PreCalculus Mathematics Instructional Materials Scoring Rubric

Introduction:

The following Instructional Materials Scoring Rubric for Mathematics is designed to score materials in the following categories:

- Instructional Focus
- Math Practices
- Aspects of Rigor
- Accessibility Features

Scoring:

Each section is to be scored using a 0, 1, or 2. For all sections, except for Rigor, use the following rubric when deciding on the appropriate rating:

- 0: The metric is not present within the material.
- 1: The metric is present within the material. The intent and/or frequency component of the metric is not fully met.
- 2: A rating of 2 indicates the metric is present and all aspects of the metric are fully met.

For Rigor:

- 0: The standard is not instructionally present within the material.
- 1: The standard is instructionally present but does not have an instructional focus on the indicated type of rigor.
- 2: The standard is instructionally present and has a clear instructional focus on the indicated type of rigor.

Note: Some standards appear under multiple aspects of rigor (i.e., Conceptual Understanding, Procedural Fluency, or Application). When scoring these standards, only score the part of the standard relevant to that aspect of rigor, which is identified by a bold, italics, larger font.

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Gateway: The publisher must provide a Tennessee standards alignment guide as a part of the scope and sequence for the material. If this gateway is not met, the materials will not be scored.

Instructional Focus				
	0	**1**	**2**	**Evidence**
Connections to content from prior grades are clearly identified and explicitly related to grade-level work.				
Materials embed a minimum of 3 tasks in every unit. Each task has multiple entry-points and can be solved using a minimum of 2 solution strategies and/or representations.				
Materials give students opportunities to work problems within each lesson. Each problem set: • Covers the full breadth of the standard(s) covered in the lesson • Is aligned to on grade level expectations as identified in the standard(s)				
Teacher resources indicate common student misconceptions in every unit and provide guidance on how to instructionally address the identified misconceptions.				
Materials provide educative supports (e.g., adult level explanations of the standards and strategies) in every lesson for teachers to ensure standards are taught accurately and to the appropriate level of rigor (i.e., conceptual understanding, procedural fluency, and application) as indicated by the standards.				
Materials develop student understanding of multiple representations (i.e., concrete, representational, abstract) for relevant standards which are identified in the state's Instructional Focus Documents.				
Materials include problems and activities in every unit that connect two or more grade level standards in a domain (e.g., 4.OA.A.1 and 2.OA.A.2).				
Materials include problems and activities in every unit that connect two or more grade level domains. (e.g., 4.MD.A.2 and 4.OA.A.3)				
Materials provide opportunities for students to participate in a spiraled review in every unit.				
			Total	

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Mathematical Practices				
Math Practices/Literacy Skills for Math Proficiency	**0**	**1**	**2**	**Evidence**
Materials embed the eight math practice standards in every unit.				
Math practice standards are clearly identified in both teacher and student materials.				
Materials use appropriate math vocabulary which is aligned to the grade level standards.				
Materials support students in discussing and articulating mathematical ideas. Within each lesson students either write or verbally justify their thoughts.				
			Total	

Accessibility Features				
Digital Materials	**0**	**1**	**2**	**Evidence**
All lessons within the materials are available in digital form and include a printable option.				
In every lesson, materials include recommended supports, accommodations, and modifications for Students with Disabilities and English Language Learners that will support their regular and active participation in accessing on grade level material (e.g., modifying vocabulary words within word problems, sentence starters, etc.).				
			Total	

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Gateway: The publisher must provide a Tennessee standards alignment guide as a part of the scope and sequence for the material. If this gateway is not met, the materials will not be scored.

Aspects of Rigor				
Conceptual Understanding: The materials support the intentional development of students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or clusters.	**0**	**1**	**2**	**Evidence**
P.N.NE.A.1 Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them.				
P.N.NE.A.2 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★				
P.N.NE.A.3 Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e.				
P.N.NE.A.4 Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not.				
P.N.NE.A.5 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.				
P.N.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.				
P.N.CN.A.2 Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$.				
P.N.CN.A.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.				
P.N.CN.A.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				
P.N.CN.A.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this				

representation for computation (for example, $(-1 + 3i)^3 = 8$ because $(-1 + 3i)$ has modulus 2 and argument 120°).				
P.N.CN.A.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				
P.N.CN.B.7 Extend polynomial identities to the complex numbers (for example, rewrite $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.				
P.N.CN.B.8 Solve quadratic equations with real coefficients that have complex solutions.				
P.N.CN.B.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.				
P.N.VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, $\vec{V}$).				
P.N.VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				
P.N.VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.				
P.N.VM.B.4 Add and subtract vectors.				
P.N.VM.B.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.				
P.N.VM.B.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.				
P.N.VM.B.4.c Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				
P.N.VM.B.5 Multiply a vector by a scalar.				
P.N.VM.B.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise (e.g., as $c(\boldsymbol{v}_x, \boldsymbol{v}_y) = (c\boldsymbol{v}_x, c\boldsymbol{v}_y)$.				

P.N.VM.B.5.b Compute the magnitude of a scalar multiple ***cv*** using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. Compute the direction of ***cv*** knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of ***cv*** is either along ***v*** (for $c > 0$) or against ***v*** (for $c < 0$).				
P.N.VM.B.6 Calculate and interpret the dot product of two vectors.				
P.N.VM.C.7 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				
P.N.VM.C.8 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				
P.N.VM.C.9 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				
P.N.VM.C.10 Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.				
P.A.S.A.1 Demonstrate an understanding of sequences by representing them recursively and explicitly.				
P.A.S.A.2 Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.				
P.A.S.A.3 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist.				
P.A.S.A.3.a Determine whether a given arithmetic or geometric series converges or diverges.				
P.A.S.A.3.b Find the sum of a given geometric series (both infinite and finite).				
P.A.S.A.3.c Find the sum of a finite arithmetic series.				
P.A.S.A.4 Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.				
P.A.S.A.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle.				
P.A.REI.A.1 Represent a system of linear equations as a single matrix equation in a vector variable.				

P.A.REI.A.2 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).				
P.A.REI.A.3 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.				
P.A.REI.A.4 Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology.				
P.A.REI.A.5 Solve systems of nonlinear inequalities by graphing.				
P.A.PE.A.1 Graph curves parametrically (by hand and with appropriate technology).★				
P.A.PE.A.2 Eliminate parameters by rewriting parametric equations as a single equation.★				
P.A.C.A.1 Display all of the conic sections as portions of a cone.				
P.A.C.A.2 Know and write the equation of a circle of given center and radius using the Pythagorean Theorem.				
P.A.C.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.				
P.A.C.A.4 From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics.				
P.A.C.A.5 Transform equations of conic sections to convert between general and standard form.				
P.F.BF.A.1 Understand how the algebraic properties of an equation transform the geometric properties of its graph (for example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation such as translations, stretches, reflections, and changes in periodicity and amplitude).				
P.F.BF.A.2 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.				
P.F.BF.A.3 Compose functions (for example, if *T(y)* is the temperature in the atmosphere as a function of height, and *h(t)* is the height of a weather balloon as a				

function of time, then *T(h(t))* is the temperature at the location of the weather balloon as a function of time).				
P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.				
P.F.BF.A.5 Find inverse functions (including exponential, logarithmic, and trigonometric).				
P.F.BF.A.5.a Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $1 / f(x)$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type.				
P.F.BF.A.5.b Verify by composition that one function is the inverse of another.				
P.F.BF.A.5.c Read values of an inverse function from a graph or a table, given that the function has an inverse.				
P.F.BF.A.5.d Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.				
P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.				
P.F.IF.A.1 Determine whether a function is even, odd, or neither.				
P.F.IF.A.2 Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★				
P.F.IF.A.4 Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational).				
P.F.IF.A.5 Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$.				
P.F.IF.A.6 Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur.				
P.F.IF.A.7 Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior.				

P.F.IF.A.8 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (for example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$).				
P.F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.				
P.F.TF.A.2 Convert from radians to degrees and from degrees to radians.				
P.F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and explain how to use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.				
P.F.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				
P.F.TF.A.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.				
P.F.GT.A.1 Interpret transformations of trigonometric functions.				
P.F.GT.A.2 Determine the difference made by choice of units for angle measurement when graphing a trigonometric function.				
P.F.GT.A.3 Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes.				
P.F.GT.A.4 Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions.				
P.F.GT.A.5 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.				
P.F.GT.A.6 Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions.				
P.F.GT.A.7 Graph the inverse trigonometric functions and identify their key characteristics.				
P.F.GT.A.8 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.				
P.G.AT.A.1 Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles.				

P.G.AT.A.2 Derive the formula *A = 1/2 ab sin(C)* for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.				
P.G.AT.A.3 Derive and apply the formulas for the area of sector of a circle.				
P.G.AT.A.4 Calculate the arc length of a circle subtended by a central angle.				
P.G.AT.A.5 Prove the Laws of Sines and Cosines and use them to solve problems.				
P.G.AT.A.6 Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (such as surveying problems and resultant forces).				
P.G.TI.A.1 Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle.				
P.G.TI.A.2 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				
P.G.PC.A.1 Graph functions in polar coordinates.				
P.G.PC.A.2 Convert between rectangular and polar coordinates.				
P.G.PC.A.3 Represent situations and solve problems involving polar coordinates.★				
P.S.MD.A.1 Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real-world phenomena and to make predictions.				
P.S.MD.A.2 Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data.				
P.S.MD.A.3 Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.				
Procedural Skill and Fluency: The materials provide intentional opportunities for students to develop procedural skills and fluencies, especially where called for in specific content standards or clusters	**0**	**1**	**2**	**Evidence**
P.N.NE.A.1 Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them.				
P.N.NE.A.2 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★				

P.N.NE.A.3 Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and *e*.				
P.N.NE.A.4 Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not.				
P.N.NE.A.5 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.				
P.N.CN.A.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.				
P.N.CN.A.2 Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$.				
P.N.CN.A.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.				
P.N.CN.A.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				
P.N.CN.A.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation (for example, $(-1 + 3i)^3 = 8$ because $(-1 + 3i)$ has modulus 2 and argument 120°).				
P.N.CN.A.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				
P.N.CN.B.7 Extend polynomial identities to the complex numbers (for example, rewrite $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.				
P.N.CN.B.8 Solve quadratic equations with real coefficients that have complex solutions.				
P.N.CN.B.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.				
P.N.VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, $\vec{V}$).				

P.N.VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				
P.N.VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.				
P.N.VM.B.4 Add and subtract vectors.				
P.N.VM.B.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.				
P.N.VM.B.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.				
P.N.VM.B.4.c Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				
P.N.VM.B.5 Multiply a vector by a scalar.				
P.N.VM.B.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise (e.g., as $c(\boldsymbol{v}_x, \boldsymbol{v}_y) = (c\boldsymbol{v}_x, c\boldsymbol{v}_y)$.				
P.N.VM.B.5.b Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).				
P.N.VM.B.6 Calculate and interpret the dot product of two vectors.				
P.N.VM.C.7 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				
P.N.VM.C.8 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				
P.N.VM.C.9 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				
P.N.VM.C.10 Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.				

P.A.S.A.1 Demonstrate an understanding of sequences by representing them recursively and explicitly.				
P.A.S.A.2 Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.				
P.A.S.A.3 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist.				
P.A.S.A.3.a Determine whether a given arithmetic or geometric series converges or diverges.				
P.A.S.A.3.b Find the sum of a given geometric series (both infinite and finite).				
P.A.S.A.3.c Find the sum of a finite arithmetic series.				
P.A.S.A.4 Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.				
P.A.S.A.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined, for example, by Pascal's Triangle.				
P.A.REI.A.1 Represent a system of linear equations as a single matrix equation in a vector variable.				
P.A.REI.A.2 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).				
P.A.REI.A.3 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.				
P.A.REI.A.4 Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology.				
P.A.REI.A.5 Solve systems of nonlinear inequalities by graphing.				
P.A.PE.A.1 Graph curves parametrically (by hand and with appropriate technology).★				
P.A.PE.A.2 Eliminate parameters by rewriting parametric equations as a single equation.★				
P.A.C.A.1 Display all of the conic sections as portions of a cone.				
P.A.C.A.2 Know and write the equation of a circle of given center and radius using the Pythagorean Theorem.				

P.A.C.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.				
P.A.C.A.4 From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics.				
P.A.C.A.5 Transform equations of conic sections to convert between general and standard form.				
P.F.BF.A.1 Understand how the algebraic properties of an equation transform the geometric properties of its graph (for example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation such as translations, stretches, reflections, and changes in periodicity and amplitude).				
P.F.BF.A.2 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.				
P.F.BF.A.3 Compose functions (for example, if *T(y)* is the temperature in the atmosphere as a function of height, and *h(t)* is the height of a weather balloon as a function of time, then *T(h(t))* is the temperature at the location of the weather balloon as a function of time).				
P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.				
P.F.BF.A.5 Find inverse functions (including exponential, logarithmic, and trigonometric).				
P.F.BF.A.5.a Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $1/f(x)$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type.				
P.F.BF.A.5.b Verify by composition that one function is the inverse of another.				
P.F.BF.A.5.c Read values of an inverse function from a graph or a table, given that the function has an inverse.				
P.F.BF.A.5.d Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.				

P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.				
P.F.IF.A.1 Determine whether a function is even, odd, or neither.				
P.F.IF.A.2 Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★				
P.F.IF.A.4 Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational).				
P.F.IF.A.5 Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$.				
P.F.IF.A.6 Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur.				
P.F.IF.A.7 Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior.				
P.F.IF.A.8 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (for example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$).				
P.F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.				
P.F.TF.A.2 Convert from radians to degrees and from degrees to radians.				
P.F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and explain how to use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.				
P.F.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				
P.F.TF.A.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.				
P.F.GT.A.1 Interpret transformations of trigonometric functions.				
P.F.GT.A.2 Determine the difference made by choice of units for angle measurement when graphing a trigonometric function.				

P.F.GT.A.3 Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes.				
P.F.GT.A.4 Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions.				
P.F.GT.A.5 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.				
P.F.GT.A.6 Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions.				
P.F.GT.A.7 Graph the inverse trigonometric functions and identify their key characteristics.				
P.F.GT.A.8 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.				
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P.G.AT.A.3 Derive and apply the formulas for the area of sector of a circle.				
P.G.AT.A.4 Calculate the arc length of a circle subtended by a central angle.				
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P.G.TI.A.2 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				
P.G.PC.A.1 Graph functions in polar coordinates.				
P.G.PC.A.2 Convert between rectangular and polar coordinates.				
P.G.PC.A.3 Represent situations and solve problems involving polar coordinates.★				

P.S.MD.A.1 Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real-world phenomena and to make predictions.				
P.S.MD.A.2 Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data.				
P.S.MD.A.3 Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.				
Applications: The materials support the intentional development of students' ability to utilize mathematical concepts and skills in engaging applications, especially where called for in specific content standards or clusters.	**0**	**1**	**2**	**Evidence**
P.N.NE.A.1 Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them.				
P.N.NE.A.2 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★				
P.N.NE.A.3 Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e.				
P.N.NE.A.4 Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not.				
P.N.NE.A.5 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.				
P.N.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.				
P.N.CN.A.2 Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$.				
P.N.CN.A.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.				
P.N.CN.A.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				

P.N.CN.A.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation (for example, $(-1 + 3i)^3 = 8$ because $(-1 + 3i)$ has modulus 2 and argument 120°).				
P.N.CN.A.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				
P.N.CN.B.7 Extend polynomial identities to the complex numbers (for example, rewrite $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.				
P.N.CN.B.8 Solve quadratic equations with real coefficients that have complex solutions.				
P.N.CN.B.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.				
P.N.VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, $\vec{V}$).				
P.N.VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				
P.N.VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.				
P.N.VM.B.4 Add and subtract vectors.				
P.N.VM.B.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.				
P.N.VM.B.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.				
P.N.VM.B.4.c Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				
P.N.VM.B.5 Multiply a vector by a scalar.				
P.N.VM.B.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise (e.g., as $c(\boldsymbol{v}_x, \boldsymbol{v}_y) = (c\boldsymbol{v}_x, c\boldsymbol{v}_y)$.				

P.N.VM.B.5.b Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).				
P.N.VM.B.6 Calculate and interpret the dot product of two vectors.				
P.N.VM.C.7 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				
P.N.VM.C.8 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				
P.N.VM.C.9 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				
P.N.VM.C.10 Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.				
P.A.S.A.1 Demonstrate an understanding of sequences by representing them recursively and explicitly.				
P.A.S.A.2 Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.				
P.A.S.A.3 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist.				
P.A.S.A.3.a Determine whether a given arithmetic or geometric series converges or diverges.				
P.A.S.A.3.b Find the sum of a given geometric series (both infinite and finite).				
P.A.S.A.3.c Find the sum of a finite arithmetic series.				
P.A.S.A.4 Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.				
P.A.S.A.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle.				
P.A.REI.A.1 Represent a system of linear equations as a single matrix equation in a vector variable.				

P.A.REI.A.2 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).				
P.A.REI.A.3 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.				
P.A.REI.A.4 Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology.				
P.A.REI.A.5 Solve systems of nonlinear inequalities by graphing.				
P.A.PE.A.1 Graph curves parametrically (by hand and with appropriate technology).★				
P.A.PE.A.2 Eliminate parameters by rewriting parametric equations as a single equation.★				
P.A.C.A.1 Display all of the conic sections as portions of a cone.				
P.A.C.A.2 Know and write the equation of a circle of given center and radius using the Pythagorean Theorem.				
P.A.C.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.				
P.A.C.A.4 From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics.				
P.A.C.A.5 Transform equations of conic sections to convert between general and standard form.				
P.F.BF.A.1 Understand how the algebraic properties of an equation transform the geometric properties of its graph (for example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation such as translations, stretches, reflections, and changes in periodicity and amplitude).				
P.F.BF.A.2 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.				
P.F.BF.A.3 Compose functions (for example, if *T(y)* is the temperature in the atmosphere as a function of height, and *h(t)* is the height of a weather balloon as a				

function of time, then *T(h(t))* is the temperature at the location of the weather balloon as a function of time).				
P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.				
P.F.BF.A.5 Find inverse functions (including exponential, logarithmic, and trigonometric).				
P.F.BF.A.5.a Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $1 / f(x)$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type.				
P.F.BF.A.5.b Verify by composition that one function is the inverse of another.				
P.F.BF.A.5.c Read values of an inverse function from a graph or a table, given that the function has an inverse.				
P.F.BF.A.5.d Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.				
P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.				
P.F.IF.A.1 Determine whether a function is even, odd, or neither.				
P.F.IF.A.2 Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★				
P.F.IF.A.4 Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational).				
P.F.IF.A.5 Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$.				
P.F.IF.A.6 Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur.				
P.F.IF.A.7 Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior.				

P.F.IF.A.8 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (for example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$).				
P.F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.				
P.F.TF.A.2 Convert from radians to degrees and from degrees to radians.				
P.F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and explain how to use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.				
P.F.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				
P.F.TF.A.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.				
P.F.GT.A.1 Interpret transformations of trigonometric functions.				
P.F.GT.A.2 Determine the difference made by choice of units for angle measurement when graphing a trigonometric function.				
P.F.GT.A.3 Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes.				
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