TN Department of Education

Fifth Grade Mathematics

Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade-level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Levels 4: Performance at these levels demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in the same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee Mathematics Standards will most likely be able to do in a classroom setting.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for levels 3 and 4.

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Operations and Algebraic Thinking

Standard 5.OA.A.1 (Supporting Content)
Use parentheses and/or brackets in numerical expressions and evaluate expressions having these symbols using the conventional order (Order of Operations).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Calculate with whole numbers using the four operations. Calculate addition and subtraction of fractions with like denominators and/or multiplication of whole number by a fraction.	Evaluate two-step expressions with parenthesis using order of operations with whole numbers. Use the distributive property to evaluate expressions. Use the commutative and the associative properties to add or multiply while evaluating expressions	Evaluate multi-step expressions with parenthesis using order of operations that may include adding and subtracting fractions with unlike denominators and multiplying a fraction by a whole number and a fraction by a fraction. Use the parenthesis when needed by the context to evaluate an expression.	Determine when it is helpful to add grouping symbols in order to solve equations and word problems. Determine which equation is true using the order of operations when given two equations. Accurately complete an error analysis of an evaluated expression. Write a context for the expression when given an expression.

Instructional Focus Statements

Level 3:

Building off of computation work in grade 4 which includes working with all 4 operations with whole numbers and addition and subtraction with like denominators with fractions, grade 5 students will begin to work more formally with expressions. In order to help students reason about the order in which operations need to be performed, students should explore the use of parenthesis by solving a variety of multi-step problems that make connections to the properties of addition and multiplication.

This standard is not about teaching dependence on mnemonic phrases like PEMDAS but is about understanding the order of operations conceptually. In grade 5, this work should be viewed as exploratory rather than for attaining mastery. Expressions with grouping symbols at this stage should not be more complex than the use of the associative or distributive properties. Seeing these multi-step expressions in context can aid in building student understanding of why it works in the conventional order. For example, Addison bought a game for $20 and 3 shirts for $7 each. How much did she spend? Prompting students to write and solve an expression to solve problems such as these help students to model order of operations. Context is key in aiding students understanding of how order of operations work. As students explore expressions such as 20 x 3+7, for efficiency, they would do the repeated addition first for 20 x 3 then add the 7 in the same way one would evaluate 20 + 7 x 3 by doing 7 x 3 then adding 20 unless the context said, "We bought 7 drinks at $3 each and a pizza for $20" now the parenthesis are needed to "undo" the conventional order of operations. All of this work is building a foundation for grade 6 and beyond as students will begin to look at expressions and be able to describe them in terms of their parts.

Level 4:

At this level, requiring students to reason as to which equation is true when evaluated or writing a context for a given expression allows students to begin to think about how the grouping of numbers and its operations affects the size of the number. Asking students to respond to an incorrectly evaluated expression and give reasoning on why it is incorrect is laying the foundation for work in grade 6 where students are interpreting expressions not just evaluating them.

Standard 5.OA.A.2 (Supporting Content)
Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Utilizes correct vocabulary associated with the 4 operations (terms such as less than, added to, product, quotient, etc.).	When given a two-term expression, translate it into words (e.g., 4 x 3 can be expressed as "the product of 4 and 3"). Identify models of multiple-term expressions (e.g., 3 x (4 + 7) modeled would be (4 + 7) repeated 3 times).	Reason when given an expression such as 3 x (124 + 16) that it is three times as much as 124 + 16. Write the numerical expression when given an expression in words. Given the numerical expression, translate it into words.	When given an expression, identify more than one equivalent written form. For example, (25 ÷ 5) - 2 could be represented as the quotient of 25 and 5 minus 2or 2 less than 25 divided by 5.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

This standard is an extension of standard 5.OA.A.1 by having students write and interpret numerical expressions. Having students move from word form to expression and from expressions to word form will reinforce their understanding of order of operations. As this is a standard that is laying the foundation for the Algebra work in future coursework, this standard is exploratory rather than mastery. Therefore, the expressions should be more complex than the work that one would do in the application of the associative or distributive property. As students are reasoning about the size of an expression in comparison to another expression (e.g., 3 x (124 + 16) is three times larger than 124 + 16 allows students to use their conceptual understanding of multiplication. This will lay the foundation for later work using variables in expressions, specifically standards 6.EE.A.2 and 6.EE.A.3, where students can recognize that 3*X* means 3 times larger than *X*.

Level 4:

Vocabulary is vital for laying the foundation of this standard. Students should have exposure to phrases such as less than, in which the order of the expression will matter. For example, six less than the product of two and four. Students must also be aware that often there is more than one way to interpret an expression and that all of the phrases associated with it will yield a correct response when evaluated. Students must learn to make sense of the situation when relating it to a given expression.

Standard 5.OA.B.3 (Supporting Content)

Generate two numerical patterns using two given rules. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences.

5.OA.B.3a Identify relationships between corresponding terms in two numerical patterns. For example, observe that the terms in one sequence are twice the corresponding terms in the other sequence.

5.OA.B.3b Form ordered pairs consisting of corresponding terms from two numerical patterns and graph the ordered pairs on a coordinate plane.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the rule that generated the pattern, given a numerical pattern. Choose a set of ordered pairs from two given numerical patterns.	Identify all patterns that follow the given rule, given a rule and a set of generated patterns. Form a set of ordered pairs from two given numerical patterns.	Identify the relationship between corresponding terms in two numerical patterns. Generate two numerical patterns, form ordered pairs consisting of corresponding terms from the patterns, and graph the ordered pairs on a coordinate plane.	Explain the relationship between corresponding terms in two numerical patterns. Represent a real world problem by generating two numerical patterns, form ordered pairs consisting of corresponding terms from the patterns, and graph the ordered pairs on a coordinate plane. Identify the relationship, if one exists, between the corresponding terms in the two numerical patterns.

Instructional Focus Statements

Level 3:

This standard builds on the previous work from Grade 4 with patterns (standard 4.OA.C.5) and includes skills that go beyond simply solving the problem and identifying a pattern. In Grade 5, students are given two rules and generate the terms in the respective sequences. Students should identify, interpret, record, and graph ordered pairs on a coordinate plane, limited to the first quadrant. This standard explicitly states to generate two numerical patterns given two rules. Students may need to begin with a mathematical problem that requires one rule before moving to problems that require two rules. In both, students should be able to make a list of a table of the generated patter(s) and graph the corresponding ordered pairs in the generated list. Students should be able to identify relationships between corresponding terms in two numerical patterns and explain the relationship using precise mathematical vocabulary. For example, as stated in the standard, students should be able to observe that the terms in one sequence are twice the corresponding terms in the other sequence.

As students are plotting order pairs for the first time in the first quadrant, it is important to understand that the x-coordinate moves left and right across the x-axis and the y-coordinate moves up and down the y-axis. It is a common misconception for students to get the x and y-coordinates mixed up. Therefore, it is important that students understand in the ordered pair that the first number indicates the horizontal distance traveled along the x-axis from the origin and the second number indicates the vertical distance traveled along the y-axis. Students should also understand that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). As students grasp this understanding, they should also be able to explain the difference of location of (2, 4) and (4, 2). Students should be given ample opportunity to label and graph points when working with this standard.

Additionally, it is important to discuss the graph and how it relates to the problem situation. This lays the foundation for future work with proportional relationships in grade 6 and grade 7 and extends to course work with functions.

Level 4:

As students extend their understanding of representing real-world and mathematical problems by locating and graphing points on the coordinate plane (positive numbers only) using two given rules, they should also be given two rules within a contextual situation. Students may need to begin with a contextual problem that requires one rule before moving to contextual problems that requires two rules. For example, Carrie makes $2, she makes $3 more each day than she does the day before. How much does she make on the fourth day? In this example with one rule, students should be able to generate the pattern and graph the ordered pairs that represent the contextual situation and determine that the total amount that Carrie makes on the fourth day is $11. As students develop an in-depth understanding of contextual problems with one rule, they should then work with contextual problems that contain two rules. For example, Lou and Tom are riding bikes. Lou rides 1.5 miles every 10 minutes and Tom rides 2.0 miles every 12 minutes. Graph the distance the boys ride. Who traveled farther at 60 minutes? Students should be able to graph the contextual situation and determine the distance that each travels in 60 minutes and determine that Tom travels further than Lou in 60 minutes. Students should be able to explain the relationship between corresponding terms in the two numerical patterns. Additionally, students should be able to create real-world and mathematical problems which require generating two numerical patterns, locate and graph points in the coordinate plane (positive numbers only) and interpret the x and y value of the ordered pair with respect to the context. Students should provide multiple representations of their solution and explain their reasoning using precise mathematical language.

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Number and Operations in Base Ten

Standard 5.NBT.A.1 (Major Work of the Grade)
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. Identify the value of each digit (including tenths, hundredths, and thousandths) in a multi-digit number.	Identify that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right. Write multiplication equations to represent/compare the relationships between place values. Demonstrate relationships between place values such as it takes 10 tenths to make a whole, 10 hundredths to make 1 tenth, 10 thousandths to make 1 hundredth.	Identify that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right and 1/10 of what it represents in the place to its left. Write multiplication and division equations to represent/compare relationships between the values of the same digit when it is located in different place values. Compare the value of a digit in one number to the value of a digit in another number.	Generate a multi-digit number that has a specified digit 10 times greater than that same digit in a provided multi-digit number and explain the reasoning behind the generated number. Generate a multi-digit number that has a specified digit 1/10 the value of the same digit in a provided multi-digit number and explain the reasoning behind the generated number.

Instructional Focus Statements

Level 3:

In grade 4, students learned that a digit in one place represents 10 times what it represents in the place to its right. The instructional focus for this standard should be discovery learning that a digit in one place represents 1/10 of what it represents in the place to its left. This should be done by using manipulatives and pictorial representations where students can see that it takes ten groups of one place to make the place to the left or has to be divided into ten groups (a tenth) to make the place to the right. It is important for students to use their understanding of multiplication and division while developing and solidifying this understanding. Instruction should not focus on tricks of adding/removing zeroes as the digit moves place values; instruction should focus on the increase/decrease being multiplicatively related. This standard is a cornerstone standard for students as they develop their conceptual understanding of the base ten number system. Instruction on this standard should come after students understand the digits in a number represent the number of groups of a particular place value (standard 5.NBT.A.3). For example, students should recognize that in 33.33, the first 3 after the decimal represents 3 groups of tenths (3 x 0.1) or 3 tenths and the second 3 after the decimal represents three groups of hundredths (3 x 0.01) or three hundredths. As they explore this, they realize that it takes ten hundredths to make a tenth and that a tenth must be cut into ten pieces to make a hundredth. This is the foundation for understanding that place values are multiplicatively related to ten.

Level 4:

At this level, the focus of instruction should be for students to move beyond recognizing when a digit is ten times the value of a digit to its right or 1/10 the value of a digit to its left to generating numbers such that this occurs. Additionally, students should be able to explain why this occurs using appropriate mathematical vocabulary.

Standard 5.NBT.A.2 (Major Work of the Grade)

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Multiply whole numbers by 10, 100, 1000, etc. Explain that an exponent denotes how many times the base number 10 is used as a factor. For example, 10^3 means 10 x 10 x 10.	Write a number multiplied by a power of 10 in multiple ways. For example, 5,000= 5 x 10 x 10 x 10 = 5 x 1000 = 5 x 10^3. Explain patterns when a number is multiplied by a power of ten.	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Multiply and divide decimals by powers of 10 given in standard or exponential form. Use exponential notation to represent powers of 10, and identify the value of powers of 10 given in exponential form.	Write very large numbers using powers of 10. Write multi-digit whole numbers in expanded form using exponents. Conduct an error analysis of an exponent being used incorrectly and explain the error.

Instructional Focus Statements

Level 3:

The use of whole number exponents to denote powers of 10 is introduced in grade 5. Students will gain an understanding of why multiplying or dividing by a power of 10 shifts the digits of a whole number or decimal the appropriate number of places to the left or right. Patterns in the number of zeros in products of a whole number and a power of 10, as well as the location of the decimal point in products of decimals and powers of 10 can be explained in terms of place value. Students may connect their understanding of multiplication and exponentiation by relating to their understanding of, and computations with decimals in terms of multiples rather than powers.

This standard includes multiplying by powers of 10, including 10^2 which is 10 x 10 = 100, and 10^3 which is 10 x 10 x 10 = 1,000. Students should recognize that the power of ten denotes the number of times 10 is used as a factor. Students should have experiences working with the pattern of the number of zeros in the product when multiplying whole numbers by powers of 10 and compare that to what happens when a decimal is multiplied by a power of 10 as shown in the example. Example: $2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$. Students should reason that the exponent above the 10 indicates that you are multiplying or making the number 10 times greater that many times when you multiply by a power of 10. The decimal point does not move, the numbers are shifting around the decimal point.

A common misconception can occur as students talk about "adding zeros" to the number when multiplying. When multiplying whole numbers, this invented strategy appears to work. However, when multiplying decimals, this misconception will cause errors in the computation. By focusing on the number being multiplied getting that many times larger, the conceptual understanding is at the heart of student thinking. For example, in 25 x 10 = 250 (a student might say 25 and add/attach/append a zero) but in 2.5 x 10 = 25 this same rule does not apply (students following the rule would write 2.5 x 10 = 2.50). Instead, students should consider that 250 is ten times larger than 25 and 25 is ten times larger than 2.5

Level 4:

Provide opportunities for students to write whole numbers in expanded form using exponents. Students should explain why this works by connecting it to their previous experiences with whole numbers in expanded form. Provide opportunities for students to analyze errors and misconceptions and explain why this error probably occurred and how it should be corrected.

Standard 5.NBT.A.3 (Major Work of the Grade)

Read and write decimals to thousandths using standard form, word form, and expanded form (e.g., the expanded form of 347.392 is written as 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000)). Compare two decimals to thousandths based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the value of a digit in a multi-digit number with models. Compare two decimals to the hundredths place (when the same number of digits are in each number), using the symbols >, <, or =.	Read and write numbers to the thousandths place in at least one form (standard, word, or expanded). Compare two decimals to the hundredths place using the symbol >, <, or = with numbers that have one decimal place to numbers with two decimal places using models and/or fraction equivalence to explain their comparison.	Read and write decimals to thousandths using standard form, word form, and expanded form. Compare two decimals to thousandths based on meanings of the digits in each place, using the symbols >, <, or =.	Compare more than two decimals by ordering a series of numbers from least to greatest or greatest to least. Generate two decimals and compare using the symbols >, <, or =, then justify using models and/or fraction equivalence to explain their comparison.

Instructional Focus Statements

Level 3:

Students should build on their work from grade 4, where they worked with both decimals (through hundredths) and fractions interchangeably. This standard references expanded form of decimals with fractions included. In previous grades, student learned that they can create whole numbers in a variety of ways. For example, 234 is 2 hundreds, 3 tens, and 4 ones or 1 hundred, 13 tens, and 4 ones, etc. This same understanding will be extended to decimal numbers as they first work to make sense of the digits as representing groups of the value of the digit. Students should explain 0.523 is 5 groups of tenths (5 x 1/10) + 2 groups of hundredths (2 x 1/100) and 3 groups of thousandths (3 x 1/1000) but could also be 4 groups of tenths (4 x 1/10) + 12 groups of hundredths (12 x 1/100) and 3 groups of thousandths (3 x 1/1000). Additionally, 0.458 can be written as 4 tenths + 5 hundredths + 8 thousandths. This understanding is important for students when learning to read decimal numbers correctly. Experiences where they create that number with only thousandths grids helps them to see that 4 tenths, 5 hundredths and 8 thousandths is equivalent to 458 thousandths and why it is read that way.

Students build on the understanding they developed in grade 4 to now read, write, and compare decimals to thousandths. In previous grades, students make comparisons based on the values of the digits in each number. This understanding is extended to making comparisons based on the value of the decimal places. The decimal place value names are new learning. In grade 4, students did not learn about the tenths or hundredths place. Now, they connect to prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. In grade 4, students represented 0.72 as seventy-two hundredths or 72/100 because 7/10 + 2/100 = 70/100 + 2/100 = 72/100. Students will also need opportunities to work with concrete and pictorial representations of equivalent decimals such as 0.5 = 0.50 = 0.500 or 0.34 = 0.340. Instruction should be based on conceptual understanding and not adding or removing zeroes. Students need concrete models and number lines to extend this understanding of decimals to the thousandths place. Models may include base ten blocks, decimal grids, place value charts, drawings, manipulatives, etc. Students should read decimals using fractional language and write decimals in fractional form, as well as in expanded form. This helps support student understanding of equivalence (i.e., 0.1 = 0.10 = 1/10 = 10/100). The conceptual understanding from this standard is a building block for standard 5.NBT.A.1 as students connect what each place value represents to the fact that as digits move places, they become ten times greater or become a tenth of the value of the previous place.

Level 4:

Solve real-world problems where a data set is presented and have students order and compare the numbers in order to analyze the data. Students should be able to generate decimal numbers to compare using the symbols >, <, or =.

Standard 5.NBT.A.4 (Major Work of the Grade)
Round decimals to the nearest hundredth, tenth, or whole number using understanding of place value.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use place value understanding to round multi-digit whole numbers to the nearest whole number. Demonstrate the two whole numbers a number would fall between using a number line.	Use place value understanding to round multi-digit numbers to the nearest whole number. Round numbers to the nearest tenth and/or hundredth using a number line.	Use place value understanding to round multi-digit numbers to the nearest hundredth, tenth, or whole number.	Create a situation where it would make sense to round a number to the nearest tenth instead of whole or vice versa.

Instructional Focus Statements

Level 3:

In grade 5, it is important that students build off of the conceptual understanding of rounding developed in grades 3 and 4 when students rounded whole numbers. Rounding to the place farthest to the left is typically the easiest for students and is often the most applicable for use in estimation. Rounding to a place in the middle of the number may be more challenging for students, and it is important to continue emphasizing that conceptually, rounding is deciding which number the number to be rounded is closest to.

To aid in teaching this concept, counting routines can be beneficial in determining between what two numbers the number to be rounded falls making it easier to find the one it's closer to on a number line. For example, start at 4,570 and count by tens or start at 3.4 and count by tenths or 3.42 and count by hundredths. Once students can determine which two tenths or hundredths, etc. a number is between, it is important that students also make sense of the midpoint.

For students to solidify their conceptual understanding of rounding, students are able to visually see this best when utilizing a number line. It is imperative that students understand conceptually as opposed to being presented a set of static rules to be applied when rounding.

Level 4:

Students should be explaining the connection between place value and rounding. Additionally, they should be able to explain using appropriate mathematical vocabulary how to round a single number to multiple places and articulate when each might be more useful. For example, in a situation such as jumping 1.72 meters, I can round the number to the nearest whole (2 meters) or the nearest tenth (1.7 meters), so students should share contexts for when each of these would be helpful and why one might be more helpful (e.g., I need about 2 meters above my head so I don't hit my head but in a high jump competition 1.7 meters is more accurate for my score).

Standard 5.NBT.B.5 (Major Work of the Grade)
Fluently multiply multi-digit whole numbers (up to three-digit by four-digit factors) using appropriate strategies and algorithms.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Accurately multiply whole numbers (up to one-digit by four-digit factors) using a model and an equation. Accurately and efficiently multiply whole numbers (up to two-digit by two-digit factors) using a model and an equation.	Accurately multiply whole numbers (up to two-digit by four-digit factors) using at least one appropriate strategy or algorithm.	Accurately, fluently, and efficiently multiply whole numbers (up to three-digit by four-digit factors) using at least one appropriate strategy or algorithm.	Explain the connections that exist between place value and standard multiplication algorithms. Analyze sample work and justify why an algorithm or strategy is correct or incorrect.

Instructional Focus Statements

Level 3:

Fluency involves a mixture of just knowing some answers, finding some answers by using patterns, finding some answers by employing strategies, and finding some answers using algorithms and knowing which is the most efficient and why. In previous grades, students have been exposed to multiple strategies as they developed their conceptual understanding of multiplication. These strategies should be generalized when multiplying multi-digit whole numbers. In grade 4, students developed a conceptual understanding of multiplication using concrete representations such as an area model and array diagrams. In grade 5, students should understand the relationship that exists between concrete representations and abstract representations such as standardized algorithms.

Instruction should focus on making sense of the operation by connecting what they know to new learning. For example, students should note that 3,247 x 5 can be represented as 3,247 groups of 5 or 5 groups of 3,247 and use this to represent an equal group model or an area model. It is important that students work with partial products and also that they develop a deep understanding of how the distributive property can be visualized in an area model. This understanding is imperative as students develop an understanding of using standardized algorithms as a strategy for multiplication. Utilizing the distributive property allows numbers to be decomposed into base ten units, products of the units to be computed, and then those products to be combined. This simplifies multiplication for students so that they are multiplying a single digit by a multiple of 10, 100, 1000, which is a concept that is introduced in grade 3. This method also extends and is particularly helpful when generalizing multiplication algorithms to working with three and four-digit factors. Instruction should focus on place value language when using the algorithms. For example, when solving 247 x 348, students would say 8 groups of 7, 8 groups of 40 and 8 groups of 200 as they work through the problem. Students must connect area models and array diagrams to the algorithms as they solidify their conceptual understanding of multiplication strategies. Providing worked out problems for students to examine for accuracy and for connections between strategies is an important instructional strategy. Students should have opportunities to connect their previous work with the meanings of multiplication to problem solving situations and use estimations to ask themselves if their answer is reasonable.

Level 4:

Students should be challenged to make connections not only within the algorithms for multiplication, but also to make connections between other strategies. Students should be able to verbalize why the algorithms and strategies work. At this level, students should be able to look at a problem containing an error, find the error, fix the error, and explain the mathematical mistake that has been made.

Standard 5.NBT.B.6 (Major Work of the Grade)
Find whole-number quotients and remainders of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Identify solutions to mathematical problems involving division when given rectangular arrays or area models.	Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors when the divisor is a multiple of 10. Illustrate or explain solutions to mathematical problems involving whole number quotients and remainders with up to four-digit dividends and one-digit divisors by using equations, rectangular arrays, and/or area models.	Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors. Illustrate or explain solutions to mathematical problems involving division with whole-number quotients and remainders with up to four-digit dividends and two-digit divisors by using equations, rectangular arrays, and/or area models. Solve division problems using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. These could include, but are not limited to, partial quotients, and area models.	Explain the connections that exist between place value and division algorithms. Analyze sample work and justify why an algorithm or strategy is correct or incorrect.

Instructional Focus Statements

Level 3:

The primary instructional focus for division at grade 5 should be solidifying a student's conceptual understanding of what it means to divide in both a partitive and quotitive context (see standard 3.OA.A.2) while extending the set of strategies developed in grade 4 with standard 4.NBT.B.6. Students extend their understanding of what it means to divide with a single-digit number to discover how a two-digit divisor impacts the strategies they have previously utilized. Instruction should also focus on invented strategies paying particularly close attention to place value strategies as they will be crucial when students work with the standard algorithm in grade 6. Students are not expected to master one particular strategy over another but they should use the strategy that makes the most sense to them.

With invented strategies, it is crucial that students refer to the place value of numbers as opposed to focusing of on the digits for both the dividend and divisor. For example, in the number 1,245 students should view this as 1000 + 200 + 40 + 5 as opposed to 4 discrete digits. If this connection is not made, students will struggle to develop a conceptual understanding of division.

While the focus of instruction should be primarily on strategies, it is important to note that as students begin dividing with two-digit divisor they may need to experience a concrete learning stage using manipulatives. Base ten blocks are a particularly helpful tool as students visualize the operation of division. Once students understand and can explain division with concrete manipulatives, they are ready to progress extending to further developing the strategies worked with in grade 4. It is important that the student's level of understanding drive how they interact with division expressions. Accepting direct modeling as a necessary developmental phase allows students who are not ready for more efficient methods a way to explore the same problems as classmates who have progressed beyond this stage.

Estimation becomes more relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted. Strategies such as partial quotients and the area model allow division to continue when the original estimate must be adjusted.

As students continue working with remainders, they should focus on identifying the greatest number less than the given dividend that the divisor will evenly divide into. This can be a cognitively complex task for students in grade 5 as it is drawing on both their understanding of multiplication and division simultaneously. Instruction should be scaffolded in a way so that students work first with smaller more familiar numbers in order to develop their conceptual understanding prior to moving to larger less familiar numbers. The remainder should be reported as a whole number. The context determines the most appropriate way to report the remainder. A variety of contextual situations should be provided in which students must determine if the remainder is the answer, if the remainder is dropped, or if the quotient should be one more because of the remainder should all be included.

Level 4:

Instruction at this level should focus on students verbalizing the process of division and providing justification for why the strategy being used works. Students should be familiar with and able to use a wide variety of different strategies for division, make connections between the various methods for division, and describe how they are connected. Additionally, students should be able to look at a problem containing an error, find the error, fix the error, and explain the mathematical mistake that has been made. Students should also be able to model and/or describe a model for both quotitive and partitive division for any given problem. Within the model, they should be able to identify the parts of the model that represent the dividend, divisor, and quotient.

Standard 5.NBT.B.7 (Major Work of the Grade)

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies (Limit division problems so that either the dividend or the divisor is a whole number).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize place values of numbers after a decimal point. Proficiently use at least one strategy for solving whole number computation with all four operations.	Use concrete models or drawings (such as decimal grids) to add and subtract decimals to hundredths. Use strategies based on place value and properties of operations to add and subtract decimals to hundredths.	Use concrete models or drawings (such as decimal grids) to add, subtract, multiply, and divide decimals to hundredths. Use strategies based on place value and properties of operations to add, subtract, multiply, and divide decimals to hundredths. Explain the connection that exists between pictorial representations and computational strategies when adding, subtracting, multiplying, and dividing decimals to hundredths. Determine if a solution is reasonable using estimation strategies when adding, subtracting, multiplying, and dividing decimals to hundredths.	Solve multi-step problems that involve adding, subtracting, multiplying, and dividing decimals to hundredths. Use estimation strategies to evaluate if a given solution is reasonable to a mathematical and contextual problem and justify using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

This standard introduces decimal computation. Students have learned computation with whole numbers using all four operations and should now generalize strategies to apply them to decimal computation. Instruction should begin by developing a conceptual understanding of decimal computation by concrete strategies allowing students to understand the part to whole relationship with regard to place value. For example, students can using base-ten blocks or grid paper models to relate those models to written equations. In earlier grades students used composing and decomposing strategies to apply operations to whole numbers. In this standard students should be able to use composing and decomposing strategies to apply operations to decimal numbers. It is important that conceptual understanding is built on place value rather than to simply line up the decimal points and compute. For example 499+59 could be solved as 500+58 through compensation in the same way 2.9+3.5 can be solved as 3+3.4 through the same compensation strategy. Another example is when students use counting up as a strategy to rewrite 44 - 22.86 = ___ as 22.86 + ____ = 44. Computation models should be limited to results containing thousandths. Models resulting in computations beyond thousandths are outside of the intended scope of grade 5 which focuses on understanding place value using models rather than on using algorithms.

Students should connect previous experiences with the meaning of multiplication and division of whole numbers to multiplication and division of decimals. Students need time to make explicit connections between concrete and pictorial representations to equations involving decimal numbers. When beginning decimal division, instruction be scaffolded to use examples of dividing a decimal by a whole number and progressing to dividing a whole number by tenths and hundredths. Instruction should be limited to having either the dividend or divisor as a decimal but not both. Connections to whole number division will be imperative here. For example, 423 ÷ 4 could be interpreted as how many groups of 4 are in 423 in the same way 6 ÷ .02 could be interpreted as how many groups of 2 hundredths are in 6. There is no expectation for the standard algorithm for division until grade 6.

Attention should be given to the situation types for all operations when using contextual problems. Students should be able to determine if a solution is reasonable using estimation strategies when adding, subtracting, multiplying, and dividing decimals to hundredths. As students make connections between concrete and student-invented representational strategies and apply estimation strategies they should be able to explain their reasoning using precise mathematical vocabulary.

Level 4:

Instruction at this level should include opportunities for students to solve multi-step real-world problems using any of the four operations with a combination of whole numbers and decimal values limited to hundredths. However, some students may begin to extend their understanding of place value to include calculations beyond thousandths such as the product of hundredths by hundredths is in the ten thousandths. Students should be able to use estimation strategies to evaluate if a given solution is reasonable to a mathematical and contextual problem and justify using precise mathematical vocabulary.

Numbers and Operations-Fractions (NF)

Standard 5.NF.A.1 (Major Work of the Grade)
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
For example, $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *(In general* $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$*.)*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract fractions (including mixed numbers) with unlike denominators where one denominator is a factor of the other and a visual fraction model is provided.	Add and subtract fractions (including mixed numbers) with unlike denominators where the denominators are 6 or less and one is not a factor of the other and a visual fraction model is provided.	Add and subtract fractions (including mixed numbers) with unlike denominators.	Add and subtract multiple fractions (including mixed numbers) with unlike denominators where all denominators are unique and no more than one is a factor of another. When provided a visual fraction model, generate the addition or subtraction equation it represents, and explain why. Create a variety of visual fraction models to represent the same given unit fraction. Create visual fraction models to represent a fraction $\frac{a}{b}$ accurately.

Instructional Focus Statements

Level 3:

In grade 4, students' added fractions with denominators of 10 and 100 as an introductory standard to adding fractions with unlike denominators. Students should interact with multiple, hands-on modeling situations in order for them to internalize a wide variety of strategies so that they develop a deep conceptual understanding of what it means to add and subtract fractions. In previous grades, students learned multiple strategies when adding and subtracting whole numbers. These strategies are still applicable when students are working with fractions. Students should encounter problems that encourage them to employ these strategies.

Exposure to tools like, but not limited to, fraction strips, fraction circles, and number lines are imperative in order for students to build conceptual understanding of addition and subtraction of fractions with unlike denominators.

Students should understand that when considering two unlike denominators, each denominator can be subdivided into fractional parts represented by the other fraction. For example, in $\frac{1}{3} + \frac{3}{4}$, one third can be subdivided into fourths, and the fourths can be subdivided into thirds, rendering both as twelfths. It is valuable for students to model this process in order to develop a true conceptual understanding of how like denominators are formed. This understanding evolves over time as students realize that the product of the addend's denominators always produce a common denominator for fractions. It is not a necessity for students to find the least common denominator when adding and subtracting fractions. Students then generalize and develop an understanding that $\frac{a}{b} + \frac{c}{d} = \frac{(ad+bc)}{bd}$. This realization is a by-product of the conceptual understanding developed through implementing a variety of modeling strategies as well as a variety of addition and subtraction strategies.

By the end of grade 5, students should have a variety of strategies that they can employ in order for a student to efficiently add and subtract fractions.

Level 4:

As students develop the ability to efficiently compute with fractions, instruction should evolve to have them work with multiple addends and subtrahends with increasing variance in denominators.

Standard 5.NF.A.2 (Major Work of the Grade)

Solve contextual problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, *by observing* $\frac{3}{7}<\frac{1}{2}$.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve contextual problems referring to the same whole that involve adding and subtracting fractions with like denominators. A visual fraction model may or may not be provided.	Solve contextual problems referring to the same whole that involve adding fractions and/or mixed numbers with unlike denominators where the denominators are 6 or less and one is not a factor of the other and a visual fraction model is provided. Solve contextual problems referring to the same whole that involve subtracting fractions and/or mixed numbers with unlike denominators where regrouping is not required and the denominators are 6 or less and one is not a factor of the other when a visual fraction model is provided.	Solve contextual problems referring to the same whole involving addition and subtraction of fractions with unlike denominators. A visual fraction model may or may not be provided. Use benchmark fractions and number sense of fractions to estimate the solution to contextual problems involving addition and subtraction of unlike denominators.	Assess the reasonableness of answers to contextual problems referring to the same whole involving addition and subtraction of unlike denominators by using benchmark fractions and number sense. Create a story context for adding and/or subtracting fractions referring to the same whole with fractions with unlike denominators.

Instructional Focus Statements

Level 3:

Students should interact with contextual problems that elicit adding and subtracting fractional amounts with increasing rigor over the course of instruction. Contextual problems should be framed in multiple ways. A good resource is the “Common Addition and Subtraction Situations” document embedded in the Tennessee mathematics standards.

Additionally, students should be interacting with situations that require them to elicit number sense in order to estimate solutions using knowledge of benchmark fractions. The ability for a student to employ number sense is an invaluable skill. Students should be expected to develop this skill with fractions in the same rigor as number sense is built with whole numbers in previous grades.

Students should justify the reasonableness of solutions to contextual problems by employing number sense coupled with their understanding of benchmark fractions.

Level 4:

Students should justify the reasonableness of solutions to contextual problems by employing number sense coupled with their understanding of any fractions. Students should engage in conversations where they justify their solutions using mathematical, sound reasoning while employing estimation strategies.

Standard 5.NF.B.3 (Major Work of the Grade)

Interpret a fraction as division of the numerator by the denominator. *For example,* $\frac{3}{4}$ *= 3 ÷4 so when 3 wholes are shared equally among 4 people, each person has a share of size* $\frac{3}{4}$*.*

Solve contextual problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve contextual problems involving partitive division of whole numbers leading to answers in the form of fractions or mixed numbers where the dividend is larger than the divisor and a visual model is provided.	Solve contextual problems involving partitive division of whole numbers leading to answers in the form of fractions or mixed numbers where the dividend is larger than the divisor.	Interpret a fraction as division of the numerator by the denominator. Solve contextual problems involving both partitive and quotitive division of whole numbers leading to answers in the form of fractions or mixed numbers.	Assess the reasonableness of answers to contextual problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using number sense. Create a story context for dividing whole numbers leading to answers in the form of fractions or mixed numbers.

Instructional Focus Statements

Level 3:

Students should go through a "self-discovery process" in order to learn that a fraction can be interpreted as division. One way to accomplish this discovery is by using visual fraction models. Additionally, students can employee their understanding of division as equal sharing in order to build their conceptual understanding.

Students should interact with contextual problems that elicit dividing whole number amounts with increasing rigor over the course of instruction. Contextual problems should be framed in multiple ways. A good resource is the *Table 2 Common multiplication and division situations* document embedded in the Tennessee mathematics standards. Instruction should evolve from students modeling partitive situations to students modeling quotitive situations. Modeling may involve visual fraction models, such as fraction strips or number lines.

A partitive division problem is one in which you know the total number of groups, and you are trying to find the number of items in each group. A quotitive division problem is one in which you know the number of items in each group, and you are trying to find the total number of groups. Additionally, by the end of grade 5, students should be able to represent the contextual problem with an equation.

Level 4:

Students should justify the reasonableness of solutions to contextual problems by employing number sense coupled with their understanding of benchmark fractions. Students should engage in conversations in which they defend their solutions using mathematical, sound reasoning.

Standard 5.NF.B.4 (Major Work of the Grade)

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or a fraction by a fraction.

5.NF.B.4a Interpret the product $\frac{a}{b}$ x q as *a* x (*q* ÷ *b*) (partition the quantity *q* into *b* equal parts and then multiply by *a*). Interpret the product $\frac{a}{b}$ x q as (*a* x *q*) ÷ *b* (multiply *a* times the quantity *q* and then partition the product into *b* equal parts). *For example, use a visual fraction model or write a story context to show that* $\frac{2}{3}$ *x 6 can be interpreted as 2 x (6 ÷ 3) or (2 x 6) ÷ 3. Do the same with* $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)

5.NF.B.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Multiply a whole number by a fraction. Find the area of a rectangle with whole number lengths by tiling. A visual representation may or may not be provided.	Multiply a fraction by a whole number. Find the area of a rectangle with fractional lengths by tiling when a visual representation is provided.	Multiply a fraction by a fraction. Interpret the product ($\frac{a}{b}$) *x q* as *a* parts of a partition of *q* into *b* equal parts (*a x q* ÷*b*). Find the area of a rectangle with fractional lengths by tiling or by multiplying side lengths, and provide a visual representation to model the multiplication. Represent fractional products as rectangular areas.	Find the area of rectilinear shapes with fractional side lengths. Explain and provide a visual model to show why $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. When provided a visual fraction model, generate the multiplication equation it represents and explain why.

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Instructional Focus Statements

Level 3:

In grade 4, students multiplied a whole number by a fraction, in order to connect the concepts of multiplication by a whole number and repeated addition. Grade 5 students will extend their understanding to multiply a fraction by a whole number. Students can develop conceptual understanding of this process by partitioning a whole number into equal fractional amounts. Thus, for example, $\frac{1}{3}\ x\ 4$ (one third of 4) is one part when 4 is partitioned into three equal parts.

As students then begin thinking about multiplying fractions by fractions, the instructional focus should hinge on developing strategies that enable students to multiply fractions in flexible ways. While it is not explicitly called out within the standard, student learning should be supported with visual fraction models and number lines. Simply memorizing the rules is not sufficient.

Students should practice tiling rectangles with unit squares with fractional side length. Students need the opportunity, for example, to look at a square with side lengths of $\frac{1}{5}$ of a unit and truly conceptualize why it has an area of $\frac{1}{25}$ of a square unit. (This seems very counterintuitive for students.) Students need the opportunity to interact with area models to build conceptual understanding, which should extend beyond applying rules they may have learned for calculating area in the third grade.

Level 4:

As students develop the ability to efficiently multiply fractions utilizing multiple strategies, instruction should evolve to have students explain the underpinnings of the mathematics using sound, mathematical reasoning.

Students who have a strong conceptual understanding of finding the area of rectangles with fractional side lengths can be challenged to find areas of rectilinear shapes comprised exclusively of rectangles. This will provide an integrated application of not only multiplying fractions, but also can reinforce addition and subtraction of fractions with unlike denominators.

Standard 5.NF.B.5 (Major Work of the Grade)
Interpret multiplication as scaling (resizing).
5.NF.B.5a Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, know if the product will be greater than, less than, or equal to the factors.*
5.NF.B.5b Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product less than the given number; and relate the principle of fraction equivalence $\frac{a}{b} = \frac{a \times n}{b \times n}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Compare the size of a product to the size of one factor on the basis of the size of the other factor, when both factors are whole numbers.	Compare the size of a product to the size of one factor on the basis of the size of the other factor, when one factor is a whole number and one factor is a fraction between 0 and 1.	Compare the size of a product to the size of one factor on the basis of the size of the other factor. Explain why multiplying a number by a fraction > 1 is greater than the given number. Explain why multiplying a given number by a fraction < 1 results in a product smaller than the given number.	Relate the principle of fractional equivalence to the effect of multiplying a fraction by 1. Provide examples of two factors whose resultant product is: • Greater than both factors • Greater than one factor but less than the other factor • Less than both factors and then explain the mathematics behind why this occurs.

Instructional Focus Statements

Level 3:

Students need the opportunity to develop the understanding that when multiplying fractions, the product either gets larger or smaller depending on the value of the factors. This is a very conceptual standard. Students should be discovering the relationships that exist between the factors and the products, not simply presented a set of rules to memorize. This is not a standard for which students should necessarily be showing their computational work.

Instead, it is a standard for which students should be explaining their answer either verbally or in written form. Scaling is a concept that will show up many more times in a student's grades 6-12 scholastic career (e.g., ratios, proportions, scale drawings, similarity, etc.). It is imperative that a conceptual understanding of this important mathematical concept is developed by the end of grade 5.

Level 4:

Relating the principal of fractional equivalence to the effect of multiplying a fraction by 1 is not procedurally difficult for students to grasp. True conceptual understanding is more difficult. This should be developed with modeling using visual fraction models, particularly number lines. This idea will be present in grades/courses beyond grade 5, so it is imperative that students understand and can verbalize the result of multiplying with fractions.

Additionally, students need the opportunity to interpret and explain that the overarching theme of multiplication as scaling.

Standard 5.NF.B.6 (Major Work of the Grade)
Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve real-world word problems involving multiplication of a whole number by a fraction when a visual fraction model is provided.	Solve real-world word problems involving multiplication of a fraction by a whole number when a visual fraction model is provided.	Solve real-world word problems involving multiplication of fractions and mixed numbers, and represent them using visual fraction models and equations.	Justify solutions to real-world problems involving multiplication of fractions and mixed numbers by providing visual fraction models or equations. Create a story context for multiplication of fractions and mixed numbers.

Instructional Focus Statements

Level 3:

Students need opportunities to use modeling strategies while solving real-world problems. It is imperative that mathematical modeling is the guiding pathway for the solution to the problem and that it does not become an afterthought to the problem.

Students should interact with contextual problems that elicit multiplying fractional amounts with increasing rigor over the course of instruction. Contextual problems should be framed in multiple ways. A good resource is the "common multiplication and division situations" document embedded in the Tennessee mathematics standards. Additionally, by the end of grade 5, students should be able to represent the contextual problem with an equation.

Level 4:

Students should justify the reasonableness of solutions to contextual problems by employing number sense coupled with their conceptual understanding of scaling. Students should engage in conversations in which they justify their solutions using mathematical, sound reasoning. Additionally, students should be encouraged to develop real-world problems to match a provided expression involving the multiplication of fractions.

Standard 5.NF.B.7 (Major Work of the Grade)
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

5.NF.B.7a Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, use visual models and the relationship between multiplication and division to explain that* $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $(\frac{1}{12}) \times 4 = \frac{1}{3}$.

5.NF.B.7b Interpret division of a whole number by a unit fraction and compute such quotients. *For example, use visual models and the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \times (\frac{1}{5}) = 4$.

5.NF.B.7c Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$ *cup servings are in 2 cups of raisins?*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Divide a whole number by a unit fraction when a quotitive visual fraction model is provided.	Divide a unit fraction by a whole number when a partitive visual fraction model is provided. Solve real-word problems involving quotitive division of whole numbers by unit fractions when a visual fraction model is provided.	Divide whole numbers by unit fractions and unit fractions by non-zero whole numbers, and represent the mathematics by providing a visual fraction model. Solve real-world problems involving partitive and quotitive division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, and represent them using visual fraction models and equations.	When provided a visual fraction model, generate the division equation it represents and explain why. Conceptually explain using sound mathematical reasoning how to divide a whole number by a unit fraction and a unit fraction by a whole number. Create a partitive and a quotitive story context for the same division problem involving division of a unit fraction by a non-zero whole number or a whole-number by a unit fraction.

Instructional Focus Statements

Level 3:

The focus of these standards is dividing exclusively using whole numbers and unit fractions. The inclusion of a unit fraction makes it much easier for students to accurately model division of fractions. Students need the opportunity to develop their conceptual understanding of dividing with unit fractions and whole numbers from their understanding of division with strictly whole numbers that was built over grades 3-4. It is not naturally intuitive for students to understand why 3 divided by $\frac{1}{3}$ is 9 unless the connection is made back to the foundational understanding of division. Rote memorization of procedural steps is not enough for students to develop the conceptual understanding needed in further grades/courses.

Additionally, in real-world problems, it is important that students be exposed to both partitive and quotitive situations as they are modeled differently. These problems should increase in rigor over the course of instruction. A good resource is the "common multiplication and division situations" document embedded in the Tennessee mathematics standards.

Level 4:

The instructional focus should hinge on students demonstrating in multiple ways that they truly conceptually understand what It means to divide using whole numbers and unit fractions. Students should be able to demonstrate understanding by using visual fraction models to model the solution to an equation, creating an equation to represent a visual fraction model provided for them, creating real-world problems to be solved by a provided equation, and explaining why the standard algorithm for division with fractions works. This is not the place to provide an algorithm and ask students to memorize it. Doing so will have negative impacts on student performance in later grades/courses.

Numbers and Operations-Measurement and Data (MD)

Standard 5.MD.A.1 (Supporting Content)

Convert customary and metric measurement units within a single system by expressing measurements of a larger unit in terms of a smaller unit. Use these conversions to solve multi-step real-world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). *For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters.*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Convert to find equivalent measurements when reference tools are provided. Complete a conversion table or chart to show equivalent measurements and solve simple problems involving conversions from larger units to smaller units. Understand relative size of units in customary and metric systems.	Connect an understanding of place value relationships to relationships between customary and metric measurement units within a single system. Solve two-step real-world problems involving conversions of larger to smaller units within a single system using a variety of problem types involving distances, intervals of time, liquid volumes, masses of objects and money	Convert from a larger metric unit of measurement to a smaller metric unit of measurement using conversion factors and multiplication, including metric units of length, mass, and liquid volume. Convert from a larger customary unit of measurement to a smaller customary unit of measurement using conversion factors and multiplication, including customary units of length, weight, and liquid volume. Convert money from a larger unit to smaller unit using conversion factors and multiplication. Use conversations of larger to smaller units within a single system to solve real-world multi-step	Solve complex tasks where conversions from larger units to smaller units within the same system of measurements are required to find the solution.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		problems involving distances, intervals of time, liquid volumes, masses of objects and money including problems involving simple fractions or decimals.	

Instructional Focus Statements

Level 3:

In grade 5, students build on their prior knowledge of related measurement units to determine equivalent measurements. Students learned both metric and customary units of length in grade 2. In grade 3, students learned metric units of mass and liquid volume. In grade 4, learned both metric and customary units for length, weight, and mass, but did not learn conversions. Students need hands on experiences to establish the equivalent units of measure and to connect to the measurement vocabulary. Students should relate the metric system to our place value system. For example, students connect that the hundredths are ten times bigger than the thousandths, and a centimeter is ten times bigger than a millimeter. Students also realize that a liter is 1000 times greater than a milliliter and that a milliliter is one thousandth of a liter. Students should recognize that multiplication can be used to find equivalent units when converting units and generalize that this works in all types of measurements because it takes more of the smaller units to make up the larger unit. This is the same relationship that students make in finding equivalent fractions as they relate the size of the pieces. When forming an equivalent amount, the number of pieces/units increases as the size of the pieces/units decrease. The state TNReady assessment provides a reference table for some conversions. In grade 6, students will learn to make conversions from smaller units to larger units.

Level 4:

Instruction prior to level 4 might require students to complete the operation and then convert their answer for a final solution. Instruction at level 4 should focus on tasks that require students to convert multiple solutions in order to find the end solution. Requiring students to think about the units that will be converted prior to finding the end result, requires decision making on what should be converted and why.

Standard 5.MD.B.2 (Supporting Content)

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8), where the horizontal scale is marked off with appropriate units. Measure objects to one-eighth of a unit including length, mass, and liquid volume.	Use appropriate vocabulary when working with line plots and fractional measurements. Use approximations when measurements are between 1/2, 1/4, or 1/8 units.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) Use operations on fractions for this grade to solve problems involving information presented in line plots. (Fraction operations might include: addition and subtraction of like and unlike fractions or mixed numbers, multiplication of fractions or mixed numbers by fractions or whole numbers, and whole numbers divided by unit fractions)	Interpret the data from the line plot and solve multi-step problems, including problems that allow for the redistribution of the data. Collect data, create a line plot, and create contextual problems that can be solved using the data. Analyze and compare results of line plots measuring different attributes (such as length, weight and liquid volume) of the same object.

Instructional Focus Statements

Level 3:

Students begin their work with line plots in grade 2 as they create line plots from data provided in whole number units with an interval of one. In grade 3, students begin measuring the lengths of objects to the nearest quarter inch and then create a line plot from the data. In grade 4, students are expected to create line plots with measurements to the nearest eighth inch and solve problems using the fractions. The difference between grade 4 and grade 5 standards are the types of problems students are solving with the data. In grade 5, students solve fraction operations with addition and subtraction with like and unlike fractions or mixed numbers, multiply fractions or mixed numbers, and divide whole numbers by unit fractions. Instruction should be alongside fraction operations lessons. Students need to both analyze and create line plots. In grade 6, students build on their understanding of line plots by representing data in box plots, pie charts, and stem plots.

Level 4:

In level 4, students should demonstrate their ability to interpret and analyze the data from the graph and use a multi-step approach to redistribute the data evenly. This is not requiring students to understand or use the term "average,” which it not presented until grade 6.

Standard 5.MD.C.3 (Major Work of the Grade)

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.C.3a Understand that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume.

5.MD.C.3b Understand that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a cube with side length 1 unit as a “unit cube”. Identify that a solid figure can be packed with cubes. Identify that volume is an attribute of 3-deminsional figures.	Identify that a cube with side length 1 unit is called a “unit cube” and that it has a volume of "one cubic unit". Recognize that a solid figure can be packed without gaps to find the volume.	Identify that a cube with side length 1 unit is called a “unit cube” and that it has a volume of "one cubic unit" and that it can be used to find the volume of a solid figure. Identify that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Explain why a solid figure packed with cubes such that there are either gaps or overlaps using *n* unit cubes does not have a volume of *n* cubic units Identify if the volume represented is less than or greater than the actual volume of the solid figure and explain why when given a solid figure packed with cubes in such a way that there are gaps or overlaps.

Instructional Focus Statements

Level 3:

The major emphasis for measurement concepts in grade 5 is volume. Volume not only introduces a third dimension and thus a significant challenge to students’ spatial structuring, but also complexity in the nature of the materials measured. Solid units and liquid units act differently. That is, solid units are “packed,” such as cubes in a three-dimensional array, whereas a liquid “fills” three-dimensional space, taking the shape of the container. “Packing” volume is more cognitively difficult than iterating a unit to measure length or measuring area by tiling. It is important to note that the metric unit structure (liters and milliliters) for liquid measurement may be psychologically one-dimensional for some students. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube and understand why it is important to pack a solid figure in such a way that there are not gaps or overlaps and connect the number of cubes to the overall volume of the figure.

Level 4:

In order to deepen a student's conceptual understanding of volume, they should be working with both solid and liquid volumes in order to develop an understanding of the similarities and difference in the two. Additionally, students should move beyond simply working with volumes that are packed correctly with unit cubes to those that are not and be challenge to explain why the volume would be incorrect when calculated by counting unit cubes in these instances. Students should be able to explain the effect of both overlaps and gaps on the overall volume calculation for the solid figure.

Standard 5.MD.C.4 (Major Work of the Grade)
Measure volume by counting unit cubes, using cubic centimeters, cubic inches, cubic feet, and improvised units.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure volumes by counting unit cubes of a figure that is n units long, 1 unit in width, and 1 unit in height.	Measure volumes by counting unit cubes when a pictorial representation showing all cubes is provided.	Measure volumes by counting unit cubes when a pictorial representation is provided which may or may not show all of the unit cubes.	Measure the volume of a given solid figure by filling in units cubes with no gaps or overlaps. Explain the relationship that exist between working with liquid and solid volume.

Instructional Focus Statements

Level 3:

It is important that students experience accurately packing cubes into right rectangular prisms and subsequently counting the cubes to determine the volume. Additionally, students can build right rectangular prisms from cubes and see the layers as they build. While seemingly the same thing, both emphasize a different aspect of volume. The first supports the importance of packing without gaps or overlaps and the second can be used as a catalyst for developing an understanding of the $V = B \times h$ formula when working with standard 5.MD.C.5.

Students should be comparing the volumes of right rectangular prisms that have different dimensions. Such experiences help students extend their spatial structuring from two to three dimensions. By learning how to both mentally decompose and recompose a right rectangular prism built from cubes into layers, students develop an understanding of what volume is beyond a formula in which to plug numbers.

Another complexity of volume often taken for granted is the connection between the individual cubes that are packing the space and units of volume. Often students can determine that a box can be filled with 24 centimeter cubes, or they will be able to build a box with 24 cubes, but they still think of the 24 cubes as individuals and do not conceptually understand that the cubes are representing units of volume.

Level 4:

One of the more complex aspects of working with volume when it is newly introduced is developing an understanding of the unique differences that exist between solid and liquid volume. As liquid volume is fluid, students may not respond confidently and correctly when asked to fill a graduated cylinder marked in cubic centimeters with the amount of liquid that would fill a box constructed with cubic centimeter boxes. That is, they have not yet connected their ideas about filling volume with those concerning packing volume. This is an important connection for students to make as they develop a true conceptual understanding of volume. Comparing and discussing the volume-units and what they represent can help students learn a general, complete, and interconnected conceptualization of volume as filling three-dimensional space.

Standard 5.MD.C.5 (Major Work of the Grade)
Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume of right rectangular prisms.
5.MD.C.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent whole-number products of three factors as volumes *(e.g., to represent the associative property of multiplication).*
5.MD.C.5b Know and apply the formulas $V = l \times w \times h$ and $V = B \times h$ (where B represents the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems.
5.MD.C.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose $V = l \times w \times h$ as the formula that represents finding volume of a rectangular prism.	Find the volume of a right rectangular prism with whole-number side lengths packed with unit cubes when the pictorial representation shows all unit cubes. Solve simple real-world and mathematical problems involving calculating the volume of right rectangular prisms with whole number edge lengths when the formula to calculate volume is provided.	Find the volume of a right rectangular prism by packing it with unit cubes and equate this to finding volume by multiplying the edge lengths. Solve real world and mathematical problems involving calculating the volume of right rectangular prisms with whole number edge lengths including solving problems necessitating finding missing edge lengths when the volume is provided. Find volumes of solid figures composed of two non-overlapping	Explain the mathematical connections that exist between the various ways to find volume. Find volumes of solid figures composed of two non-overlapping right rectangular prisms when a pictorial representation without all dimensions labeled is provided. Solve real world problems involving calculating the volume of two non-overlapping right rectangular prisms.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		right rectangular prisms when a pictorial representation with all dimensions labeled is provided.	

Instructional Focus Statements

Level 3:

As students transition from counting cubes as their primary strategy for calculating volume to using the traditional formulas for calculating volume of rectangular prisms, students should discover the connection that exists between the two processes developing a conceptual understanding of why the formulas are what they are and why they work. That is, they understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism. They see that volume is a derived attribute that, once a length unit is specified, can be computed as the product of three length measurements or as the product of one area and one length measurement. Then, students can learn the volume formulas for right rectangular prisms as efficient methods for computing volume, maintaining the connection between these methods and their previous work with computing the number of unit cubes that pack a right rectangular prism.

Students then use these competencies to find the volumes of right rectangular prisms with edges whose lengths are whole numbers and solve real-world and mathematical problems involving such prisms. Students also recognize that volume is additive in much the same way that area was seen as additive in grade 3. They use this understanding to find the total volume of solid figures composed of two right rectangular prisms.

Level 4:

To demonstrate a deep conceptual understanding of volume, students should be able to provide an explanation as to the connections that exist between counting unit cubes and the formulas for calculating the volume of a rectangular prism. Additionally, students should be working with real-world and mathematical problems of increasing complexity including those involving calculating volume of two rectangular prisms where they must use deductive reasoning to determine some of the measurements of the prisms. Additionally, students could be challenged to find the volume of two overlapping rectangular prisms. Students should be challenged to apply their conceptual understanding to challenging problems that have the aspects of mathematical modeling that they will experience in the high school courses.

TN Department of Education

Geometry (G)

Standard 5.G.A.1 (Supporting Content)

Graph ordered pairs and label points using the first quadrant of the coordinate plane. Understand in the ordered pair that the first number indicates the horizontal distance traveled along the x-axis from the origin and the second number indicates the vertical distance traveled along the y-axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Plot points on a number line where hash marks are provided for each whole number and only benchmark numbers are labeled. Choose the ordered pair associated with the plotted point when given a point graphed on a coordinate plane.	Identifies the key components of the coordinate plane (x-axis, x-coordinate, y-axis, y-coordinate and origin). Identify the ordered pair associated with the plotted point when given a point graphed on a coordinate plane. Graph points on the x- or y- axis. Draw a coordinate axis.	Graph the point when given coordinates of a point located in the first quadrant of the coordinate plane. Explain that in an ordered pair the first number indicates the horizontal distance traveled along the x-axis from the origin and the second number indicates the vertical distance traveled along the y-axis. Draw and accurately label a coordinate axis.	Identify points that have the same x- or y-coordinates when given a variety of plotted points on a coordinate plane. Create a shape on a coordinate plane. Write a set of instructions, including a set of coordinates, which allow another students to accurately recreate the shape.

Instructional Focus Statements

Level 3:

Students use number lines as an introduction to the idea of a coordinate plane. They realize that a coordinate plane is constructed of a horizontal and vertical number line. Students should understand that just as points on the number line can be located by their distance from 0, ordered pairs can be used to locate and plot points on the coordinate plane in relationship to a combination of horizontal and vertical distances from the origin. Developing a student's understanding of the vocabulary that accompanies the coordinate plane should play a significant role in instruction. Necessary vocabulary includes coordinate plane, quadrant, point, line, axis (x and y), horizontal, vertical, intersection, origin, ordered pairs, and coordinate. As students are plotting order pairs for the first time in the first quadrant, it is important for them to understand that the x-coordinate moves left and right across the x-axis and the y-coordinate moves up and down the y-axis. It is a common misconception for students to get the x- and y-coordinates mixed up. Therefore, it is important that students understand that when looking at an ordered pair the first number indicates the horizontal distance traveled along the x-axis from the origin and the second number indicates the vertical distance traveled along the y-axis from the origin. Students should also understand that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). As students grasp this understanding, they should also be able to explain the difference of location for (2, 4) and (4, 2). Students should be given ample opportunity to label and graph points when working with this standard. Instruction for this standard should be integrated with instruction for standard 5.OA.B.3.

It is also important to note that there are two different ways of viewing the point (2, 3). First, think of the point as instructions: "right 2, up 3". The second is seeing the point as defined by being a distance 2 from the y-axis and a distance 3 from the x-axis. In these two descriptions the 2 is first associated with the x-axis, then with the y-axis. Students need to develop this flexibility in thinking.

Students are connecting ordered pairs of coordinates to points on the grid. This allows coordinate pairs to eventually be seen as numerical objects. In subsequent grades this will allow a single coordinate to be operated upon as a single mathematical entity despite that fact that it is composed of 2 parts.

A variety of opportunities should be provided for students to apply these experiences to integrate coordinate graphing with real-world situations such as map location skills. This allows for a nice integration with standard 5.G.A.2. It is imperative that students have a deep understanding of the coordinate plane as this is the foundation for graphing for the middle grades and high school.

Level 4:

As students will be using the coordinate system well into high school, an extensive understanding of how coordinates work with whole numbers is vital. Instruction at this level should focus on helping students make explicit connections between coordinate graphing and real-world problems that increase in rigor over time. Students should be able to not only work with coordinates on the coordinate plane, but also explain the process of graphing a coordinate point using precise mathematical vocabulary. Having students demonstrate their understanding by constructing their own images and providing a list of the related coordinate points, allows them to integrate plotting points alongside providing the coordinates for those points.

Standard 5.G.A.2 (Supporting Content)

Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Graph ordered pairs on the coordinate plane.	Choose an ordered pair that represents a given contextual situation. Choose a correct real-world interpretation for a given point graphed in the first quadrant. (Restricted as seeing the x and y axis as representative of distance.) Represent a real-world or mathematical problem by graphing or interpreting points in the coordinate plane when both the x and y axes are restricted to representing distance.	Represent real-world and mathematical problems by locating and graphing points in the first quadrant of the coordinate plane. Interpret coordinate values of points in the context of the situation.	Using data embedded in a real-world situation, graph the data, answer questions requiring interpretation of the data, and draw conclusions based on the data presented.

Instructional Focus Statements

Level 3:

In standard 5.G.A.1, students begin developing an understanding of how to graph ordered pairs and label points using the first quadrant of the coordinate plane. The focus for this standard is to build upon standard 5.G.A.1 by representing data and interpreting real-world data and mathematical problems in terms of their context.

Initially, students need opportunities to work on developing their conceptual understanding of what a graphed point means in terms of a real-world context. This will build from a student's understanding of simply identifying the coordinates to now being able to connect the values of the x- and y-coordinates to the meaning attached to each axis.

Mapping locations as ordered pairs on the coordinate plane is a common real-world situation that is used for this standard. Keep in mind that this contextual situation is an example of a context where the x- and y-axis is representative of distance (left/right and/or up/down). It is important that students are also introduced to situations where the x- and y-axis have unique quantitative measures (e.g., time and height.)

One caution, as students have not developed an understanding of rates and ratios, the focus for this standard should not involve predicting answers based on an understanding of rates. For example, if Melissa earns $10 for every hour worked, how much would she earn in 5 hours would not be an appropriate question at grade 5 as it requires students to have some knowledge of rates and ratios.

Level 4:

As students solidify their understanding, they should be able to extract data from a real-world situation, graph the data, answer questions requiring interpretation of the data, and draw conclusions based on the data presented. Ultimately, students need to experience rich problems with increasing rigor over time. Developing these critical thinking patterns will be invaluable to students in subsequent grades and courses.

Standard 5.G.A.3 (Supporting Content)
Classify two-dimensional figures in a hierarchy based on properties. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify parallel and perpendicular lines in a two-dimensional figure. Identify right, acute, and obtuse angles in a two-dimensional figure.	Classify two-dimensional figures based on the presence or absence of parallel and/or perpendicular lines. Classify two-dimensional figures based on the presence or absence of angles of a specified size.	Classify two-dimensional figures by understanding that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. Determine similarities and differences between categories and subcategories of two-dimensional figures.	Provide non-examples for given categories of two-dimensional figures. Explain whether or not a shape belongs to a particular category using precise mathematical vocabulary.

Instructional Focus Statements

Level 3:

In grades 3 and 4, students experienced classifying and categorizing shapes (standards 3. G.A.1, 3.G.A.2, and 4.G.A.2). In Grade 5 the instructional focus of this standard is for students to reason about attributes of figures and understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, a polygon is a closed figure formed with straight lines and all polygons including squares, rectangles, triangles, etc., are also closed, straight-lined figures. One common misconception for students is that figures an only belong to a single category. For example, students must understand that when classifying a square, it is not just a square, it can also be classified as a rectangle, rhombus, parallelogram, quadrilateral and also a polygon. Grade 5 students will study attributes including, sides, angles and symmetry. Instruction should focus on sorting figures where the students determine the categories. Using tools such as flow-charts and Venn diagrams can show the hierarchy of figures based on properties.

Students should use precise mathematical vocabulary such as attribute, category, subcategory, hierarchy, properties, two-dimensional, and specific names of polygons in explanations and student discourse as opposed to memorizing definitions of terms.

Level 4:

Justification plays a particularly important role in understanding and categorizing geometric figures. Posing questions such as "How do you know that is true?", "Will that always be true?", and "Is your statement true for every rectangle?" can help students to develop higher levels of geometric reasoning. Based on these types of questions, students should be able to provide examples and non-examples of figures when given the categories or attributes.