13.  $1,000 \text{ ft}^3$
14. 19,747.2 gal
15. 11,743.6 gal
16. 1,937.69 gal
17.  $1,256 \text{ ft}^2$
18. 80 ft
19.  $166.67 \text{ yd}^3$
20. 1.5 ft
21. 90 ft
22. 17,952 gal
23. 19,747.2 gal
24. 558,055.87 gal
25. 46,750 gal
26.  $43.96 \text{ ft}^2$
27.  $7,398.87 \text{ ft}^2$
28. 446,444.70 gal

## Section 9

### Flow and Velocity



# Velocity & Flow

## Velocity

- The speed at which something is moving
- Measured in

○  $ft/min$   $ft/sec$   $miles/hr$  etc

$$Velocity = \frac{distance}{time}$$

## Example 1

- Blue dye is placed in a sewer line at a manhole. Three (3) minutes later, the dye appears in a manhole 125 feet down stream. What is the velocity of the flow in ft/min?

$$Velocity = \frac{distance}{time}$$

## Flow

- The volume of water that flows over a period of time

- $\frac{volume}{time}$

- Measured in

- $ft^3/sec$     $ft^3/min$     $gal/day$     $MG/D$

$$Flow = (Area)(Velocity)$$

$$Q = AV$$

## Example 2

- Water is flowing at velocity 3 ft/sec through a channel that is 2 feet wide and 18 inches deep. What is the flow in cubic feet per second?

$$Q = AV$$



## Example 3

- Determine the flow in ft<sup>3</sup>/sec through a 5 foot pipe that is flowing full at a velocity of 4.5 ft/sec.

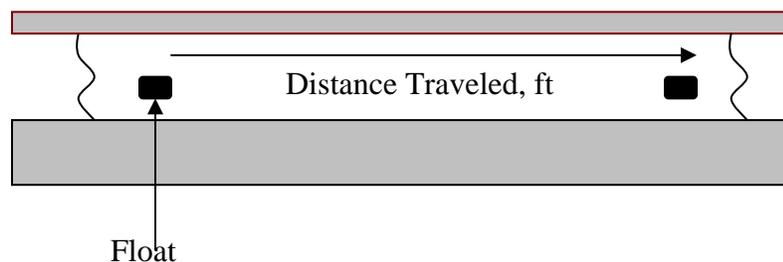
$$Q = AV$$



## Basic Math for Water and Wastewater Flow and Velocity

### Velocity

1. A cork is placed in a channel and travels 370 feet in 2 minutes. What is the velocity of the wastewater in the channel, ft/min?
  
2. A float travels 300 feet in a channel in 2 minutes and 14 seconds. What is the velocity in the channel, ft/sec?
  
3. The distance between manhole #1 and manhole #2 is 105 feet. A fishing bobber is dropped into manhole #1 and enters manhole #2 in 30 seconds. What is the velocity of the wastewater in the sewer in ft/min?



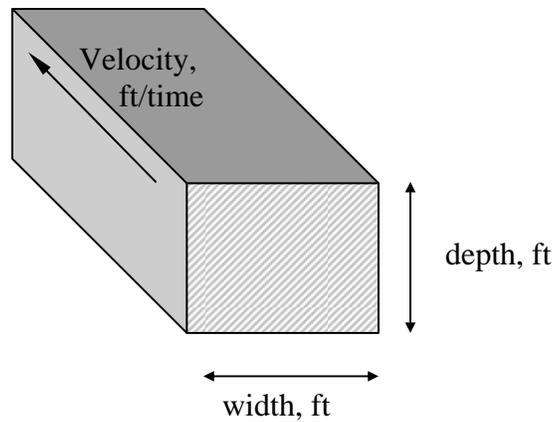
$$\text{Velocity} = \frac{\text{Distance Traveled, ft}}{\text{Duration of Test, min}}$$

$$= \text{ft/min}$$

3.) 210 ft/min

2.) 2.2 ft/sec

1.) 185 ft/min



$$Q = (A) (V)$$

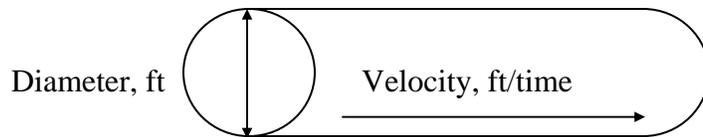
$$\text{ft}^3/\text{time} = (\text{ft})(\text{ft}) (\text{ft}/\text{time})$$

#### Flow in a channel

4. A channel 48 inches wide has water flowing to a depth of 1.5 feet. If the velocity of the water is 2.8 ft/sec, what is the flow in the channel in cu ft/sec?
  
5. A channel 3 feet wide has water flowing to a depth of 2.5 feet. If the velocity through the channel is 120 feet/min, what is the flow rate in cu ft/min? in MGD?
  
6. A channel is 3 feet wide and has water flowing at a velocity of 1.5 ft/sec. If the flow through the channel is 8.1 ft<sup>3</sup>/sec, what is the depth of the water in the channel in feet?

6.) 1.8 ft

5.) 900ft<sup>3</sup>/min; 9.7 MGD4.) 16.8 ft<sup>3</sup>/sec



$$Q = (A) (V)$$

$$\text{ft}^3/\text{time} = \text{ft}^2 (\text{ft}/\text{time})$$

$$Q = (0.785) (D)^2 (\text{vel})$$

$$\text{ft}^3/\text{time} = (\text{ft})(\text{ft}) (\text{ft}/\text{time})$$

Flow through a full pipe

7. The flow through a 2 ft diameter pipeline is moving at a velocity of 3.2 ft/sec. What is the flow rate in cu ft/sec?
  
8. The flow through a 6 inch diameter pipeline is moving at a velocity of 3 ft/sec. What is the flow rate in ft<sup>3</sup>/sec?
  
9. The flow through a pipe is 0.7 ft<sup>3</sup>/sec. If the velocity of the flow is 3.6 ft/sec, and the pipe is flowing full, what is the diameter of the pipe in inches?
  
10. An 8 inch diameter pipeline has water flowing at a velocity of 3.4 ft/sec. What is the flow rate in gpm?

10. 532.4 gpm

9. 6 in

8. 0.59 ft<sup>3</sup>/sec

7. 10.05 ft<sup>3</sup>/sec

## Basic Math for Water and Wastewater FLOW RATE

$$Q = AV$$

1. A channel is 3 feet wide with water flowing to a depth of 2 feet. If the velocity in the channel is found to be 1.8 fps, what is the cubic feet per second flow rate in the channel?
2. A 12-inch diameter pipe is flowing full. What is the cubic feet per minute flow rate in the pipe if the velocity is 110 feet/min?
3. A water main with a diameter of 18 inches is determined to have a velocity of 182 feet per minute. What is the flow rate in gpm?
4. A 24-inch main has a velocity of 212 feet/min. What is the gpd flow rate for the pipe?
5. What would be the gpd flow rate for a 6" line flowing at 2 feet/second?

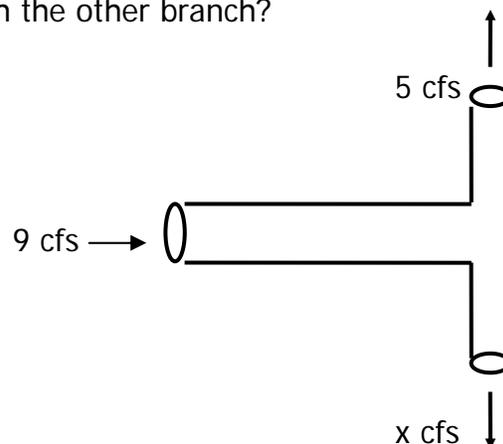


10. A cork placed in a channel travels 30 feet in 20 seconds. What is the velocity of the cork in feet per second?
11. A channel is 4 feet wide with water flowing to a depth of 2.3 feet. If a float placed in the channel takes 3 minutes to travel a distance of 500 feet, what is the cubic-foot-per-minute flow rate in the channel?

#### FLOW

12. The average velocity in a full-flowing pipe is measured and known to be 2.9 fps. The pipe is a 24" main. Assuming that the pipe flows 18 hours per day and that the month in question contains 31 days, what is the total flow for the pipe in MG for that month?

13. The flow entering the leg of a tee connection is 9 cfs. If the flow through one branch of the tee is 5 cfs, what is the flow through the other branch?



## ANSWERS:

## Flow and Velocity

1. 185 ft/ min
2. 2.24 ft/sec
3. 210 ft/min
4. 16.8 cfs
5. 9.69 MGD
6. 1.8 ft
7. 10.05 cfs
8. 0.59 cfs
9. 6 in
10. 532.4 gpm

## Flow Rate

- |                               |                                   |
|-------------------------------|-----------------------------------|
| 1. 10.8 ft <sup>3</sup> /sec  | 8. 9.47 MGD                       |
| 2. 86.35 ft <sup>3</sup> /min | 9. 120 ft/min                     |
| 3. 2,404.50 gpm               | 10. 1.5 ft/sec                    |
| 4. 7,170,172.42 gpd           | 11. 1,533.33 ft <sup>3</sup> /min |
| 5. 253,661.76 gpd             | 12. 136.83 MG                     |
| 6. 7,926.93 gpm               | 13. 4 ft <sup>3</sup> /sec        |
| 7. 9.13 MGD                   |                                   |

### More Velocity and Flow Problems

1. A float travels 500 ft in a channel in 5 minutes and 22 seconds. What is the velocity in ft/sec?
2. A cork is placed in a channel and travels 50 ft in 9 seconds, what is the velocity in ft/min?
3. A car travels at a speed of 60 mph, what is the velocity in ft/sec?
4. The distance between a manhole A and manhole B is 400 ft. A float is dropped into manhole A and enters manhole B in 2 minutes and 30 seconds. What is the velocity of the water in ft/min?
5. A garden snail travelled 15 inches in 10 minutes, what is the snail's velocity in ft/min?
6. A channel 3 ft wide has water flowing to a depth of 11 inches. If the velocity of the water is 3.2 ft/sec, what is the flow through the channel in  $\text{ft}^3/\text{sec}$ ?

7. A channel 30 inches wide has water flowing at a depth of 2 ft. If the length of the channel is 5,000 ft and the velocity through the channel is 2.5 ft/sec, what is the flow through the channel in  $\text{ft}^3/\text{sec}$ ?
  
8. A channel is 2.5 ft wide and the water is flowing at a velocity of 3 ft/sec. If the flow through the channel is measured to be  $6.4 \text{ ft}^3/\text{sec}$ , what is the depth of the water in the channel in ft?
  
9. A channel is 3 ft wide and the water is flowing at a velocity of 210 ft/min. If the water is 6 inches deep in the channel, what is the flow through the channel in gpm?
  
10. A channel is 24 inches wide and has water to a depth of 18 inches. If the water is flowing at a velocity of 2.9 ft/sec, what is the flow rate in cubic feet/min?
  
11. The flow through a channel is 100 gpm. If the channel is 3 ft wide and has water to a depth of 2 ft, what is the velocity of the water in ft/sec?

12. The flow through a 3 ft diameter pipeline is moving at a velocity of 4 ft/sec. What is the flow through the pipe in cubic feet/sec?
13. The flow through a 10 inch diameter pipe is moving at a velocity of 2 ft/sec. What is the flow rate in cubic ft/sec?
14. A 6 inch diameter pipe has water flowing at a velocity of 120 ft/min. What is the flow rate in gpm?
15. The flow through a pipe is  $0.82 \text{ ft}^3/\text{sec}$ . If the velocity of the flow is 1.5 ft/sec, and the pipe is flowing full, what is the diameter of the pipe in inches?
16. A 2 ft main has water flowing at a velocity of 4.1 ft/sec. What is the flow through the pipe in gph?

17. A 3 ft diameter main has just been installed. According to the Design Criteria for the State of Tennessee, the minimum flushing velocity is 2.5 ft/sec. If the main is flushed at a velocity of 3 ft/sec, how many gallons per minute will be flushed from the hydrant?
18. A pipe has a diameter of 24 inches. If the pipe is flowing full, and the water is known to flow a distance of 200 ft in 3 minutes, what is the flow rate for the pipe in MGD?
19. What is the flow rate in gpd for a 6 inch main flowing at a velocity of 220 ft/min?
20. If the flow through a 10 inch diameter pipe is 3.2 MGD, what is the velocity of the water in ft/sec?
21. The flow through a pipe is 320 gpm. If the velocity through the pipe is 3.6 ft/sec what is the diameter of the pipe in inches?

22. A certain pipe has a diameter of 10 inches. If the water in the pipe is known to travel 200 yds in 3 minutes, what is the flow rate for the pipe in gpd?

**Answers**

- |                              |                                |
|------------------------------|--------------------------------|
| 1. 1.55 ft/sec               | 12. 28.3 ft <sup>3</sup> /sec  |
| 2. 333.3 ft/sec              | 13. 1.089 ft <sup>3</sup> /sec |
| 3. 88 ft/sec                 | 14. 176 gpm                    |
| 4. 160 ft/min                | 15. 10 in                      |
| 5. 0.125 ft/min              | 16. 346,671 gph                |
| 6. 8.8 ft <sup>3</sup> /sec  | 17. 9,512 gpm                  |
| 7. 12.5 ft <sup>3</sup> /sec | 18. 2.25 MGD                   |
| 8. 0.853 ft                  | 19. 465,046.56 gpd             |
| 9. 2,356 gpm                 | 20. 9.09 ft/sec                |
| 10. 522 ft <sup>3</sup> /min | 21. 6 in                       |
| 11. 0.037 ft/sec             | 22. 1,174,266.53 gpd           |