TN Department of Education

Fourth Grade Mathematics

Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at these levels demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in the same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. The provided evidence of learning statements are examples of what students will most likely be able to do and do not represent an exhaustive list.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

Operations and Algebraic Thinking

Standard 4.OA.A.1 (Major Work of the Grade)
Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Interpret the factors and product in a given whole number multiplication equations using the mathematical language of groups and objects.	Choose a comparison statement that represents a given multiplication equation. Choose a multiplication equation that represents a verbal statement of multiplicative comparison.	Interpret multiplication equations as comparisons. Represent a verbal statement of multiplicative comparison as a multiplication equation.	Represent complex, verbal statements of multiplicative comparison as equations.

Instructional Focus Statements

Level 3:

In grade 3 students worked with contextual situations involving multiplication with equal groups/repeated addition and area/arrays. In grade 4, instruction should focus on developing multiplicative reasoning so that students develop an understanding of multiplication as a comparison. It is important that students relate multiplicative reasoning to an iterative process of making multiple copies. When students see the mathematical equation 5 x 3 = 15, for example, they should view 5 as the scalar factor (number of copies) and the 3 as the multiplicative unit (size of the group being copied).

Students understanding of multiplication is enhanced when they have opportunities to think about and model it in different ways. Phrases such as “twice as many cups” or “five times as much money” motivate students to develop a concept of multiplication that builds on their informal understanding of these situations and helps them interpret other descriptions of multiplication.

As students interact with contextual problems, models that represent the situation will also be helpful to students in understanding this meaning of multiplication. Additionally, they help students differentiate between the various types of multiplication situations.

Level 4:

Students at this level can be challenged with representing verbal complex statements that involve multiplicative comparison as equations. This challenges students to think through exactly what is said in order to accurately capture the mathematics in an equation. This type of thinking will support student development of algebraic thinking in subsequent grades.

Standard 4.OA.A.2 (Major Work of the Grade)

Multiply or divide to solve contextual problems involving multiplicative comparison, and distinguish multiplicative comparison from additive comparison. For example, school A has 300 students and school B has 600 students: to say that school B has two times as many students is an example of multiplicative comparison; to say that school B has 300 more students is an example of additive comparison.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Multiply or divide to solve simple contextual problems involving multiplicative comparison when provided instructional supports such as visual models.	Choose contextual problems that involve multiplicative comparisons. Multiply or divide to solve contextual problems that have been identified as involving a multiplicative comparison.	Multiply or divide to solve contextual problems involving multiplicative comparison. Distinguish between multiplicative comparison and additive comparison.	Multiply or divide to solve two-step contextual problems involving multiplicative comparison.

Instructional Focus Statements

Level 3:

In previous grades, students worked with tape diagrams/bar models and other models to represent additive comparison situations. Additionally, in grade 3, students worked with contextual problems involving equal groups/repeated addition and area/arrays and identified these with the operation of multiplication. Instruction for this standard is twofold in that it must focus on helping students differentiate multiplicative comparison from additive comparison while also expanding the understanding to a new way to demonstrate a multiplicative relationship. In multiplicative comparison problems, one factor identifies the quantity in one group, while the other factor is the scalar factor. This standard builds on standard 4.OA.A.1.

The relational language in multiplicative compare problems is difficult for students, especially those for whom English is a second language. Students make sense of both the language and the relationships the language implies by discussing and modeling these problems. Comparing and contrasting the language of additive relationships (more than, less than) with that of multiplicative relationships (times more than, times as many) will be helpful to students. Students need a variety of problems to model and discuss. In multiplicative comparison problems, either the product is unknown, the factor (size of each group) is unknown, or the factor (number of groups) is unknown.

Instruction should intentionally provide mixed additive and multiplicative comparison situations to help students distinguish between the two types of comparisons. Students must be given opportunities to decide what they know from the information in the problem and help focus on the question to help make sense of the problem. Visual models using concrete materials, pictures, words and numbers can help students relate the information in the problem to the mathematics of the situation.

Level 4:

As students deepen their understanding of contextual situations and become proficient at determining the difference between additive and multiplicative comparison problems, they can be challenged with two-step contextual problems that involve multiplicative comparison situations. This will help support their understanding as they interact with multi-step contextual problems involving all for operations in standard 4.OA.A.3.

Standard 4.OA.A.3 (Major Work of the Grade)

Solve multi-step contextual problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve two-step contextual problems using the four operations in which the unknown is in a variety of positions. Represent two-step contextual problems using equations with a letter standing in for the unknown quantity.	Interpret remainders in one-step division contextual problems. Represent two-step contextual problems with an equation with a letter standing for the unknown quantity. Use an estimation strategy to assess the reasoning of answers in two-step contextual problems.	Solve multi-step contextual problems posed with whole numbers using any combination of the four operations. Interpret remainders in multi-step contextual problems. Represent multi-step contextual problems with an equation using a letter for the unknown quantity. Assess the reasonableness of answers using estimation strategies.	Use mental computation and estimation strategies to present a reasonable solution to a given contextual multi-step problem, solve the contextual problem including a representation using equations with a letter standing in for the unknown quantity, and compare the original estimation and the actual answer providing mathematical justification for any discrepancies. Create multi-step contextual problems using the four operations.

Instructional Focus Statements

Level 3:

In grade 3, students solved two-step contextual problems with all four operation. In grade 4, students build on these experiences to extend student thinking as they solve multi-step contextual problems using all four operations including for the first time problems in which remainders must be interpreted.

In transitioning all students to working with multi-step, multi-operation contextual problems, instruction should initially focus on problems involving smaller, familiar numbers allowing students to focus on the conceptual understanding of multiple operations within the problem as opposed to focusing on computation with less familiar numbers. Initially, instruction involving problems which require interpreting remainders should focus on helping students understand why the remainder must be interpreted and how that interpretation effects their solution as opposed to focusing on correct solutions.

It is important to call out that students should continue to use manipulatives, multiple strategies, and written equations when solving multi-step contextual problems. To demonstrate their understanding, they should be able to explain the connections between the visual representation and the equation(s) that represents the problem. Additionally, students should be encouraged to use multiple strategies and make connections between each strategy. For example, students may write individual equations for each step in a multi-step problem or write all steps in one equation. This is a good opportunity for students to compare their work to others and explain why both are correct or in some cases incorrect and explain the connection between the two strategies. The instructional focus should be more on students understanding multi-step problems and sense making as opposed to simply getting a correct answer.

Teaching key words to associate with addition, subtraction, multiplication, and division should not be an instructional focus. Instruction should focus on developing an understanding of what operation is needed to solve the problem rather than focusing on key words that sometimes, but not always, associate with the operation.

Instruction should also focus on encouraging students to assess the reasonableness of their answers. Students should use estimation strategies and mental computations as they consider reasonableness. One beneficial instructional strategy is for students to estimate a solution prior to solving the problem.

Level 4:

As students deepen their understanding of multi-step contextual problems, they should be able to represent these problems with a mathematical drawing, diagram, and equations with a letter for the unknown number. They should be able to explain their thinking using multiple representations and make connections between the visual representations and their equations. Students should be able to use mental computation and estimation strategies to present a reasonable solution, solve the problem, and then compare the original estimation and the actual answer providing mathematical justification for any discrepancies.

Additionally, students should be able to create their own multi-step contextual problem and explain the solution. When doing so, students should use visual presentations, equations, and precise mathematical vocabulary.

Standard 4.OA.B.4 (Supporting Content)

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose one factor pair for a given whole number in the range 0-100. Identify a prime whole number in the range 0-20. Identify a composite whole number in the range 0-20.	Choose all factor pairs for a given whole number in the range of 1-100. Choose a whole number in the range of 1-100 that is a multiple of a given one-digit number. Mathematically explain the difference between prime and composite numbers.	Find all factor pairs for a whole number in the range of 1-100. Recognize and explain why a whole number is a multiple of each of its factors. Determine whether a whole number in the range of 1-100 is a multiple of a given one-digit number. Determine whether a whole number in the range of 1-100 is prime or composite.	Find all factor pairs for a whole number in the range of 1-100 and explain how one factor pair can be used to identify others.

Instructional Focus Statements

Level 3:

In grade 3, students developed an understanding that any given number can have multiple factors as they developed fluency with multiplication within 100. For example, they noticed that 36 can be represented as 4 x 9 as well as 6 x 6. In grade 4, students build on this understanding as they find all factor pairs for any whole number in the range 1-100.

Initially, instruction should focus on having students work with manipulatives such as tiles to build arrays and help them identify factors. These arrays can help students find factors as they reorganize the arrays in different configurations. As students experiment with this, they will begin to develop organization for testing factors and ensuring that they have all the factor pairs. Eventually students develop a systematic process for finding factor pairs so they can do this for any number. As students have had less experience working with larger numbers such as 51, 69, and 93, having a systematic approach will lessen the chance that factor pairs will be missed. Having a systematic approach does not mean that instruction should focus on procedures with no connection. It is important that students discover and develop a process that works for them. Students should also begin connecting how factor pairs that yield the same product are related to one another. For example, how are 9 x 4 and 6 x 6 related to one another? These realizations should help students as they generated factor pairs for more unfamiliar numbers.

Through exploration, students should discover patterns that exist between products and factor pairs. They may discover some of the following trends:

- All even numbers are multiples of 2 so all even numbers have a factor of 2.
- All factors of 10 end in a zero have 10 as a factor and are a multiple of 10.
- All even numbers that can be halved twice (with a whole number result) are multiples of 4 and have a factor of 4.
- All numbers ending in 0 or 5 are multiples of 5 and have a factor of 5.
- If the sum of the digits is divisible by 3 then the number has a factor of 3.
- If 2 and 3 are factors of a number, then 6 is a factor of the number.

These number patterns to help students develop a deeper understanding of factors, multiples, primes and composites.

As students notice that there are some numbers with only two factors, they begin to differentiate between prime and composite numbers. Students should investigate whether numbers are prime or composite by through a variety of activities such as building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g., 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number) and by finding factors of the number and noting that all multiples of a prime number are all composite numbers.

Level 4:

Students at this level should be challenged to find all factor pairs for a whole number in the range of 1-100 and follow that with an explanation of how one factor pair can be used to identify others. For example, students not only recognize that 9 x 4 and 6 x 6 are factor pairs for 36, but also discover and can mathematically explain the relationship that 3 x 3 =9 and 2 x 2 =4 thus using these alongside the associative property 9 x 4 = (3 x 3) x (2 x 2) = (3 x 2) x (3 x 2) = 6 x 6. This flexibility of thinking helps set a foundation for algebraic thinking in subsequent grades.

Standard 4.OA.C.5 (Supporting Content)

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify arithmetic patterns in growing and repeating number patterns, and explain the pattern using properties of operations.	Generate the next number or shape when given a pattern that follows a given rule. Choose apparent features of a given pattern that are not necessarily explicit.	Generate a number or shape pattern that follows a given rule. Self-identify features of the pattern that are not necessarily explicit.	Generate a number or shape pattern that follows more than one rule. Identify the rule that is being used to generate a given number of shape pattern. After identifying a pattern, determine if a number would belong in the sequence without generating the entire pattern. Identify features of the pattern that are not necessarily explicit and explain why those features exist.

Instructional Focus Statements

Level 3:

In many instances, patterns are a way in which students connect with the world around them. They have looked at and extended patterns as a way to build spatial recognition skills since they were very young. Patterns are one way in which children look for and find mathematical beauty in and make sense of the world around them. Students interacted with patterns in grade 3 where they identified arithmetic patterns and explained discovered patterns using properties of operations. Instruction for grade 4 should focus on students being able to generate a number or shape pattern that follows a

given rule. The more challenging piece of this standard for students is the expectation that they identify apparent features of the pattern that were not explicit in the rule itself. This portion requires students to use the same level of curiosity developed in grade 3 to look beyond the pattern provided to discover new relationships that exist within the members of the pattern. For example, in a pattern such as 21, 25, 29… following the rule “add four”, students might notice that the members in the pattern are all odd numbers and that each number is an increasing multiple of 4.

As many mathematical concepts build on patterns and students ability to identify those patterns, the more experience children have with looking for and making sense of patterns the better prepared they are to access concepts requiring patterned thinking in subsequent grades. As students notice and wonder about the patterns they see in the tables and in number sequences, continue to ask questions such as, " How do you know?" and "Does that always work?" to push their thinking about patterns.

Level 4:

As students develop a deeper conceptual understanding of patterns and the mathematics behind them, they should be able to work with increasingly more complex arithmetic patterns involving multiple rules. Additionally, they should be challenged to identify multiple features of given patterns that are not explicitly part of the rule. Students to be challenged to look for the mathematical relationships that exist within the initial rule and new relationships that are discovered. For example, they should be challenged to think about and justify why a pattern such as 21, 25, 29… following the rule “add four”, has members that are all odd numbers and why they are also increasing multiples of 4. The ability to identify and extend patterns is critical as students begin to think algebraically in subsequent courses.

Numbers and Operations in Base Ten (NBT)

Standard 4.NBT.A.1 (Major Work of the Grade)

Recognize that in a multi-digit whole number (less than or equal to 1,000,000), a digit in one place represents 10 times as much as it represents in the place to its right. *For example, recognize that 7 in 700 is 10 times bigger than the 7 in 70 because 700 ÷ 70 = 10 and 70 x 10 = 700.*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Multiply a one-digit whole number by a multiple of 10.	Multiply a power of 10 by 10.	Identify that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	Generate a multi-digit number that has a specified digit 10 times greater than that same digit in a provided multi-digit number and explain the reasoning behind the generated number.

Instructional Focus Statements

Level 3:

The instructional focus for this standard should be discovery learning leading students to the realization that a digit in one place represents 10 times what it represents in the place to its right. It is important for students to use their understanding of both multiplication and division while developing and solidifying this understanding. This standard is a cornerstone standard for students as they develop their conceptual understanding of the base ten number system. There are multiple standards in subsequent grades that hinge on a student not only knowing this standard, but also conceptually understanding it as well.

Level 4:

At this level, the focus of instruction should be for students to move beyond recognizing when a digit is ten times the value of a digit to its right to generating numbers such that this occurs. Additionally, students should be able to explain why this occurs using appropriate mathematical vocabulary.

Standard 4.NBT.A.2 (Major Work of the Grade)

Read and write multi-digit whole numbers (less than or equal to 1,000,000) using standard form, word form, and expanded form (e.g., the expanded form of 4256 is written as 4 x 1000 + 2 x 100 + 5 x 10 + 6 x 1). Compare two multi-digit numbers based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the word form of the number or vice versa, given a multi-digit whole number. Compare two three-digit numbers using >, =, and < symbols to record the results of comparisons.	Choose the expanded form of the number or vice versa, given a multi-digit whole number. Compare two multi-digit numbers which do not have the same number of digits using <, >, and = symbols.	Read multi-digit whole numbers written in standard form or word form. Given a multi-digit number in any form (standard form, word form, or expanded form) write the numeral in the other two forms (numeric form, word form, or expanded form). Compare two multi-digit numbers using >, =, and < symbols.	Generate a six digit number with a different digit in each place (i.e. 123,456) in any form (standard form, word form, or expanded form) and write the numeral in the other two forms (numeric form, word form, or expanded form). Order several multi-digit numbers from least to greatest or greatest to least when at least two of the numbers have the same number of digits. Explain how to compare numbers using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

Students began developing a conceptual understanding of what it means to compare numbers in previous grades. In grade 4, students should build upon and extend their previous understanding to include a wide range of multi-digit numbers. It is important that students apply their previously built

conceptual understanding of number comparisons and that they make comparisons based on the values of the digits in each number. They should be using tools such as number lines coupled with their place value understanding to provide mathematically accurate justifications for their solutions.

As to reading and writing multi-digit numbers in multiple forms, special attention should be paid to developing understanding of the commas that appear in numbers. Students need to understand the purpose of the commas in both how to read and write numbers. Additionally, expanded form is new for students at this grade. This is the first time that students will work with numerical values written in this way. Instruction should build from the decomposition of numbers students have used as strategies for working with numeric operations. For example, students should begin decomposing a number like 5,234 into 5,000 + 200 + 30 +4 as a scaffold prior to completing writing a number in expanded form. Students should be able to not only write numbers in expanded form, but also explain when expanded form might be useful.

Level 4:

Instruction at this level should focus on solidifying students' conceptual understanding of comparing numbers. This can be accomplished by having students order a set of 3 or more numbers where the number sets become increasingly more rigorous over time and students providing a mathematical justification for their thought process. In general, students should be able to explain their thinking process for ordering and comparing numbers using appropriate mathematical vocabulary.

For reading and writing numbers in multiple forms, students should be able to connect the similarities and differences between each of the three forms and explain the need and use for each of the three using appropriate mathematical vocabulary.

Standard 4.NBT.A.3 (Major Work of the Grade)

Round multi-digit whole numbers to any place (up to and including the hundred-thousand place) using understanding of place value.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Round multi-digit whole numbers to the nearest 10.	Use place value understanding to round to the highest place value. Round multi-digit whole numbers to the nearest 100.	Round multi-digit whole numbers to any place value.	Explain how rounding a six digit number to one place differs from rounding to a different place using a number line as a model to justify their thinking. Create a situation where it would make sense to round a multi-digit number to tens place instead of hundreds place or vice-versa.

Instructional Focus Statements

Level 3:

In grade 4, it is important that students build off of the conceptual understanding of rounding developed in grade 3. As in grade 3, rounding to the place furthest to the left is typically the easiest for students and is often the most applicable for use in estimation. Rounding to a place in the middle of the number may be more challenging for students. Due to the uniformity of the base ten number system, the same methods work thus teachers need to help students make this connection. That said, it is important to continue emphasizing that conceptually rounding is deciding which number the number to be rounded is closest to. For students to solidify their conceptual understanding of rounding, students are able to visually see this best when utilizing a number line. It is imperative that students understand conceptually as opposed to being presented a set of static rules to be applied when rounding.

Level 4:

Students should be explaining the connection between place value and rounding. Additionally, they should be able to explain using appropriate mathematical vocabulary how to round a single number to multiple places and articulate when each might be more useful.

Standard 4.NBT.B.4 (Major Work of the Grade)

Fluently add and subtract within 1,000,000 using appropriate strategies and algorithms.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Accurately add within 100 using the standard algorithm. Accurately subtract within 100 using the standard algorithm.	Accurately add multi-digit whole numbers when it is not necessary to compose any numbers using the standard algorithm. Accurately subtract multi-digit whole numbers when it is not necessary to decompose any numbers using the standard algorithm.	Accurately add multi-digit whole numbers using the standard algorithm. Accurately subtract multi-digit whole numbers using the standard algorithm.	Explain the connections that exist between place value and the standard algorithms for addition and subtraction. Explain why the standard algorithm for addition works using appropriate mathematical vocabulary. Explain why the standard algorithm for subtraction works using appropriate mathematical vocabulary. Explain the connections between the standard algorithm of addition and the standard algorithm for subtraction.

Instructional Focus Statements

Level 3:

Fluency involves a mixture of just knowing some answers, finding some answers by using patterns (e.g., “adding 0 yields the same number”), finding some answers by employing strategies, and finding some answers using algorithms and knowing which is the most efficient and why. In previous grades,

students have been exposed to multiple strategies as they developed their conceptual understanding of addition and subtraction. These strategies should be generalized when adding and subtracting within 1,000,000 in grade 4. In grade 4, students should be able to understand and explain the standard algorithms for both addition and subtraction by applying what they have previously learned in order to conceptually understand the inner workings of the algorithms. It is important that students not only know how to use the algorithm, but also that they can understand and explain why the algorithms work. Additionally, students should understand and be able to explain the connection between previous learned addition and subtraction strategies and the standard algorithm.

Level 4:

Students should be challenged to make connections not only within the algorithm for exclusively addition or subtraction, but also to make connections between the two standard algorithms. Students should be able to verbalize why the algorithms work. At this level students should be able to look at a problem containing an error, find the error, fix the error, and explain the mathematical mistake that has been made.

Standard 4.NBT.B.5 (Major Work of the Grade)

Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Multiply within 100 using strategies such as the relationship between multiplication and division or properties of operations. Multiply a whole number of up to three digits by a one-digit whole number when a rectangular array or area model is provided.	Multiply a whole number of up to four digits by a one-digit whole number when a rectangular array or area model is provided. Multiply a two-digit whole number by a two-digit whole number when a rectangular array or area model is provided.	Multiply a whole number of up to four digits by a one-digit whole number. Multiply two two-digit numbers. Illustrate and explain multiplication using equations, rectangular arrays, or area models.	Illustrate and explain multiplication using equations, rectangular arrays, and area models.

Instructional Focus Statements

Level 3:

In working with multiplication, students should use methods that they understand and that they can explain. In grade 4, students are developing a conceptual understanding of the process of multiplication. Visual representations such as an area model and array diagrams reinforce for students what is mathematically occurring. Ideally in subsequent grades students will see the standard algorithm for multiplication as a summary of their conceptual understanding developed in grade 4.

It is important that students work with partial products and develop a deep understanding of how the distributive property and multiplication can be connected. This understanding will be crucial for students when they begin working with and trying to understand the standard algorithm in subsequent grades. Utilizing the distributive property allows numbers to be decomposed into base ten units, products of the units to be computed, and then those products to be combined. This simplifies multiplication for students so that they are multiplying a single digit by a multiple of 10, 100, 1000, which is a concept that is introduced in grade 3. This method also extends and is particularly helpful when working with two-digit by two-digit multiplication.

Students can connect area models and array diagrams to numerical work in order to help develop their conceptual understanding of multiplication methods.

Level 4:

Instruction at this level should focus on students verbalizing the process that they are using for multiplication. Students should be familiar with and able to use a wide variety of different strategies for multiplication, make connections between the various methods for multiplication, and describe how they are connected. Additionally, students should be able to look at a problem containing an error, find the error, fix the error, and explain the mathematical mistake that has been made.

Standard 4.NBT.B.6 (Major Work of the Grade)

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Divide within 100 using strategies such as the relationship between multiplication and division or properties of operations.	Find whole-number quotients with up to four-digit dividends and one-digit divisors when the solution does not generate a remainder.	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Illustrate or explain solutions to mathematical problems involving whole number quotients and remainders with up to four-digit dividends and one-digit divisors by using equations, rectangular arrays, or area models.	Illustrate and explain solutions to mathematical problems involving whole number quotients and remainders with up to four-digit dividends and one-digit divisors by using equations, rectangular arrays, and area models.

Instructional Focus Statements

Level 3:

The instructional focus for division at grade 4 should be on developing a student's conceptual understanding of not only what it means to divide a multi-digit number by a single digit number in both a partitive and quotitive context, but also providing a set of strategies that can be employed in order to divide a multi-digit number by a single digit number.

The initial instructional focus should be on partitive division incorporating partitioning beginning with the largest place in the dividend. For example, 145 divided by 5 should be viewed as partitioning each of 100, 40, and 5 into 5 equal groups in that order and then finding the sum within 1 partition to determine the quotient. It is very important that students recognize the digit 1 within this number as having a value of 100. If this connection is not made, students will not develop a conceptual understanding of division. One other note, students should be allowed time to develop a conceptual

understanding of division without remainders in both partitive and quotitive situations before the concept of remainders is introduced. Quotitive division may be more difficult to model with larger dividends and single digit divisors. For example, in the previous example, 145 divided by 5, modeling the quotitive understanding would entail creating as many qroups of 5 as possible and then counting to realize that 29 groups could be made.

When students do begin working with remainders, they should focus on identifying the greatest number less than the given dividend that the divisor will evenly divide into. This can be a cognitively complex task for grade 4 students as it is pulling on both their understanding of multiplication and division simultaneously. Instruction should be scaffolded in a way so that students work first with smaller, more familiar numbers in order to develop their conceptual understanding prior to moving to larger less familiar numbers. It is important to note, that decimal notations resulting from division are not appropriate at grade 4 as decimals are not a focus until subsequent grades. The ultimate instructional focus in grade 4 should be centered on students conceptually understanding the process of division.

Level 4:

Instruction at this level should focus on students verbalizing the process of division and providing justification for why the strategy being used works. Students should be familiar with and able to use a wide variety of different strategies for division, make connections between the various methods for division, and describe how they are connected. Additionally, students should be able to look at a problem containing an error, find the error, fix the error, and explain the mathematical mistake that has been made. Students should also be able to model and/or describe a model for both quotitive and partitive division for any given problem. Within the model, they should be able to identify the parts of the model that represent the dividend, divisor, and quotient.

Numbers and Operations-Fractions (NF)

Standard 4.NF.A.1 (Major Work of the Grade)

Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{axn}{bxn}$ or $\frac{a \div n}{b \div n}$ by using visual fraction models, with attention to how the number and size of the parts differ, even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

For example, $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize and model simple equivalent fractions with denominators of 2, 3, 4, 6, and 8.	Recognize and model equivalent fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Generate equivalent fractions with denominators of 2, 3, 4, 6, and 8. Choose fractions equivalent to $\frac{a}{b}$ that are written in the form $\frac{n \times a}{n \times b}$.	Generate equivalent fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \div n}{b \div n}$.	Provide a model and use it to explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$. Provide a model and use it to explain why a fraction *a*/*b* is equivalent to a fraction $\frac{a \div n}{b \div n}$.

Instructional Focus Statements

Level 3:

Students should develop an understanding of how to use area models and number lines to reason about fractional equivalence. They should discover the multiplicative relationship between equivalent fractions and multiplying the numerator and denominator by the same non-zero number. Students should understand and explain that this corresponds to physically partitioning each unit fraction piece into *n* smaller equal pieces. The whole is then partitioned into n times as many pieces, and there are n times as many smaller unit fractions pieces as the original fraction. This fundamental property can also be presented in terms of division. Be careful not to over-emphasize the importance of simplifying fractions. There is no mathematical reason why fractions must be written in simplified form.

Level 4:

Students should be providing logical justification using multiple visual fraction models in order to explain fractional equivalence. Students should supply sound justification that demonstrates an understanding of how using multiplication relates to the use of division when creating equivalent fractions. Additionally, they should have a logical explanation of how partitioning is involved.

Standard 4.NF.A.2 (Major Work of the Grade)

Compare two fractions with different numerators and different denominators by creating common denominators or common numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Compare two fractions with the same numerator or same denominator with symbols >, =, or <.	Compare two fractions with different numerators and different denominators with symbols >, =, or < when a visual representation is provided.	Compare two fractions with different numerators and different denominators by creating common numerators, comparing common denominators, and using benchmark fractions. Record the comparison of two fractions with symbols >, =, or <. Select a justification for why two fractions are <, >, or = to each other.	Provide a justification for why two fractions are <, >, or = to each other. Create a problem to represent when two seemingly equivalent fractions would not be equal and explain why (e.g., provide a situation when $\frac{2}{4}$ would not equal $\frac{1}{2}$ and explain why). Order more than 2 fractions from least to greatest or greatest to least using appropriate symbols and provide a justification.

Instructional Focus Statements

Level 3:

Students employ their understanding of equivalence of fractions as a stepping stone for developing an understanding of how to compare fractions. There are multiple ways to look at these comparisons. Students may consider like denominators as one method. Additionally, it is important to build an understanding of how using benchmark fractions such as $\frac{1}{2}$ and 1 can provide a foundation and another method for fractional comparisons. It is important to encourage students to employ number sense as they are comparing fractions as opposed to a rote set of memorized procedural steps.

Additionally, students should continue building on the idea that fractions are numbers, which was emphasized in grade 3, coupled with a conceptual understanding of how students compared whole numbers using number lines. The end result will be students developing a conceptual understanding of fractional comparisons. Number lines are a valuable tool in building a conceptual understanding of comparing fractions.

Level 4:

Students should move beyond simply comparing two fractions to ordering a set with increasing variance in denominators and providing justification both verbally and in written form for why they have ordered fractions in a particular way. Holistically, students should be demonstrating a deep conceptual understanding of how fractions are compared and should be able to describe multiple strategies that can be used to reason about the size of fractions.

Standard 4.NF.B.3 (Major Work of the Grade)

Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. *For example,* $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.

4.NF.B.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.B.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$), recording each decomposition by an equation. Justify decompositions by using a visual fraction model.

4.NF.B.3c Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.B.3d Solve contextual problems involving addition and subtraction referring to the same whole and having like denominators.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Choose one way to decompose a fraction into a sum of fractions with the same denominator. Represent a fraction $\frac{a}{b}$ on a number line diagram when a pre-partitioned number line in b equal parts is provided, and choose an addition equation to represent the fraction as a sum of unit fractions. Add and subtract fractions with like denominators when both fractions are less than 1 when a visual representation of the addition/subtraction is provided. Solve one-step contextual problems involving addition or subtraction of	Choose multiple ways to decompose a fraction into a sum of fractions with the same denominator. Decompose a fraction into a sum of fractions with the same denominator in only one way. Add and subtract mixed numbers with like denominators when composing or decomposing a whole number is not required and a visual representation is provided. Solve one-step contextual problems involving addition or subtraction of simple, like-denominator fractions referring to the same whole when	Explain that addition of fractions is the joining of parts referring to the same whole and that subtraction is separating parts referring to the same whole with and without a visual fraction model to justify the explanation. Decompose a fraction into a sum of fractions with the same denominator in more than one way and record the decomposition by an equation. Add and subtract mixed numbers with like denominators where composing or decomposing whole numbers is required.	Provide an example and demonstrate why when adding and subtracting fractions, the fractions must refer to the same whole. Solve multi-step contextual problems involving addition and/or subtraction of like-denominator fractions referring to the same whole where composing or decomposing whole numbers are required.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
simple, like-denominator fractions less than 1, referring to the same whole when composing or decomposing a whole number is not required, and a visual representation is provided.	composing or decomposing a whole number is not required, and a visual representation is provided.	Solve one-step and two-step contextual problems involving addition or subtraction of like-denominator fractions referring to the same whole.	

Instructional Focus Statements

Level 3:

Students should build their understanding of adding and subtracting fractions from their previous understanding of adding and subtracting whole numbers. For example, the sum of 2 and 3 can be seen as the length of the segment obtained by joining two segments of length 2 and 3, so the sum of $\frac{1}{4}$ and $\frac{5}{4}$ can be seen as joining together two segments of length $\frac{1}{4}$ and $\frac{5}{4}$. Number lines are a natural fraction model to use as an extension of the understanding of adding and subtracting whole numbers on number lines.

The simple understanding of addition as "putting together" allows students to see how fractions are built from unit fractions. Students can then compose and decompose fractions in various ways using unit and non-unit fractions with the same denominator in order to find the sum. They can then use this fundamental understanding in order to add and subtract fractions with like denominators. It is important that students model addition and subtraction with visual fraction models including, but not limited to, number lines and area models. Additionally, there are numerous strategies based on the properties of operations that students learned when adding and subtracting whole numbers in previous grades. Students should be able to employ these same strategies when adding and subtracting fractional amounts.

In order to rewrite a mixed number as an equivalent fraction, students should see $2\frac{1}{3}$ as $2 + \frac{1}{3} = \frac{6}{3} + \frac{1}{3}$, representing the mixed number as an addition of fractions with like denominators as opposed to a unique rote process that holds no conceptual meaning to students. Decomposing a fraction greater than 1 into an equivalent mixed number is a matter of decomposing the fraction into the sum of a whole number expressed as a fraction and a fraction less than 1. For example, $\frac{12}{8}$ is equivalent to $\frac{8}{8} + \frac{4}{8}$ or $1\frac{4}{8}$. Rote procedures should not be used, as they do not develop conceptual understanding for students. Students should interact with contextual problems that elicit adding and subtracting fractional amounts with like denominators, with increasing rigor over the course of instruction. Contextual problems should be framed in multiple ways. A good resource is the "common addition and subtraction situations"

document embedded in the Tennessee mathematics standards. Additionally, students should be encouraged to model the mathematics using visual fraction models.

Level 4:

Students should be demonstrating a strong conceptual understanding of what it means to add and subtract fractions by justifying their work with multiple types of visual fraction models. Students should also be able to employ a number of addition and subtraction strategies when working with fractions. Additionally, students should be able to explain both verbally and in writing when and how mixed numbers need to/can be composed or decomposed for addition and subtraction.

Students should justify the reasonableness of solutions to contextual problems by employing number sense coupled with their understanding of benchmark fractions. Students should engage in conversations in which they justify their solutions using logical mathematical reasoning.

Standard 4.NF.B.4 (Major Work of the Grade)

Apply and extend previous understandings of multiplication as repeated addition to multiply a whole number by a fraction.

4.NF.B.4a Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times \frac{1}{4}$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \times \frac{1}{4}$.

4.NF.B.4b Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a whole number by a fraction. *For example, use visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \times \frac{a}{b} = \frac{n \times a}{b} = (n \times a) \times \frac{1}{b}$.)

4.NF.B.4c Solve contextual problems involving multiplication of a whole number by a fraction (e.g., by using visual fraction models and equations to represent the problem). *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be 4 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Choose $a \times \frac{1}{b}$ as an accurate representation of $\frac{a}{b}$ when a visual fraction model is provided.	Choose $a \times \frac{1}{b}$ as an accurate representation of $\frac{a}{b}$. Choose a multiple of $\frac{1}{b}$ to represent a multiple of $\frac{a}{b}$. Solve one-step contextual problems involving multiplication of a whole number by a fraction when a visual fraction model is provided.	Provide a visual fraction model to show a representation of $\frac{a}{b} = a \times \frac{1}{b}$. Provide a multiple of $\frac{1}{b}$ to represent a multiple of $\frac{a}{b}$. Multiply a whole number by a fraction, and provide a visual fraction model to justify their answer. Solve one and two-step conceptual problems involving multiplication of a whole number by a fraction, and represent the problem with a visual fraction model.	Solve multi-step contextual problems involving multiplication of a fraction by a whole number, and represent the solution with an equation or visual fraction model. Justify solutions to contextual problems involving multiplication of a whole number and a fraction by providing visual fraction models or equations. Create a story context for multiplication of a whole number by a fraction.

Instructional Focus Statements

Level 3:

Students should apply their understanding of multiplication developed in grade 3, as the number of objects represented in groups, to multiply a whole number by a fraction in grade 4. They must see that the operation of multiplication is the same regardless of whether both of the factors are whole numbers or if one of the factors is a fraction. Thus, they should view $3 \, x \, \frac{1}{4}$ as three groups of $\frac{1}{4}$ or $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. This then becomes an application of addition with like denominators and reinforces grade-level understanding. Students should model multiplication on a number line just as they modeled 2 x 4 on a number line in grade 3. Additionally, students learned multiple multiplication strategies with whole numbers in the grade 3. They should make the connection that the previously learned strategies will work with a fractional amount in the same way that they worked with whole number factors.

Students need the opportunity to use modeling strategies while solving real-world problems. It is imperative that the modeling drives the solution to the problem and does not become an afterthought to the problem.

Students should interact with contextual problems that elicit multiplying a whole number by a fractional amount with increasing rigor over the course of instruction. Contextual problems should be framed in multiple ways. A good resource is the "Common Multiplication and Division Situations" document embedded in the Tennessee Mathematics Standards.

Level 4:

Students should be able to explain the connections that exist between multiplying with whole number factors and multiplying when one factor is a fraction and the other a whole number, providing multiple visual fraction models that support their argument.

Students should justify the reasonableness of solutions to contextual problems by employing number sense coupled with their understanding of the meaning of multiplication. Students should engage in conversations in which they justify their solutions using mathematical, sound reasoning.

Additionally, students should be encouraged to develop real-world problems to match a provided expression involving the multiplication of a whole number and a fraction.

Standard 4.NF.C.5 (Major Work of the Grade)

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express,* $\frac{3}{10}$ *as* $\frac{30}{100}$ *and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Model a fraction with a denominator of 10 using an area model and a number line model. Model a fraction with a denominator of 100 using an area model and a number line model.	Express a fraction with a denominator of 10 as an equivalent fraction with denominator 100 when a visual fraction model is provided. Add two fractions when one has a denominator of 10 and the other has a denominator of 100 when provided a visual fraction model.	Express a fraction with denominator 10 as an equivalent fraction with denominator 100. Add two fractions when one has a denominator of 10 and the other has a denominator of 100 and model the mathematics with a visual fraction model.	Add multiple fractions when some have a denominator of 10 and some have a denominator of 100 and model the mathematics with a visual fraction model.

Instructional Focus Statements

Level 3:

Students should exclusively be developing an understanding of adding fractions with denominators of 10 and 100. It is imperative that students develop a conceptual understanding of the importance of representing the two fractions with the same denominator.

Decimal fractions with denominators of 10 and 100 arise very naturally from money-based situations. This can provide a concrete context for determining that $\frac{2}{10}$ (2 dimes) is the same as $\frac{20}{100}$ (20 pennies). By the end of grade 4, students should be able to add decimal fractions by converting them to fractions with like denominators. This is the precursor for adding fractions with unlike denominators in grade 5.

Level 4:

Students should be able to extend their understanding to add multiple fractions with denominators of 10 and 100, model them using a variety of visual fraction models, and provide both a written and verbal justification of how to add fractions with denominators of 10 and 100. Additionally, students should be able to provide an explanation as to why it is important for the denominators to be the same in order to add tenths and hundredths.

Standard 4.NF.C.6 (Major Work of the Grade)

Read and write decimal notation for fractions with denominators 10 or 100. Locate these decimals on a number line.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Locate a decimal in tenths on a provided subdivided number line.	Locate a decimal in hundredths on a provided subdivided number line.	Use decimal notation for fractions with denominators 10 or 100. Locate a decimal in tenths or hundredths on a number line.	Locate multiple decimals in tenths and hundredths on a number line.

Instructional Focus Statements

Level 3:

Students learn that fractions with denominators of 10 and 100 can be written with a decimal point. Students should be locating these on number lines. Additionally, students should model decimals in both tenths and hundredths using visual fraction models.

Level 4:

Students should be focused on accurately placing tenths on a number line. Students should be exposed to accurately placing hundredths as well as approximating the position of hundredths on a number line. Students should be placing multiple decimals written as a combination of tenths and hundredths on the same number line.

Standard 4.NF.C.7 (Major Work of the Grade)

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Compare two decimals when both numbers are written exclusively to the tenths place, and record the results of comparisons with the symbols >, =, or <.	Compare two decimals written to the tenths or hundredths place when both numbers are written to the same decimal place, and record the results of comparisons with the symbols >, =, or <.	Compare two decimals written to the hundredths place, and record the results of comparisons with the symbols >, =, or <, and provide a justification for the conclusion. Explain why decimal comparisons are only valid when the two decimals refer to the same whole.	Order two or more decimals expressed in tenths and hundredths from least to greatest or greatest to least using appropriate symbols, and provide a justification. Provide an example of a situation generating two decimals which cannot be compared because they do not refer to the same whole.

Instructional Focus Statements

Level 3:

Students should use fractional reasoning in order to compare fractions. Thus, when comparing 0.3 and 0.45, students should first visualize these as $\frac{3}{10}$ and $\frac{45}{100}$ and see that $\frac{3}{10} < \frac{45}{100}$ because $\frac{30}{100} < \frac{45}{100}$. Additionally, it is important for students to demonstrate this with visual fraction models.

Students should develop a conceptual understanding of comparing decimals, such as understanding why 0.3 is less than 0.45. Students should not use procedural tricks, which may lead to future misconceptions. Additionally, place value understanding with decimals is developed in grade 5. Thus, in grade 4, students should not be relying on strategies employing place value understanding.

Level 4:

Students should extend their understanding beyond comparing two decimals to ordering two or more decimals and provide a justification and fraction models to validate their mathematical reasoning. Students should continue to demonstrate a conceptual understanding of the connection that exists between fractions and decimals. They should be able to verbalize that just as fractions must refer to the same whole in order for comparisons to be made, decimals must also refer to the same whole in order to compare them.

Measurement and Data (MD)

4.MD.A.1 (Supporting Content)

Measure and estimate to determine relative sizes of measurement units within a single system of measurement involving length, liquid volume, and mass/weight of objects using customary and metric units.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure liquid volumes using standard units of milliliters and liters. Measure the mass of objects using standard units of grams and kilograms. Determine what attributes can be measured for real-world objects. Measure the length of objects in inches using rulers marked with halves and fourths of an inch.	Estimate liquid volumes using standard units of milliliters and liters. Estimate the mass of objects using standard units of grams and kilograms. Identify the larger unit of measure when given two units within the same system of measurements (i.e. one yard is larger than one foot.)	Measure and estimate lengths, liquid volumes, and weight using standard customary units. Measure and estimate lengths, liquid volumes, and mass using standard metric units.	Measure either length, liquid volume, or mass/weight of an object using two different units of measure each from a different system of measurement. Compare and contrast the two measurements and discuss the relative size of the units chosen.

Instructional Focus Statements

Level 3:

Instruction should include opportunities for students to estimate and measure objects using each of the different units as appropriate. Once students have multiple experiences estimating and measuring, students can then draw from these experiences to answer questions that may be considered abstract. In grade 2, students estimated and measured objects' lengths in customary and metric units and in grade 3 students estimated and measured objects' masses and/or liquid volumes in metric units. In grade 4, students should build on their prior understandings of measurement to know metric and customary units for each length, weight/mass, and liquid volume. Instruction should focus on making determinations about what measurement unit is

appropriate for certain situations. For example, students should be able to determine what customary or metric unit would be used to measure the volume of a bathtub or a coffee cup.

As students measure using the metric system, connections should be made to the place value system. In grade 4, students are introduced to decimal numbers and this standard also supports the development of this understanding. For measuring mass, this will most often involve reading a digital scale; grams and kilograms are a part of the metric system, and the scales will read mass typically to the nearest tenth of a unit. One important note, within the science standards, it is not necessary at this grade level for students to distinguish the difference between weight and mass. Thus, this distinction should not be drawn in mathematics either. For liquid volume, students should be able to read a vertical number line in order to measure the liquid volume. Additionally in grade 4, students work with 10ths in the denominator. As students report measurements, it is appropriate to report these units as either fractions or decimals, keeping in mind the limitation for denominators for grade 4. It is imperative that students understand that benchmark measurements can be utilized in estimating units. The mathematical focus of this piece is using estimation to get to an application of multiplication. Students should also be able to identify the larger unit of measure when given 2 units within the same system of measurements. For example, students should be able to identify that one yard is larger than one foot. Students are not expected to use conversions in grade 4 as this is a grade 5 expectation.

Level 4:

As students solidify their understanding of measuring and estimating length, liquid volume, and mass/weight of objects using customary and metric units, they should be able to explain their thinking using verbal and written representations. Students should interact with estimations grounded in using benchmark measure(s) with increasing rigor over time. Students should also be able to measure either length, liquid volume, or mass/weight of an object using two different units of measure each from a different system of measurement and then compare and contrast the two measurements with respect to the relative size of the units chosen. In grade 4, it is important that students develop a strong conceptual understanding of comparing and contrasting different types of measurements to set a foundation for grade 5 when students begin to convert measurements.

4.MD.A.2 (Supporting Content)
Solve one- or two-step real world problems involving measurement with all four operations within a single system of measurement including problems involving simple fractions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve one-step real world problems involving measurement with addition and subtraction with exclusively whole number measures.	Solve one-step real world problems involving measurement with addition and subtraction including problems involving simple fractions not exceeding the grade-level computational bounds for fractions. Solve one-step real world problems involving measurement with multiplication and division with exclusively whole number measures. Solve simple two-step real world problems involving measurement with all four operations with exclusively whole number measures.	Solve two-step real world problems involving measurement with all four operations including problems involving simple fractions that can be solved within the grade-level computational bounds for fractions.	Create and solve one- and two-step real world problems involving measurement with all four operations including problems involving simple fractions that can be solved within the grade-level computational bounds for fractions.

Instructional Focus Statements

Level 3:

Students should build on their knowledge of standard 4.MD.A.1 to understand that measurements can be used in contextual situations and integrate the use of problem-solving strategies from standard 4.OA.A.3. Instruction should involve using any combination of the four operations to solve one or two-steps problems. For example, students should be able to answer the problem "John has 32 water jugs he needs to fill with water. Each jug can hold 12 L of water. How many L of water will he need to fill all the jugs?" by setting up a problem to show that a total of 384 L of water is needed to fill the jugs. It is

important to give attention to all of the situation types and use computations appropriate for grade 4. Problems should not involve converting units as this is a grade 5 expectation. Additionally, fraction computation should support the fraction standards for grade 4 (i.e. adding and subtracting with common denominators, multiplying a whole number and a fraction).

Level 4:

As students develop a deeper understanding of solving one- and two-step real-world problems involving measurement, they should be able to explain their thinking about the strategies they selected. Problems should increase with rigor over time including problems that involve multiple measurements. Students should also be able to make connections between concrete, representational, and abstract strategies used to solidify their understanding. Instructionally, students can also advance their thinking by creating their own real-world problems and provide multiple representations to explain their thinking. Students can also be provided with problems created by others so they can analyze and critique their peers' work.

4.MD.A.3 (Supporting Content)
Know and apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the area of a rectangle by tiling. Find the perimeter of a rectangle.	Generate a rectangle with the same area but a different perimeter when given a rectangle. Generate a rectangle with the same perimeter but a different area when given a rectangle.	Know and apply the area and perimeter formulas in real world and mathematical problems. Find the missing length when given the area or perimeter of a rectangle.	Find areas and perimeters of rectilinear figures in real world and mathematical problems when all measurements may not be provided.

Instructional Focus Statements

Level 3:

In grade 3, instruction on area was integrated with the instruction of multiplication as students were modeling multiplication using array models and tiling. They discovered that area of rectangles can be found by tiling the rectangle and counting the square units or by multiplying the length and the width measurements. Students also learned that perimeter is a linear measure and developed ways to find the perimeter of polygons. Holistically, the focus for instruction in grade 3 for both area and perimeter was developing conceptual understanding. It was not an expectation that the formulas for calculating area and perimeter be introduced to students.

In grade 4, students are introduced to the formula for these operations. It is important to connect to the third grade understanding to self-discover the formulas as opposed to memorizing them. Students should also discover that there is more than one formula for perimeter of rectangles (2l + 2w) or 2(l + w) or (w + l + w + l). They should be challenged to think about why there different representations and justify why they are equivalent. Students should use this knowledge to solve real world and mathematical problems. Real world and mathematical problems should both include instances where students are challenged to:

- Find an unknown side length for a rectangle when provided the rectangle's perimeter and one side length.
- Create a rectangle with the same area but a different perimeter than a provided rectangle. Critically think to answer the following resulting from a real world situation: What are all possibilities? What might be the best possibility for this given situation?

- Create a rectangle with the same perimeter but a different area than a provided rectangle. Critically think to answer the following resulting from a real world situation: What are all possibilities? What might be the best possibility for this given situation?

Level 4:

In grade 3 students learned that one can find the area of rectilinear figures by decomposing it into non-overlapping rectangles, finding the area of each rectangle, and then add the areas together to find the total area. In grade 4, students can continue to build on this idea by finding areas and perimeters of rectilinear figures when dimensions are not necessarily provided and students must identify the missing dimensions through problem solving.

4.MD.B.4 (Supporting Content)

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4), where the horizontal scale is marked off with appropriate units.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) on a pre-partitioned number line. Solve simple real-world problems involving addition and subtraction of fractions found in data already plotted on line plots.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use grade 4 fraction operations to solve problems involving information presented in line plots (fraction operations include: addition and subtraction of like denominator fractions or mixed numbers and multiplication of a whole number by fractions).	Interpret the data from the line plot and solve multi-step problems. Collect data, create a line plot, and create contextual problems that can be solved using the data presented on the line plot.

Instructional Focus Statements

Level 3:

Students begin their work with line plots in grade 2 as they created line plots from data provided in whole number units with an interval of one. In grade 3, students began measuring the lengths of objects to the nearest quarter inch and then created a line plot from the data. In grade 4, students are expected to create line plots with measurements to the nearest eighth inch and solve problems using fractions and fraction operations appropriate for this grade level. Students need to both analyze and create line plots in grades 4 and 5. As students are still developing their conceptual understanding of operation with fractions, it is appropriate for students to use concrete manipulatives such as fraction strips to help solve problems. The instructional focus for this standard is two-fold. It should focus on making a line plot to represent fractional measurements and also focus on supporting their developing conceptual understanding of fractions by solving grade-level appropriate problems generated using a line plot.

Level 4:

In level 4 students should demonstrate their ability to interpret and analyze the data from the graph and use a multi-step approach to solve problems. Instruction should encourage students to share their thinking with one another using appropriate mathematical vocabulary including data, line plot, length, and fractions. At this level, students should be able to determine a situation in which it would be appropriate to collect fractional data, collect that data, create a line plot, and create contextual problems that can be solved using the data presented on the line plot.

4.MD.C.5 (Supporting Content)

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

4.MD.C.5.a Understand that an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle.

4.MD.C.5.b Understand that an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees and represents a fractional portion of the circle.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Understand there are 360 one-degree angles in a circle. Know that degrees are a unit of measure for angles.	Recognize right, acute, and obtuse angles. Recognize that a circle can be decomposed into different angles (e.g., there are four right angles in a circle).	Understand the relationship between an angle and a circle. Represent an angle measure *n* as *n*/360 and show that it is made up of *n* one degree angles.	Determine the adjacent angle using knowledge of the degrees of a circle when given the measurement of an angle. Find the measurements of unknown angles when given other angle measures within the circle.

Instructional Focus Statements

Level 3:

The instructional focus for this standard should be around helping students understand the connections that exist between angles and circular measurement. Instruction should be integrated with aspects of standard 4.G.A.1 as that is where students develop an understanding of drawing rays, lines, and angles. The overarching concept of connecting circles and angles is typically foreign to students. It is crucial that students are able to conceptually connect these topics as that understanding will be important in subsequent grades and courses.

Students will have to build the understanding that a circle encompasses 360 degrees. They will then connect this and their understanding of fractions to note that each degree is 1/360 of the way around a circle called a "one degree angle". Students will then be able to generalize this information to understand that an angle turning through n representations of this unit fraction would total that many degrees of measure and represent n/360 of a circle.

Students then must develop an understanding of how to measure an angle with reference to a circle. At this point, instruction should integrate with standard 4.MD.C.6 where students measure angles using a protractor. In measuring angles, students should equally experience measuring angles inscribed in circles and angles not appearing in relationship to a circle.

Level 4:

Students should be challenged to determine unknown angle measures when given one angle measure within the circle based upon their understanding of angles, including straight angles, coupled with the understanding that a circle encompasses 360 degrees.

4.MD.C.6 (Supporting Content)
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize angles as acute, obtuse, and right in order to determine which range of numbers an angle would measure.	Read a protractor to the nearest whole number. Estimate the measure of an angle.	Measure given angles to the nearest whole number. Sketch an angle when given a specific measurement.	Assess the accuracy of given angle measures and justify why they are reasonable or not.

Instructional Focus Statements

Level 3:

After solidifying an understanding of the definition of a degree when working with standard 4.MD.C.5, instruction for this standard should build on that understanding focusing on students measuring angles and sketching angles not inscribed in a circle using protractors. As with all measurable attributes, students must first recognize what attribute of the angle the protractor is specifically measuring. Students need to experience measuring a wide variety of angles oriented in a wide variety of ways in order to avoid developing misconceptions around angle measurement (e.g., misconceptions that a right angle is an angle that points to the right, or two right angles represented with different orientations are not equal in measure).

Students should use appropriate terminology to determine the type of angle being measured and use benchmark angles to help make estimates prior to measuring the angle. This is important as it will help students determine if their measures are reasonable. For example, if they measure an acute angle at 105 degrees, they realize this is not reasonable since the angle is acute and acute angles are less than 90 degrees.

Likewise, prior to sketching an angle, students should use appropriate vocabulary and visualize benchmarks for the measurement to visualize what the angle should look like. Students should also use appropriate vocabulary such as vertex, ray, degree, point, end point, etc. as they sketch angles. Sketching angles brings its own set of challenges for students. Just as students should not always be presented angles that appear in the same orientation to measure, it is important that they are sketching angles in multiple orientations as well. One particularly helpful activity for students as they are solidifying their understanding is for students to partner up, each draw an angle, and then trade with their partner measuring to check the accuracy of their partners work.

Level 4:

Instruction at this level should focus of challenging students to assess the accuracy of provided measurements and provide justification for their thinking. Students could also be challenged to think about angles with a measure greater than 180 degrees and how they might draw that representation. Connecting back to a circular representation of angles will be important as students undertake this work.

4.MD.C.7 (Supporting Content)

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Decompose an angle into two non-overlapping angles.	Determine the measure of an angle when given the angle decomposed into two non-overlapping parts and the degree measure of each of those parts.	Decompose an angle into at least two non-overlapping parts, recognize the sum of the parts is the same as the total angle measure, and find the measure of all involved angles. Solve addition and subtraction problems to find an unknown angle measures. Write an equation with a symbol to represent an unknown angle measure and then find the unknown measure.	Create angle measure problems where an angle(s) measure is unknown.

Instructional Focus Statements

Level 3:

One instructional focus for this standard should be helping students understand what it means to decompose an angle into smaller, non-overlapping parts. The second should be on then connecting how addition and subtraction can be used to find the measure of all involved angles. Instruction should connect to previous learning as students decomposed numbers in the early grades (e.g., decompose 10 into 8 + 2 and 7 + 3). They should see the parallel between decomposing angle measures and decomposing numbers into sums. Students need to develop the mathematical understanding that angle measures, like numbers, can be decomposed.

Students should recognize the part to whole relationships when operating with angles by designating the whole as 90 degrees, 180 degrees, or any angle measurement which can be decomposed into parts consisting of smaller angle measurements.

As with other measurements (i.e., length), when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the measurements of unknown angles on a diagram in real-world and mathematical problems. For example, a straight angle is decomposed into two parts, A and B. If B is 1/3 of the straight angle, what is the measurement, in degrees, of angle A? This type of problem requires students to use multiple grade level computation skills to determine the angle measure.

Level 4:

Students should be challenged with mathematical and real world problems that increase in complexity over time. Additionally, instruction at this level should provide students with opportunities to make sense of what they are learning about angles that are decomposed and the recognition of angle measure as additive by designing their own challenging problems to solve for angle measure(s).

Geometry (G)

Standard 4.G.A.1 (Supporting Content)

Draw points, lines, line segments, rays, angles (right, acute, obtuse, straight, reflex), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose an example of a point, line, line segment, or a ray. Choose a drawing that represents a set of parallel or perpendicular lines. Choose a drawing that represents a right angle.	Identify if they are parallel, perpendicular, or neither, given two lines presented in a traditional format. Identify if it is right, acute, or obtuse, given an angle presented in a traditional format. Draw a point, line, line segment, and ray.	Draw geometric objects (lines, line segments, rays, angles, perpendicular and parallel lines). Identify geometric objects (lines, line segments, rays, angles, perpendicular and parallel lines) in two-dimensional figures. Use appropriate vocabulary to describe lines, line segments, rays, angles, perpendicular and parallel lines.	Draw a geometric figure, given several attributes. Identify geometric objects (lines, line segments, rays, angles, perpendicular and parallel lines) in three-dimensional figures.

Instructional Focus Statements

Level 3:

Grade 4 is a student's first formal mathematical introduction to rays, angles, and perpendicular and parallel lines. As students are developing a conceptual understanding of rays, angles, and perpendicular and parallel lines, they should be encouraged to observe them in their environment. Students may not easily identify lines and rays as they are more abstract concepts.

Students should also have experiences where they build these geometric figures with manipulatives, draw them, identify them in isolation, and identify and compare them when they are embedded in two-dimensional figures. It is important that students are able to explain either verbally or in written form

the similarities and difference between lines, rays, and line segments. This will demonstrate that they not only know what each looks like, but also the properties that distinguish them from one another. Likewise, students should be able to do the same with all types of angles and also with parallel and perpendicular lines. In order for students to identify points, lines, line segments, rays, angles (right, acute, obtuse, straight, reflex), and perpendicular and parallel lines in two-dimensional figures, they must have developed a conceptual understanding of the properties of each.

Focusing specifically on angles, one common misconception is that students think a wide angle with short sides is smaller than a narrow angle with long sides. Students need the opportunity to compare them. One way to accomplish this is by placing angles over each other to see the difference in angle size is independent of the length of the sides.

It is important to note that students develop an understanding of angles as a geometric shape in standards 4.MD.C.5a and 4.MD.C.5b. First, it will be important to make connections between standards 4.MD.C.5 and 4.G.A.1. Additionally, the understandings gained in standard 4.MD.C.5 along with the understandings from this standard are pre-requisite skills for students being able to classify two-dimensional shapes in standard 4.G.A.2.

Level 4:

As students extend their understanding, they should be able to use a set of given attributes to draw a geometric figure. Over time the set of given attributes should increase. For example, students might be asked to draw a figure with one set of parallel sides and one set of perpendicular sides. Later, they might be asked to draw a figure that has one set of parallel sides, two sets of perpendicular sides, and one acute angle.

Instruction should shift from identifying geometric objects in two-dimensional shapes to identifying geometric objects in three-dimensional shapes. It is important that students can not only identify the geometric objects but can also justify their labeling.

Standard 4.G.A.2 (Supporting Content)

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify two-dimensional figures when given a set of attributes. Choose a right triangle.	Identify parallel and perpendicular lines in a two-dimensional figure. Identify right, acute, and obtuse angles in a two-dimensional shape. Identify right angles in a right triangle.	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines. Classify two-dimensional figures based on the presence or absence of angles of a specified size. Recognize right triangles as a category. Identify right triangles and non-right triangles.	Classify multiple two-dimensional figures and justify why they can or cannot be grouped together.

Instructional Focus Statements

Level 3:

Students must be able to identify parallel and perpendicular lines as well as types of angles in two-dimensional figures prior to working on classifying figures. Standard 4.G.A.1 is a pre-requisite skill for work on classification. Students learned in grade 3 that shapes in different categories may share attributes and that the shared attributes can define a larger category. This understanding is important for students to have as they use this to classify figures. Instruction at this level should include students sorting and classifying to see how shapes are alike and different. Students should focus on the absence of parallel sides, more than one set of parallel sides, and the presence or absence of varying angle types. Students should see a sufficient variety of examples and need ample opportunities to work with concrete models. Students should be asked to sort and classify shapes when given a specific

attribute as well as look at pre-sorted shapes and identify the attribute that was used to classify them. Students should use appropriate academic vocabulary to explain the classifications. Students should be able to classify shapes presented in various ways (i.e., drawings or verbal descriptions).

Right triangles are also a specific focus in this standard. Students should be able to recognize right angles within a right triangle. They should also be able to identify right triangles and non-right triangles.

Level 4:

Students should classify multiple two-dimensional figures with at least two common attributes and justify why the figures have been grouped or why the figures cannot be grouped together. Venn diagrams, graphic organizers, and Frayer models may be particularly helpful as an organizational tool for students as the classify figures. Students should also have opportunities to make and test conjectures around the attributes for classifying a wide variety of two-dimensional figures. For example, they might consider if an idea is true for all triangles or just specific triangles and explain their reasoning.

Standard 4.G.A.3 (Supporting Content)
Recognize and draw lines of symmetry for two-dimensional figures.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a two-dimensional figure that contains an accurately drawn line of symmetry. Fold a common two-dimensional figure to demonstrate one line of symmetry.	Fold a two-dimensional figure to demonstrate multiple lines of symmetry. Draw one line of symmetry in two-dimensional figure.	Recognize lines of symmetry in two-dimensional figures. Draw lines of symmetry in two-dimensional figures.	Create images with symmetry.

Instructional Focus Statements

Level 3:

Grade 4 is the first time symmetry is formally defined using mathematical concepts. Students should understand through hands-on experiences that if a shape can be folded on a line so that the two halves match, then it is said to have line symmetry. The fold line is called a "line of reflection" or "line of symmetry". The portion of the shape on one side of the line is reflected onto the other side. This understanding is the foundation for transformations in middle school. Instruction should include students building symmetrical designs with tools such as pattern blocks, geoboards, and dot paper. One technique that can be used to discover symmetry is using mirrors.

Instruction should begin with understanding one line of symmetry in a figure. Students should then discover that some objects have more than one line of symmetry. Again, paper folding and mirrors can be useful tools. For example, through discovery learning, students should be able to demonstrate and explain that a square has four lines of symmetry. Additionally, students should be able to identify and draw all lines of symmetry in both regular and non-regular polygons.

Level 4:

Instruction at this level should focus on students creating two-dimensional figures that are symmetric in some way. Initially, they may be asked to create a figure with a single line of symmetry. Students should be challenged to examine their figures and identify any unintended lines of symmetry. Students could then create figures with multiple lines of symmetry. As vertical symmetry is typically more intuitive for students, they should be encouraged to create images with horizontal and diagonal symmetry. One other extension is when given a partial image, students can create the other part of the figure utilizing their conceptual understanding of symmetry.