

Second Grade Mathematics Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into either four or seven levels. For grade 2, the standards that provide seven levels are congruent with the scoring rubrics for the kindergarten portfolio. Standards that only provide four levels are not included as a part of the portfolio scoring rubric.

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Levels 4-7: Performance at these levels demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in the same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. The provided evidence of learning statements are examples of what students will most likely be able to do and do not represent an exhaustive list.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3, 4, and 4-7 portfolio standards.

Operations and Algebraic Thinking (OA)

Standard 2.OA.A.1¹ (Major Work of the Grade)

Add and subtract within 100 to solve one- and two-step contextual problems, with unknowns in all positions, involving situations of add to, take from, put together/take apart, and compare. Use objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:	Students with a level 5 understanding of this standard will most likely be able to:	Students with a level 6 understanding of this standard will most likely be able to:	Students with a level 7 understanding of this standard will most likely be able to:
<p>Add and subtract within 20 to solve contextual problems, involving any of the problem types.</p>	<p>Add and subtract within 100 to solve one-step contextual problems which do not require composing or decomposing tens, using <u>two</u> different situations of add to-start unknown, take-from-start unknown, compare-smaller unknown (version with more), compare-bigger unknown (version with fewer). Represent these problems with a mathematical drawing or concrete models. Students choose a representation in order to explain their thinking to both themselves and others.</p>	<p>Add and subtract within 100 to solve one-step contextual problems which do not require composing or decomposing tens, using <u>two</u> different situations of add to-start unknown, take-from-start unknown, compare-smaller unknown (version with more), compare-bigger unknown (version with fewer). Represent these problems with a mathematical drawing, diagram, or equation with a symbol for the unknown number.</p> <p>Add and subtract within 100 to solve two-step contextual problems. Represent these problems with a</p>	<p>Add and subtract within 100 to solve one-step contextual problems which require composing or decomposing tens, using <u>three different</u> situations of add to-start unknown, take-from-start unknown, compare-smaller unknown (version with more), compare-bigger unknown (version with fewer). Represent these problems with a mathematical drawing, diagram, or equation with a symbol for the unknown number.</p> <p>Add and subtract within 100 to solve two-step contextual problems. Represent these problems with a</p>	<p>Add and subtract within 100 to solve one-step contextual problems which require composing or decomposing tens, using <u>all</u> of the different situations of add to-start unknown, take-from-start unknown, compare-smaller unknown (version with more), compare-bigger unknown (version with fewer). Represent these problems with a number line model or equation with a symbol for the unknown number.</p> <p>Add and subtract within 100 to solve two-step contextual problems. Represent these problems with a</p>	<p>Add and subtract within 100 to solve a wide variety of two-step contextual problems. Represent these problems with a single equation that encompasses both steps needed to solve the problem.</p> <p>Create two unique contextual problems that could be solved using the provided equations, given two one-step equations arising from the different situations of add to-start unknown, take-from-start unknown, compare-smaller unknown (version with more), compare-bigger</p>	<p>Create two unique contextual problems that could be solved using the provided equations, given two, two-step equations (one of which incorporates both addition and subtraction) arising from the different situations of add to-start unknown, take-from-start unknown, compare-smaller unknown (version with more), compare-bigger unknown (version with fewer).</p>

¹ Standard 2.OA.A.1 has seven levels of Evidence of Learning Statements as it is a Portfolio Standard.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:	Students with a level 5 understanding of this standard will most likely be able to:	Students with a level 6 understanding of this standard will most likely be able to:	Students with a level 7 understanding of this standard will most likely be able to:
		mathematical drawing, diagram, or equation(s) Students choose a representation in order to explain their thinking to both themselves and others.	number line model or equation(s) with a symbol for the unknown number(s).	number line model and equation(s) with a symbol for the unknown number(s).	unknown (version with fewer).	

Instructional Focus Statements

Level 3:

In grade 1, students developed an understanding of adding and subtracting within 20 through interacting with a wide variety of problem-solving situations. Students also began adding a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100) in standard 1.NBT.A.1. In grade 2, there are three significant differences in how students interact with contextual problems. The first is extending the range of numbers students use for addition and subtraction from within 20 to within 100 encompassing a much larger range of sums and differences. The second is that students are expected to be exposed to all types of common addition and subtraction situations. The table for common addition and subtraction situations is located on page 20 in the TN mathematics standards located [here](#). Finally, the standard explicitly calls out two-step problems for the first time. In previous grades, no distinction is made as to one-step versus two-step contextual problem solving situations. That said, in previous grades teachers are encouraged to push students to work with two-step contextual situations as determined by student readiness.

As students begin to work with a larger range of numbers and more complex problem solving situations, they should continue to make use of models, drawings, and multiple representations in order to demonstrate their understanding. They may employ strategies that involve counters, linking cubes, ten frames, base ten blocks, part-part-whole models, number lines, bar models, etc. In working with larger numbers, students should begin to transition to more efficient representations of problem situations, looking for and realizing that some representations are easier to use with larger numbers. For example, students should understand that using a bar model, number line, or the numeral itself may be more efficient when working with larger numbers rather than drawing out the number of objects.

In transitioning all students to working with two-step contextual problems, instruction should initially focus on problems involving smaller, familiar numbers and operations allowing students to focus on the conceptual understanding of multiple operations within the problem as opposed to focusing on computation with less familiar numbers. Additionally, it is easier for students to begin with problems that call for the same operation within the problem and then move on to working with two-step problems that involve using both addition and subtraction. It is important to call out that students should

continue to use manipulatives, multiple strategies, and written equations when solving two-step contextual problems. To demonstrate their understanding, they should be able to explain the connections between the visual representation and the equation(s) that represents the problem. Additionally, students should be encouraged to use multiple strategies and make connections between each strategy. For example, students may write individual equations for each step in a two-step problem or write both steps in one equation. This is a good opportunity for students to compare their work to others and explain why both are correct or in some cases incorrect and explain the connection between the two strategies.

Teaching key words to associate with addition and subtraction should not be an instructional focus. Instruction should focus on developing an understanding of what operation is needed to solve the problem rather than focusing on key words that sometimes, but not always, associate with the operation.

Levels 4-7:

As students deepen their understanding of operations with addition and subtraction with a larger range of numbers and two-step problems, they should be able to represent these problems with a mathematical drawing, diagram, and equation with a symbol for the unknown number. They should be able to explain their thinking of multiple representations and make connections between the visual representations as well as the problem represented as an equation. As an extension, students should be able to create their own two-step contextual problem and explain the solution. When doing so, students should use visual presentations, equations, and precise mathematical vocabulary.

Standard 2.OA.B.2² (Major Work of the Grade)

Fluently add and subtract within 30 using mental strategies. By the end of 2nd grade, know from memory all sums of two one-digit numbers and related subtraction facts.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Add and subtract within 10 using concrete objects. A context is not provided.</p>	<p>Fluently add and subtract within 10 using mental strategies. A context is not provided. Students can quickly, efficiently, and accurately produce answers without recording their thinking on paper.</p> <p>Add and subtract within 30 using concrete objects. A context is not provided.</p>	<p>Fluently add and subtract within 30 using mental strategies. A context is not provided. Students can quickly, efficiently, and accurately produce answers without recording their thinking on paper.</p>	<p>Fluently add and subtract within 30 using mental strategies. A context is not provided. Students can quickly, efficiently, and accurately produce answers without recording their thinking on paper, identify the strategy used, and explain why the strategy works.</p> <p>Analyze a given addition problem and its accompanying solution and determine if the solution is correct or incorrect. When it is correct, identify the strategy used, and explain why the strategy works. When it is incorrect, correct the mistake and explain the mathematical misunderstanding that would cause the mistake to occur.</p> <p>Analyze a given subtraction problem and its accompanying solution and determine if the</p>

² Standard 2.OA.B.2 has four levels of Evidence of Learning statements as it is not a Portfolio Standard.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
			solution is correct or incorrect. When it is correct, identify the strategy used, and explain why the strategy works. When it is incorrect, correct the mistake and explain the mathematical misunderstanding that would cause the mistake to occur.

Instructional Focus Statements

Level 3:

As stated in the introduction of the Tennessee Mathematics Standards, fluency is the ability to apply procedures accurately, efficiently, and flexibly. By the end of grade 1, students fluently added and subtracted within 20 using mental strategies and knew from memory all sums up to 10. By the end of grade 2, students should extend this understanding to fluently add and subtract within 30 using mental strategies.

Building fluency that is based on mental strategies is a process. Students begin by developing a conceptual understanding of the operations of addition and subtraction through direct modeling. The next natural progression is for students to work with student-driven, invented strategies that are deeply rooted in place value and number sense. Students began working with invented strategies with adding and subtracting within 20 in standard 1.OA.C.5. Before they reach fluency with mental strategies, students must be given the opportunity to interact with direct modeling and/or student-invented strategies in order to have the mathematical foundation needed to move along the continuum towards reaching fluency with mental strategies. This process takes time. Students should be exposed to various strategies and choose the one that is most efficient and makes the most sense to them, ultimately utilizing their strategies for mental computation as they progress in their learning. It is important to note that timed tests do not build fluency in students. Exposure to flexible thinking, explaining their thoughts, and appropriate scaffolding over time builds fluency.

As students become more fluent with adding and subtracting numbers within 30, they should start to produce answers without recording their thinking and explaining their mental thought process. Students should explain or defend their answer, such as decomposing and composing the numbers, properties of operations, place value, or describing mental images used to obtain the answer. Additionally, students should have many opportunities to practice, explain their thinking, compare and make connections with multiple strategies. Number Talks, written explanations, and selecting the strategy

that makes the most sense to them will allow students to develop a conceptual understanding to become fluent with adding and subtracting within 30 and know from memory all sums of two one-digit numbers and related subtraction facts.

One final note, algorithms for addition and subtraction are not introduced within the standards until grade 3.

Level 4:

As students develop a wider range of mental strategies that they are comfortable with and can explain, they should be able to explain the connections that exist between multiple strategies. They should also be able to, given a work sample of adding or subtracting two numbers within 30, identify if the computation is correct or incorrect, identify the strategy used, and explain why the strategy works or does not work. Students should also be able to explain what misconception took place to produce an incorrect answer. It is imperative that as students transition to using mental strategies that they are asked questions that press for the underlying mathematics and that students provide an explanation of their thinking using precise mathematical vocabulary.

Standard 2.OA.C.3 (Supporting Work)

Determine whether a group of objects (up to 20) has an odd or even number of members by pairing objects or counting them by 2s. Write an equation to express an even number as a sum of two equal addends.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Skip count by twos.</p> <p>Split a group of even numbered objects into two equal subgroups counting by ones.</p> <p>Choose representations of adding within 20 using doubles.</p>	<p>Determine whether a group of objects (up to 20) has an odd or even number of members by breaking the group of objects into two subgroups with the same number in each subgroup (even) or if it can be broken into two equal subgroups with a leftover object (odd).</p> <p>Give examples of adding within 20 using the strategy of doubles.</p>	<p>Determine whether a group of objects (up to 20) has an odd or even number of members by pairing objects or counting them by twos.</p> <p>Write an equation to express an even number as a sum of two equal addends.</p>	<p>Determine whether a group of objects (greater than 20) is an odd or even number and justify their thinking.</p> <p>Create a group of more than 20 objects that is even and explain why in more than one way.</p> <p>Create a group of more than 20 objects that is odd and explain why in more than one way.</p>

Instructional Focus Statements

Level 3:

Students develop an understanding of odd and even numbers by using concrete materials to explore and discover the unique properties held by numbers classified as even and numbers classified as odd.

Instruction should focus first on students determining if a number can be broken into two equal parts. For example, students may use counters to represent the numbers four, seven, and ten. They find that the counter representations of four and ten can be shared equally in two groups while the group of seven counters cannot be evenly split into two equal groups (i.e., one group always has at least 1 more counter than the other). Students then

begin looking for patterns as they represent additional numbers in order to self-define what it means for a number to be even and what it means for a number to be odd.

Instruction should also focus on students determining if a number is even or odd by pairing objects and looking to see if there are leftover items. For example, a student may be given 18 counters with which they make nine pairs of counters. A student may also be given 19 counters in order to discover that they can make nine pairs of counters with one leftover. This provides another avenue for students to develop an understanding of what it means for a number to be even or odd.

As students move along a continuum of learning from concrete to abstract, they can be challenge to make connections relating the concept of odd and even to the addition facts. Students are able to show that an even number can be expressed by a doubles fact (e.g., $9 + 9 = 18$). Students may even make the generalization that when two like addends are combined, the sum is always even because the addends represent two equal groups. Odd numbers can be expressed using a doubles + 1 facts (e.g., $19 = 9 + 9 + 1$ or $19 = 9 + 10$). Students are also able to show that an even number can be expressed by a sum of twos. For example, $2+2+2+2+2+2+2+2+2=18$. Likewise, an odd is a sum of repeatedly added twos plus one.

The categorization of numbers as odd or even is an important structure in our number system. All too often children are simply told that the even numbers are those that end in 0, 2, 4, 6, or 8 and odd numbers are those that in 1, 3, 5, 7, or 9. Although this is true, and students may discover this pattern and even have discussions about it, it is only an attribute of even and odd numbers rather than a definition that explains what “even” or “not even” (i.e. odd) really means.

Level 4:

Students extend their understanding of even and odd numbers by identifying if a number is even or odd for numbers greater than 20. Students make a claim if a number is even or odd and justify their reasoning with representations. Additionally students can be challenged to create a group of objects with more than 20 members with a given condition that it is either even or odd and provide multiple justifications for why the group they created can be classified as either even or odd.

Standard 2.OA.C.4 (Supporting Work)

Use repeated addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Construct an array when given up to 25 objects	Counts to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to represent the sum.	Use repeated addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	Given a rectangular array of objects with up to 5 rows and up to 5 columns, write an equation to express the total as a sum of equal addends; rearrange the objects to make a new rectangular array; write a new equation to express the total as a sum of equal addends; and provide mathematical justification for why the two equations are equivalent.

Instructional Focus Statements

Level 3:

With this standard, students should extend their understanding of addition with equal groups to discover the benefit of intentionally using repeated addition in order to determine the total number of objects in a rectangular array. This understanding is a critical building block for multiplication in grade 3.

As students initially engage with this standard, they should simultaneously develop academic vocabulary around the concept of an array while discovering efficient methods to determine the total number of objects in arrays of various sizes. Real-world examples such as a tray of doughnuts or an egg carton can be particularly helpful for students as they are initially visualizing and working with arrays. Students may count by ones or skip count by either rows or columns to determine the total number of objects. Students should be able to construct a viable argument justifying to their classmates (MP 3) how and why their counting method is efficient. One mathematical understanding that all students should discover is that skip counting by rows and skip counting

by columns for the same array will both lead to the same total count of objects in the array. This sets students up with a strong foundational understanding for what will become the commutative property of multiplication in subsequent grades.

Once students are able to skip count to find the total number of objects in arrays, they should be challenged to write an equation to represent their thinking and provide justification for how the equation matches their skip counting patterns. Ultimately, students should be able to represent the total number of objects in any given array with an equation that expresses the total as a sum of equal addends.

Instruction should integrate this standard and standard 2.G.A.2 where students partition a rectangle into rows and columns of same-sized squares and find the total number of squares. The geometry standard provides a concrete learning experience for students as they physically build arrays. Students can then find the total number of squares using the same methods as have been used with standard 2.OA.C.4. With both partitioned rectangles and arrays of objects, it is imperative that students express their thinking as a sum of repeated addends.

Level 4:

Students at this level should be challenge to discover deeper connections between rectangular arrays with the same number of objects but different numbers of rows and columns. For example, students could be challenged to find as many arrays that contain twelve objects as possible (i.e., 1 by 12, 12 by 1, 2 by 6, 6 by 2, 3 by 4, and 4 by 3), write equations with repeated addends that would represent each, and then mathematically justify why all of these yield a count of twelve objects. As students create hypotheses and test their thinking, they will interact with arrays that are larger than five rows by five columns (e.g., in the example provided students interact with a row of twelve). Extending the range allows students to make connections over all combinations. This level of flexible thinking will support students as they develop multiplication strategies in grade 3.

Numbers and Operations in Base Ten (NBT)

Standard 2.NBT.A.1 (Major Work of the Grade)

Know that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 can be represented in multiple ways as 7 hundreds, 0 tens, and 6 ones; 706 ones; or 70 tens and 6 ones).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Represent a number less than 20 as a ten and some ones.</p> <p>Represent a two-digit number as groups of tens and ones in one way.</p>	<p>Represent a two-digit number as groups of tens and ones in more than one way.</p> <p>Represent a three digit number as groups of hundreds, tens, and ones in one way.</p>	<p>Represent a three digit number as groups of hundreds, tens, and ones in more than one way.</p>	<p>Represent a three digit number as groups of hundreds, tens, and ones in more than one way. Explain why these multiple representations can represent the same number.</p>

Instructional Focus Statements

Level 3:

In grade 1, students learned to represent a two-digit number in more than one way and began to develop place value understanding involving tens and ones. In grade 2 students extend their understanding of place value and the base-ten number system to include three digit numbers. They begin looking at 10 tens as a new unit called a “hundred.” This lays the foundation for students developing an understanding of the structure of the base-ten system. In grade 4, students solidify their understanding that the base-ten number system is developed by repeatedly bundling groups of 10 and understanding that the unit associated with each place is 10 of the unit associated with the place to its right.

Representations with place value manipulatives (e.g., base ten blocks, place value chips, layered three-digit place value cards) as well as math drawings provide students the opportunity to discover connections between written three-digit numbers and place value of numbers composed of hundreds, tens, and ones. It is important for students to work with unitizable manipulatives such as unifix cubes, straws or other items so that they physically bundle groups of ten ones to make a ten and then bundle groups of ten tens to make 100 before moving to unitized manipulatives such as base ten blocks. Instruction might further progress to non-proportional manipulatives such as place value chips to represent the numbers. When using all of these

manipulatives, it is important for students to develop their understanding across the full continuum from concrete to abstract representations of the expanded number.

Through interacting with multiple representations, students develop the conceptual understanding that the digit in the hundreds place represents that many groups of 100. They further extend their understanding to realize that when there is a "0" in a place there are zero bundles of that group. Students also develop flexibility in thinking as they discover non-traditional ways to decompose numbers. They realize that a number may be represented with more than 10 tens (e.g., 428 could be represented by a student as 3 hundreds 12 tens and 8 ones). They also develop the understanding that when recording the value of the number, regrouping must take place in order to write the appropriate digit in the appropriate place (e.g., they cannot write 12 in the tens place and must "fair trade" the ten 10s in for one 100 in order to record the number's value).

Instruction for this standard can be integrated with standard 2.NBT.A.3 where students read and write numbers in various forms. Students should connect the words to the written numeral and be guided to make explicit connections between concrete and pictorial representations for place value.

As students begin to make sense of the base ten number system, they should expand their thinking to flexibly compose and decompose three-digit numbers in multiple ways. In working with numbers such as 543, students may use representations such as 5 hundreds, 4 tens, and 3 ones; 4 hundreds 14 tens, and 3 ones; or, 3 hundreds, 24 tens, and 3 ones. To advance student thinking during instruction encouraging the discovery of multiple solution paths, questions such as the following could be posed:

- "What if you only had 4 hundred flats/chips, could you still make this number?"
- "What if you only had ten rods and ones, could you still make this number?"

Once these variety of solution paths have been created, they should be shared and discussed in a whole group discussion to solidify understanding. This ability to flexibly compose and decompose numbers is foundational for students as they work with addition and subtraction problems involving regrouping.

Level 4:

At this level, students should have a good understanding of the base ten number system and compose and decompose three-digit numbers in a wide variety of ways. Students can explain why the multiple representations represent the same number.

Standard 2.NBT.A.2 (Major Work of the Grade)

Count within 1000. Skip-count within 1000 by 5s, 10s, and 100s, starting from any number in its skip counting sequence.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Count up to 100 by ones, fives, and tens starting at any number in its skip counting sequence.	Count to 120 by ones, fives, and tens, starting at any number in its skip counting sequence.	Count within 1000. Skip-count within 1000 by fives, tens, and hundreds, starting from any number in its skip counting sequence.	Identify missing number(s) in a given counting sequence within 1000 when counting by fives, tens, and hundreds starting from any number in its skip counting sequence, when the pattern (rule) is not provided. Students can describe the pattern (rule) and explain their reasoning. Skip count within 1000 by other numeric patterns (e.g., twos, threes, fours, sixes, etc.) starting from any number in its skip counting sequence.

Instructional Focus Statements

Level 3:

Students began skip counting in kindergarten to 100 by ones, fives, and tens and extended that work in grade 1 counting to 120 by ones. Grade 2 students expand the range of numbers by counting in a wide variety of ways within 1,000. Although not explicitly called out in the standard, it is important that students count backwards within 1000 as well. Additionally, they count not only by ones, but also skip count by fives, tens, and hundreds starting from any number in the counting sequence. Skip counting lays the foundation for future work with multiplication by forming groups of a given size. Skip counting is not yet true multiplication as students do not keep track of the number of groups they have counted.

Instruction should provide opportunities for students to explore and discuss patterns of numbers when they count and skip count forward and backward. For example, while using a hundreds chart or number line, students learn that the ones digit alternates between five and zero when skip counting by fives. They should be counting starting not only at five, but also starting from a different number in the counting sequence such as 150 and observing if their predicted pattern holds true. When students skip count by hundreds, they learn that the hundreds digit is the only digit that changes and that the value of the digit increases by one with each subsequent number in the counting sequence. Students should be challenged to recognize and explain why the tens place changes when counting by ones when crossing over a decade number (e.g., 349 to 350). It is essential that instruction include discussions of patterns in numbers within counting sequences coupled with student provided explanations of why mathematically those patterns occur.

Level 4:

Students at this level extend their understanding of skip counting by identifying a missing number(s) in a given counting sequence within 1000 when counting by fives, tens, and hundreds starting from numbers not in their counting sequence. Additionally, students should be able to explain how and why the patterns discovered within the counting sequence change when starting with a number not in the counting sequence (e.g., explaining why when counting by fives starting at 123, the end digit no longer alternates between 5 and 0). Students should be able to identify the counting sequence pattern (rule) and then explain their reasoning. Students should be challenged to skip count within 1000 by other numeric patterns (e.g., twos, threes, fours, sixes, etc.) starting from any number in its skip counting sequence, look for patterns within and between the sequences, and mathematically explain why those patterns occur. Extensive experiences with skip counting helps students when they begin working with multiplication in subsequent grades.

Standard 2.NBT.A.3 (Major Work of the Grade)

Read and write numbers to 1000 using standard form, word form, and expanded form.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Reads and writes numbers from 0-20.</p> <p>Writes numerals from 0-9 in word form.</p>	<p>Reads and writes numbers to 100 using standard form, word form, and expanded form.</p> <p>Given a number written in either standard form, expanded form, or word form, choose the other representations.</p>	<p>Read and write numbers to 1000 using standard form, word form, and expanded form.</p>	<p>Explain why standard form and expanded form of a number are equivalent.</p> <p>Explain how the digit zero in a number affects standard form, word form, and expanded form of a number.</p>

Instructional Focus Statements

Level 3:

As students develop an understanding that the third digit of a three-digit number represents the amount of hundreds in that number, they can expand the skill set of reading and writing numerals to 120 from grade 1 to reading and writing numbers to 1000 in standard form, word form, and expanded form in grade 2. Instruction should focus on seeing the various forms of a given number as a cohesive set discussing the similarities and differences between the forms as opposed to discrete, isolated representations.

Drawings and place value cards are particularly helpful as students are making connections between numbers in word form, standard form, and expanded form. Specifically, layered three-digit place value cards foster student made connections between written three-digit numbers and expanded form. A conceptual understanding of expanded form is essential as students utilize place value strategies to add and subtract large numbers in standard 2.NBT.7. Place value concepts provide a convenient way to compose and decompose numbers to facilitate addition and subtraction computations. A common misconception is that students do not recognize the value of the digits in 134 and write the expanded form as 1 + 3 + 4 instead of 100+30+4. Also when working with expanded form, students need to develop the understanding that a 0 represents no groups of tens or ones as opposed to simply representing a place holder.

One final note, teachers and students should be cognizant that when reading and writing whole numbers, the word “and” should not be used as this will cause confusion when students read decimal numbers in subsequent grades (e.g., 532 is stated and written as “five hundred thirty-two” as opposed to “five hundred and thirty-two”).

Level 4:

Students at this level build on their understanding of place value to articulate the meaning of zero in a number with respect to word form, standard form, and expanded form. For example, given the number 507, a student can represent the number with concrete objects or drawings, read and write the number using standard form, word form and expanded form and state that zero in this number indicates there will not be tens represented in any of these three forms of the number. It is important to differentiate this from stating that there are no tens as a student could correctly decompose 507 to 50 tens and seven ones.

The symbol 0 helps us distinguish among numbers such as 507, 57, and 570. Without 0, these numbers would have the same nonzero digits in the same order. Zero is often thought of meaning nothing, a notion that can present difficulties in understanding the value of multi-digit numbers. Zero is one of the most important digits in the base-ten system.

The book, *A Place for Zero* by Angeline Sparagna Lopresti, will connect this topic to children’s literature.

Standard 2.NBT.A.4 (Major Work of the Grade)

Compare two three-digit numbers based on the meanings of the digits in each place and use the symbols $>$, $=$, and $<$ to show the relationship.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Compare two numbers up to 10 and use the symbols $>$, $=$, and $<$ to show the relationship.</p> <p>Compare a one-digit number to a two-digit number and use the symbols $>$, $=$, and $<$ to show the relationship.</p>	<p>Compare two two-digit numbers based on the meanings of the digits in each place and use the symbols $>$, $=$, and $<$ to show the relationship.</p>	<p>Compare two three-digit numbers based on the meanings of the digits in each place and use the symbols $>$, $=$, and $<$ to show the relationship.</p>	<p>Accurately order a set of three or more three-digit numbers from least to greatest or greatest to least based on the meanings of the digits in each place and uses the symbols $>$ or $<$ to show the relationships and provide justification for the comparison.</p>

Instructional Focus Statements

Level 3:

Students developed an understanding of comparing two two-digit numbers utilizing the correct inequality symbol in grade 1. Also, in working with standards 2.NBT.1 and 2.NBT.3, students are focus on examining the amount of hundreds, tens and ones in numbers. Standard 2.NBT.A.4 builds on these understandings to compare two three-digit numbers using the appropriate inequality symbol.

As students have a base understanding of comparing two-digit numbers, instruction should focus on challenging them to discover what happens when a third digit is included as a part of the numbers they are comparing. Students should be encouraged to utilize concrete representations, place value representations, and number lines as they interact with comparing three-digit numbers. Ultimately, students should come to the realization that the number with the most hundreds is greater. If the number of hundreds is the same, the number with more tens is greater. If the number of hundreds and tens is the same, the number with more ones is greater. When students truly understand this concept, it makes sense that one would compare three-digit numbers by looking at the hundreds place first. It is important that students come by this realization through discovery and conversation as opposed to a procedural set of rules presented to them to utilize when comparing three-digit numbers. It is equally important that students are able to articulate and justify their thinking. Conversations should focus on discussing how numbers are related and how to determine which number is greater or less.

Students should have ample experiences communicating their comparisons in words before using symbols. Students were introduced to the symbols greater than ($>$), less than ($<$) and equal to ($=$) in first grade and continue to use them in grade 2 to compare numbers within 1,000. It is important for students to associate the symbols $<$ and $>$ with their real meaning. Rather than use tricks such as alligators or Pac Man, it may help students who confuse the symbols to remember that the open end of the symbol is always closest to the greater number and the closed end is always closest to the lesser number. Students could also be challenged to examine what happens to their comparisons when the order of the digits within numbers is changed.

Level 4:

Students at this level can accurately order a set of three or more three-digit numbers from least to greatest or greatest to least based on the meanings of the digits in each place and uses the symbols $>$ or $<$ to show the relationships. The student provides justification for the comparison (oral or written) by explaining the reasoning used. Students may also choose to use drawings and/or manipulatives to justify their explanations.

Standard 2.NBT.B.5 (Major Work of the Grade)

Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Accurately add or subtract two numbers within 10 using mental strategies. No context is provided.	Accurately add or subtract two numbers within 20 using mental strategies. No context is provided.	Fluently and accurately add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. No context is provided.	Fluently and accurately add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. No context is provided. Students can explain or defend the accuracy of their answer and the strategy used. Students can also be challenged to consider multiple strategies and provide justification for why one strategy is more efficient than another strategy.

Instructional Focus Statements

Level 3:

As stated in the introduction of the Tennessee Mathematics Standards, fluency is the ability to apply procedures accurately, efficiently, and flexibly. Fluency is about a student being able to flexibly think about the problem posed in order to efficiently answer by employing a strategy from their tool box that makes sense in that particular situation leading to an accurate answer. There is no one strategy that works every time for every student. Each child develops fluency from the strategies that individually work best for them.

In kindergarten and grade 1, students began to develop an understanding of the base-ten number system. In 1.NBT.C.4 students added a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100). In grade 2, students continue developing this understanding so that by the end of grade 2 they can fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. This work will be foundational as students extend their learning to adding and subtracting within 1000 using concrete models in 2.NBT.B.7. The use of place value and physical objects from previous course work should be continued in this standard. The natural progression of learning begins with direct modeling. It is important that students who still need direct modeling in order to grasp the mathematics be allowed to do so. With these students, it will be important over the course of the year to help them move from direct modeling to more strategy based approaches. Ultimately strategy based approaches are what builds fluency for students. The foundation of this standard and ultimately fluency for addition and subtraction relies on the understanding of three major concepts; computation of sums and differences of all one-digit numbers, the ability to flexibly compose and decompose numbers, and combining like units (regrouping).

Initially, students may need to use direct modeling with visual models and diagrams to help them grasp the conceptual understanding of decomposing, composing, and combining like units (regrouping). This may be accomplished with base ten blocks, linking cubes, or other hands on manipulatives. Once a student no longer needs the manipulatives, they move to more strategy based thinking. For example, as students find the sum of $36 + 27$, they should understand that the decomposition of each number is 3 tens and 6 ones + 2 tens and 7 ones, respectively. From previous work in grade 1, students should understand that the ones can be combined into a new group of ten. In this example, 7 ones and 6 ones are combined to form a new group of ten and 3 ones. It is imperative that students develop a deep understanding of combining units by using visual representations such as base ten blocks, ten frames, and bundling manipulatives first before moving to a purely strategy based approach. Students should also understand that the composition, including the newly formed unit, results in 3 tens + 2 tens + 1 ten + 3 ones, resulting in a total of 6 tens and 3 ones, or 63. It is also beneficial to provide experiences using open number lines and the hundreds chart, using benchmarks of tens to help students develop strategies for adding and subtracting. Additionally, written equations should be used in conjunction with visual representations.

As students work with multiple representations and strategies, they should be able to make connections between the representations and strategies, including written equations, and explain which strategy is most efficient for them for a particular problem. It is also important to note that this standard integrates with 2.MD.B.6 where students work with sums and differences on a number line. Number line strategies are a very visual way to help build operational fluency for students.

Students should encounter equations that are written both vertically and horizontally. Students should also make estimates when adding and subtracting to determine if their solution is reasonable. Using estimation and the understanding of reasonableness will be a foundational skill that students will use throughout future grade levels/courses. As students become fluent with addition and subtraction within 100, they should be able to accurately compute the correct answer, flexibly use multiple representations, and choose the strategy that is most efficient.

Level 4:

Students should solidify their understanding by flexibly, accurately, and efficiently adding multiple addends within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. The multiple addends or subtrahends should naturally illicit the use of different strategies. Students must explain or defend the accuracy of their answer by not only showing multiple strategies, but also by explaining why one strategy is more efficient than another strategy. As students express their thinking in written and verbal formats, they should use precise mathematical language, diagrams, and written equations.

Standard 2.NBT.B.6 (Major Work of the Grade)

Add up to four two-digit numbers using properties of operations and strategies based on place value.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:	Students with a level 5 understanding of this standard will most likely be able to:	Students with a level 6 understanding of this standard will most likely be able to:	Students with a level 7 understanding of this standard will most likely be able to:
Accurately add three or four numbers with sums less than 20 using properties of operations and strategies based on place value.	Accurately add three or four numbers with sums within 100 which do not require composing or decomposing tens, using properties of operations and strategies based on place value.	Add three or four two-digit numbers with sums within 100 which require composing or decomposing tens, using properties of operations and strategies based on place value.	Add three two-digit numbers with sums greater than 100 which require composing and decomposing tens, using properties of operations and strategies based on place value.	Add four two-digit numbers with sums greater than 100 which require composing and decomposing tens, using properties of operations and strategies based on place value.	Create three different equations each involving the sum of at least three addends and requires regrouping that add up to the provided whole number, given a whole number within 100. Explain the properties of operations or place-value based strategies that could be used to solve each of the three created expressions.	Create three different equations each involving the sum of at least four addends and requires composing tens that add up to the provided whole number, given a whole number greater than 100. Explain the properties of operations or place-value based strategies that could be used to solve each of the three created expressions.

Instructional Focus Statements

Level 3:

As students continue to expand their toolbox with strategies that can be used to efficiently add two two-digit numbers, they should gradually begin to work with a greater number of addends. In standard 2.NBT.B.6, students should add up to four two-digit numbers using properties of operations and strategies based on place value. This should be scaffolded by first introducing students to adding three two-digit numbers and encouraging the use of previously learned strategies that include using the commutative, associative, and identity properties. The commutative and associative properties are particularly helpful as they allow students to reorder a string of numbers so that they are presented in a way that makes combining the numbers more efficient for the student. For example, when adding $24 + 18 + 32$, one student might add 18 and 32 first to get 50 and then add 24 to get a total of 74. Another student might decompose the 18 to $16 + 2$ giving $24 + 16 + 2 + 32$ in order to get $40 + 2 + 32$ yielding 74. Both are correct and both demonstrate ways in which students can flexibly combine numbers, one using a property and one not.

As students use different strategies and properties, they should continue to use visual representations such as a number line as they make sense of which addend order best works for their thinking. As students begin to add with four two-digit numbers, it is easier for them to start with problems with no regrouping and over time transition to working with cases where regrouping is needed. As in previous work, students may need to use strategies that involve manipulatives and direct modeling to help them understand problems that require regrouping.

Levels 4-7:

As students solidify their understanding of adding up to four two-digit numbers, they should be flexible in using multiple strategies and representations. Students should be able to construct a viable argument (MP 3) as to why their way is the most efficient way to add the numbers. Further, they should be able to critique the reasoning of others (MP 3) as their explanations are provided. Additionally, students should be able to produce a string of numbers whose sum is a given whole number greater than 100. In doing so, they should be able to create three different equations each involving the sum of at least four addends and require composing tens that add up to the provided whole number. They should also be able to explain the properties of operations or place-value based strategies that could be used to solve each of the three created expressions.

Standard 2.NBT.B.7 (Major Work of the Grade)

Add and subtract within 1000 using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain the reasoning used.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:	Students with a level 5 understanding of this standard will most likely be able to:	Students with a level 6 understanding of this standard will most likely be able to:	Students with a level 7 understanding of this standard will most likely be able to:
Use a concrete model to add or subtract two whole numbers with sums/differences within 100 which do not require composing or decomposing tens.	Add and subtract two whole numbers with sums/differences within 1000 which do not require composing or decomposing tens or hundreds. Students should explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Add and subtract two whole numbers with sums/differences within 1000 which require composing or decomposing tens or hundreds, but not both. Students should explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Add and subtract two whole numbers with sums/differences within 1000 which require composing or decomposing both tens and hundreds. Students should explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Add and subtract three or more whole numbers with sums/differences within 1000 which require composing or decomposing both tens and hundreds. Students should explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Add four two-digit numbers with sums greater than 100 which require composing and decomposing tens, using properties of operations and strategies based on place value.	Add more than three whole numbers with a sum within 1000, where composing both tens and hundreds is required using two different strategies. Students should explain each using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Additionally, they should be able to describe how their two selected strategies are similar and how they are different.	Subtract more than three whole numbers with a difference within 1000, where decomposing both tens and hundreds is required using two different strategies. Students should explain each using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Additionally, they should be able to describe how their two selected strategies are similar and how they are different.

Instructional Focus Statements

Level 3:

In previous grades, students began to develop a conceptual understanding of adding and subtracting through a progression of three stages of learning: concrete, representational, and abstract. Instruction within grade 1 exclusively focused on understanding addition and subtraction through concrete and representational learning. In standard 2.NBT.B.5 students demonstrate fluency with addition and subtraction within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. As students build on this understanding, they should continue to use concrete and representational learning and transition to adding and subtracting a larger range of numbers within 1000 over time.

In the concrete learning stage, instruction should focus on direct modeling. Students should be physically manipulating objects or creating drawings to solve math problems. The initial focus for this standard should be on having students use direct modeling techniques and strategies to move towards representational strategies to add and subtract within 1000. When thinking about choosing numbers to add and subtract within 1000, consider that direct modeling can become very cumbersome. That said, it is important to select friendly numbers. Students should be encouraged to look for patterns in their process and answers when they are adding and subtracting as this will also cause them to think about more efficient ways of combining the numbers. It is important to note that direct modeling is a necessary developmental phase which allows children who are not ready for more efficient methods a way to explore the same problems as classmates who have progressed beyond this stage.

Base ten blocks are very helpful in helping model hundreds, tens, and ones for students who have developed the conceptual understanding that the rod representing a ten is comprised of 10 ones and a flat representing a hundred is comprised of 10 tens. As students have developed this understanding in working with addition and subtraction in grade 1 and due to the magnitude of the numbers being added and subtracted, it is appropriate to utilize base 10 blocks with students. As students are working with them, make sure to ask students to explain how they are using the blocks to check their understanding of composing and decomposing hundreds and tens.

As students are solidifying their understanding of adding and subtracting within 1000, they should build upon their own “invented strategies” developed in grade 1. Invented strategies are when students begin eliciting their understanding of composing and decomposing numbers in flexible ways in order to more quickly figure out a computation. Some examples of invented strategies are compensation, counting on, counting back, or making a ten. It is important to note that for many students, these strategies are easier, more intuitive, and quicker than the standard algorithm that is not introduced until grade 3. For example, when finding the sum of 234 and 126, students should make use of previous composing and decomposing strategies to make tens and hundreds. Decomposing the addition problem by place value as $(200 + 30 + 4) + (100 + 20 + 6)$ is one way that students may efficiently find the sum. In doing so, students should use previous understandings of regrouping and making tens and hundreds resulting in $300 + 50 + 10 = 360$. This is just one example of a student-invented strategy. Students should be encouraged to compare and contrast to find the strategy that works the best for them. Students should, over time, build on the range of numbers with which they are adding and subtracting.

Students can also subtract within 1000 by viewing a subtraction problem as an unknown addend problem, e.g., $276 + ? = 425$. Also, counting-on and adding-on methods for addition can be used for subtraction. Additionally, many students struggle with subtracting numbers that have zero tens and/or zero ones, sometimes referred to as “subtraction across zeros”. It is imperative that students conceptually understand how to compose and decompose numbers when regrouping is necessary. For example, when subtracting $400 - 274$, students should be able to “break apart” a hundred as ten tens and a ten as ten ones. This should be done with direct modeling with manipulatives and drawings for students to grasp the conceptual understanding that will be needed in future grades.

As students work with concrete and representational strategies, including student-invented strategies, they should be able to explain their reasoning, make connections to different solution paths, and explain the similarities and differences and relationship between the two using precise mathematical vocabulary.

Levels 4-7:

As students extend their understanding, they should be able to add and subtract three or more whole numbers with sums and differences within 1000 which require composing or decomposing both tens and hundreds. Students should also be able to explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Additionally, students should be able to explain, using precise mathematical vocabulary, the connections that exist between multiple strategies across both addition and subtraction. Students should be able to construct a viable argument (MP 3) to justify when strategies are more efficient.

Standard 2.NBT.B.8 (Major Work of the Grade)

Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100– 900.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Use concrete manipulatives and counting by ones to find the number that is ten more than a given two-digit number.</p> <p>Use concrete manipulatives and counting by ones to find the number that is ten less than a given two-digit number.</p>	<p>Mentally find 10 more than a given two-digit number without having to count by ones and explain the reasoning used.</p> <p>Mentally find 10 less than a given two-digit number without having to count by ones and explain the reasoning used.</p>	<p>Mentally add 10 or 100 to a given number 100–900.</p> <p>Mentally subtract 10 or 100 from a given number 100– 900.</p>	<p>Mentally add multiples of 10 or multiples of 100 and explain the reasoning used when given a number 100-900.</p> <p>Mentally subtract multiples of 10 or multiples of 100 and explain the reasoning used when given a number 100-900.</p>

Instructional Focus Statements

Level 3:

The instructional focus for this standard should center around building on the conceptual understanding students developed with standard 1.NBT.C.5 where they used mental strategies to find 10 more or 10 less than a given two-digit number. As students deepen their understanding of place value, they extend the range of numbers they work with to 100-900 mentally finding 10 more or 10 less and 100 more or 100 less than any number in that given range.

As students extend their understanding mentally finding 10 more and 10 less to a larger range of numbers, particular attention needs to be shown to developing conceptual understanding around when the required computation crossing over hundreds. This requires mentally regrouping to create a hundred which can be challenging for students. For example, a student mentally adding $295 + 10$ may struggle to regroup tens to get the third hundred in order to state the answer as 305. Before students are able to use mental strategies to solve this type of problem, they must be given the opportunity to interact with direct modeling in order to have the mathematical foundation needed to move along the learning continuum.

With this standard, finding 100 more or 100 less will most likely be the easier of the two concepts for students to grasp as the pattern is very similar to the pattern students would have discovered when mentally finding 10 more or 10 less to a two-digit number in grade 1. That said, students need to experience a wide variety of both concrete math materials and representational mathematical strategies when adding or subtracting 100 from a number in the range of 100 – 900. Students should be exposed to various strategies and then choose the one that is most efficient and makes the most sense to them. This process will allow students to grow so that they are eventually able to not only add and subtract 100 using mental strategies, but also conceptually understand the computation. This process takes time. Additionally, students need opportunities to explain their reasoning using place value understanding and patterns on the number line or extended hundreds chart as the basis for their explanation. Instruction should not focus on tricks or procedures with no mathematical connections. Instead, instruction should focus on helping students conceptually understand our place value system, discovering patterns that exist, and mathematically discussing the implications of those patterns as they relate to addition and subtraction.

Level 4:

Instruction at this level should focus on students exploring adding or subtracting multiples of 10 or 100 from a given number 100-900. Students should connect adding and subtracting multiples of 10 or 100 to skip counting. For example, to mentally add 300 to 456, students may skip count 456, 556, 656, 756 and determine that the sum is 756. Number lines may be helpful to develop a mental image of skip counting. With number lines, it is important to reinforce that number lines represent whole numbers as lengths when they are using number lines as a tool for adding and subtracting multiples of 10 and 100.

Standard 2.NBT.B.9 (Major Work of the Grade)

Explain why addition and subtraction strategies work using properties of operations and place value. (Explanations may include words, drawings, or objects.)

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify one strategy that can be used to accurately add two numbers within 100. Explain the strategy using drawings or objects.</p> <p>Identify one strategy that can be used to accurately subtract two numbers within 100. Explain the strategy using drawings or objects.</p>	<p>Identify one to two strategies that can be used to accurately add two numbers within 100. Explain why each strategy works using properties of operations and place value.</p> <p>Identify one to two strategies that can be used to accurately subtract two numbers within 100. Explain why each strategy works using properties of operations and place value.</p>	<p>Identify multiple different strategies that can be used to accurately add two numbers within 100. Explain why each strategy works using properties of operations and place value.</p> <p>Identify multiple different strategies that can be used to accurately subtract two numbers within 100. Explain why each strategy works using properties of operations and place value.</p>	<p>Choose two different strategies that can be used to accurately add the same two numbers within 100. Explain how the two strategies are related, explain the strengths and weaknesses of each strategy, and explain why one strategy is more efficient than another using appropriate mathematical vocabulary.</p> <p>Choose two different strategies that can be used to accurately subtract the same two numbers within 100. Explain how the two strategies are related, explain the strengths and weaknesses of each strategy, and explain why one strategy is more efficient than another using appropriate mathematical vocabulary.</p>

Instructional Focus Statements

Level 3:

As students solidify their learning of addition and subtraction, they should be able to explain their strategies based on their understanding of place value and the properties of addition and subtraction. Students should demonstrate this understanding using a variety of concrete methods such as place value manipulatives, hundreds charts, open number lines, and drawings. They should also use representational and student-invented strategies to demonstrate their understanding. The focus of this standard is explaining what it means to add and subtract numbers. Although this is the focus for this standard, students should also be explaining their thinking throughout all OA and NBT standards.

In contextual problems, student explanations should be focused on why they chose a particular operation to solve a problem. Students may use either addition or subtraction to solve the same problem. They should be able to explain their reasoning to why the operation they chose will solve the problem. Students should be challenged to make connections as to why other methods are also accurate for computation. For example, the addition problem $25 + ? = 34$ and the subtraction problem $34 - ? = 25$, could both be used to solve the same contextual problem. The underlying importance is for students to explain their reasoning on the method they selected and accurately solve the problem using any method of their choice.

As students use multiple representations of concrete models, representational strategies, equations, and written explanations in solving addition and subtraction problems, they should be able to make and explain connections between the multiple representations.

Level 4:

As students extend their understanding of explaining why addition and subtraction strategies work using properties of operations and place value, they should be able to add and subtract flexibly with a variety of strategies. This includes adding/subtracting one number in parts, adding by place, and changing and adjusting the number(s). Students should also be able to demonstrate more than one strategy for addition and subtraction and explain the connection between the two strategies. Additionally, students should be able to explain why one strategy is more efficient in some cases than other cases and defend their reasoning using precise mathematical vocabulary.

Measurement and Data (MD)

Standard 2.MD.A.1 (Major Work of the Grade)

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure the length of an object using non-standard units.	Measure the length of an object when given an appropriate tool such as rulers, yardsticks, meter sticks, and measuring tapes.	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes and provide an explanation as to why the measuring device chosen is the best for a given situation. Given an incorrect measure for an object, correct the error and explain why the error may have occurred.

Instructional Focus Statements

Level 3:

Grade 2 students build upon their non-standard measurement experiences in grade 1 (standard 1.MD.A.2) to measure in standard units for the first time. On the surface, measurement seems like a relatively simple concept for students to understand. However, there are conceptual and procedural concepts that must be an intentional focus of instruction. Students are used to counting discrete objects where numbers represent those objects (standard K.CC.B.4). For both this standard and the number line standard 2.MD.B.6, it is important for students to develop the understanding that the “1” on the ruler represents the distance between 0 and 1 as opposed to it representing the tick mark (an object) on the ruler. This is conceptually abstract for students. One manipulative that can help develop this understanding is the use of physical objects, typically squares that are one inch or one centimeter in length. These are often called “length-units” and can help bridge a student from the non-standard units used in grade 1 to using a ruler in grade 2. One

effective application of this manipulative is for students to use these length-unit manipulatives to build their own rulers. They provide a concrete representation to help students see the abstract concept of distance.

Other important mathematical understandings for students to develop include developing an understanding that there is no space between successive length-units, the importance of correctly aligning the zero-point on the ruler at the beginning of the total length, that the number on the ruler represents all of the accumulated distance from zero to that number, and the importance of the units being the exact same size. Often with counting discrete objects, those object may have slight variance in size. For example, when counting cars, they may not all be exactly the same and there still be eight cars, for example, on a table. In measurement, it is crucially important that the units be the same. A teacher might challenge students by lining up 4 larger squares next to 4 smaller squares to say that an object is eight units long to provide students the opportunity for discussion and discourse around if unity in size is important (MP 3).

It is important to note that grade 2 students should initially measure with a ruler that contains only whole number inches as the instructional focus for this grade is on solidifying an understanding of how to measure using standard units. Additionally, as students have not been introduced to either formal fraction or decimal notation, they are not prepared at this grade to integrate those concepts with measurement.

As students are choosing appropriate tools and units to measure objects, it is appropriate that they be allowed to measure in both inches and feet or both centimeters and meters. (MP 5) This provides two things for students. First, they develop an understanding of which is more efficient and second they begin to understand the connection between the size of a unit and the number of units needed providing a natural lead in to standard 2.MD.A.2. Students also need to develop an understanding of when a measuring tape is more useful than a ruler or yardstick. One specific example would be in measuring the circumference of a basketball. Students should be given the opportunity to measure numerous items including items that are curved or distances that are not straight lines. Conceptual understanding of measurement can also be supported through grade appropriate science activities (2.ETS2.1) and other STEM related work.

Level 4:

As students engage in measurement activities, they develop an understanding of the relative size of units within the same system and should be able to justify their choice of unit when measuring. Students may begin relating the units within a system such as noticing that there are 12 inches in a foot and incorporate that as a part of their justification for choosing a specific unit. Additionally, students should be able to look at an incorrect measure, correct the mistake, and provide an explanation for the mistake that was made.

Standard 2.MD.A.2 (Major Work of the Grade)

Measure the length of an object using two different units of measure and describe how the two measurements relate to the size of the unit chosen.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure the length of an object using non-standard units.	Measure the length of an object using two different units of measure.	Measure the length of an object using two different units of measure and describe how the two measurements relate to the size of the unit chosen.	Match the correct units of measure to the objects and justify their reasoning by describing how the measures are related, given two lengths of an object without the unit of measure.

Instructional Focus Statements

Level 3:

Grade 2 students develop an understanding of the inverse relationship that exists between the relative size of a unit and the number of units needed to measure an object. Students need opportunities to explore measurements to discover that the larger the unit, the fewer number of units in a given measurement and the smaller the unit, the more units in a given measurement. Students should measure the same object using different units and reflect on how the overall measurements relate to each other and then on how the measurements relate to the size of the unit chosen. It is important to note that activities can involve both standard and non-standard units of measure to help students develop this understanding. For example, when measuring the height of a door, if one unit selected to measure is small such as inches, students should notice that it takes many of these small units to measure the height. Conversely, when measuring the same door with a larger unit such as yards, it will take many less units to measure the height. Instruction should provide ongoing measurement experiences and activities to help students solidify their understanding that the unit used to measure is as important as the attribute being measured. This will be important when students estimate with standard units of measure in standard 2.MD.A.3. Additionally, this standard provides a good opportunity for students to attend to precision (MP 6) by labeling units when they measure.

Level 4:

Students at this level should be able to extend their understanding by matching the correct units of measure to two measurements for the same object and justify their reasoning. For example, students may be told that a cabinet is both 79 units and 2 units long. When asked which unit is meters and

which unit is inches, students should match 79 to inches and 2 to meters and be able to justify their reasoning. Additionally, students can extend their understanding of units to make inferences about the relative size of objects. For example, if students are told that a red string is 10 regular paper clips long and a blue string is 10 jumbo paper clips long, they should be able to reason that since the number of units used is the same (10) and since the units used have different sizes (jumbo and regular), that the two strings are actually different in length.

Standard 2.MD.A.3 (Major Work of the Grade)

Estimate lengths using units of inches, feet, yards, centimeters, and meters.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify an object with a length of 1 unit without measuring (i.e., identify something in the room that is 1 inch long) for at least one of inches, feet, yards, centimeters, or meters.</p> <p>Choose a reasonable estimate for a measurement provided in inches, feet, yards, centimeters, or meters. Students may only be able to choose correct answers consistently for a single unit.</p>	<p>Identify an object with a length of 1 unit without measuring (i.e., identify something in the room that is 1 inch long) for inches, feet, yards, centimeters, and meters.</p> <p>Choose a reasonable estimate for a measurement provided in inches, feet, yards, centimeters, and meters.</p>	<p>Estimate lengths using units of inches, feet, yards, centimeters, and meters.</p>	<p>Estimate the length of a given object with more than one unit of measure and justify their reasonableness for each estimate.</p>

Instructional Focus Statements

Level 3:

Instruction for this standard should not only focus on helping students be able to accurately estimate, but also on helping them understand what estimating is and the importance of estimation particularly in everyday life. It is important to note that this is the first time estimation is explicitly called out in the standards. Students will have to reason not only with the relative sizes of measurement, but also with the concept of estimation and the idea that it is okay to not have an exact answer. Both measurement and estimation vocabulary should be modeled for students using terms such as a little more than, a little less than, about, and close to.

As students are developing their conceptual understanding of estimating, they should have ample opportunities to estimate a length and then actually measure to check their accuracy. For that reason, this standard integrates well with both standards 2.MD.A.1 and 2.MD.A.2 once students have solidified their understanding of the mechanics of measuring objects. As students use standard units to measure objects, they can also begin estimating lengths

before they actually measure the length. Students should estimate the lengths of objects using inches, feet, centimeters, and meters prior to measuring. Once a student has made an estimate and measured the object, they can then reflect on the accuracy of the estimate made and consider this information for the next estimate. This iterative process not only helps them become better estimators, but also supports the development of measurement skills. Another by-product of estimating is that as students estimate, they must consider the size of the unit helping them to become more familiar with unit size in general.

Another helpful tool for students is for them to develop their own benchmarks of measure for each unit. Students may say a paperclip is about an inch or the desk is about a meter off of the floor. Visual benchmarks help students when they estimate as length is very abstract to grade 2 students. It is important that students develop their own benchmarks as one visual benchmark may work better for a student than another one.

Level 4:

As students extend this understanding of estimation, they should be challenged to estimate the length of one object using two different units of measure and justify the reasonableness of each estimate. This allows students to think flexibly about length and continue making connections between different units of measure within the same system of measure. Students may estimate that a desk is two feet long and also 24 inches long. When asked to explain their estimates, students may begin using repeated addition as a part of their justification incorporating their knowledge that there are 12 inches in a foot. Additionally, students should be challenged to critique estimates and explanations provided by their peers.

Standard 2.MD.A.4 (Major Work of the Grade)

Measure to determine how much longer one object is than another and express the difference in terms of a standard unit of length.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Visually identify which of two objects is longer/shorter.	Determine how much longer one object is than another in terms of a standard unit of length when given the measure of two objects.	Measure to determine how much longer one object is than another and express the difference in terms of a standard unit of length.	<p>Measure more than two objects to determine the length of each in terms of a standard unit of length. Students should make multiple comparative statements about the length of the objects in the set including not only which objects are longer/shorter than others, but also around specifically how much longer or shorter. Students should encounter sets that have some objects with the same length.</p> <p>Explain why it is important for objects to be measured using the same standard unit of length when determining how much longer one object is than another.</p>

Instructional Focus Statements

Level 3:

In grade 1, students learned to compare the length of objects directly and indirectly without actually measuring them. Standard 2.MD.A.4 will extend a student’s understanding when they actually measure to find the length of two objects and then calculating the actual difference in lengths of the two objects. They are moving from knowing that one is longer than the other in grade 1 to finding exactly how much longer in grade 2. Students learn that to

find the difference in length it is essential that they use the same unit to measure both objects. This standard is a natural extension of standard 2.MD.A.1. While not called out in the standard, students should be measuring in inches, feet, yards, centimeters, or meters and should only be interacting with whole number lengths.

Instructional opportunities should focus on students choosing two objects to measure, identifying an appropriate tool and unit, measuring both objects, and then determining the difference in length. Students should make a comparative statement about the differences in the length of two objects. For example, "This object is shorter by 3 centimeters" or "This object is longer by 5 inches". This also provides an opportunity for students to practice estimation (standard 2.MD.A.3) as they can provide an estimation for how much longer one object is than another before actually calculating the difference.

Instruction in measurement intersects with other strands in mathematics. Measurement situations can provide a real-world connection to the computation skills students are learning in standards 2.OA.A.1 and 2.NBT.B.7. Additionally, measurement applications offer opportunities for interdisciplinary learning in subjects such as social studies and science. For example, students could measure the daily growth of a bean sprout and determine the difference from one day to another.

Level 4:

Students should be able to measure more than two objects to determine the length of each in terms of a standard unit of length and make comparative statements about the length of the objects in the collection including not only which objects are longer/shorter than others, but also around specifically how much longer or shorter. Students should be able to justify their thinking using appropriate mathematical vocabulary (MP 3). Additionally, students should be able to explain why it is important that objects be measured with the same standard unit of length in order to provide accurate comparisons.

Students may also be challenged with a set of three paths including a zig-zag path, curved path, and straight path. Students can be challenged to find the length of each path and make comparative statements about the three paths. This will challenge them to use appropriate tools strategically (MP 5) as they determine the length of each.

Standard 2.MD.B.5 (Major Work of the Grade)

Add and subtract within 100 to solve contextual problems involving lengths that are given in the same units by using drawings and equations with a symbol for the unknown to represent the problem.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:	Students with a level 5 understanding of this standard will most likely be able to:	Students with a level 6 understanding of this standard will most likely be able to:	Students with a level 7 understanding of this standard will most likely be able to:
<p>Students should show one strategy for addition and one strategy for subtraction. Adds and subtracts within 20 to solve contextual problems involving lengths given with the same units. The problems must include both addition and subtraction. Represent these problems with mathematical drawings or concrete models.</p>	<p>Add and subtract within 100 to solve contextual problems involving lengths given with the same units which do not require composing or decomposing tens, using <u>two</u> different situations. These two situations must include both addition and subtraction. Represent these problems with mathematical drawings or concrete models.</p>	<p>Add and subtract within 100 to solve contextual problems involving lengths given with the same units which do not require composing or decomposing tens, using <u>two</u> different situations. These two situations must include both addition and subtraction. Represent these problems with a mathematical drawing and equation with a symbol for the unknown number.</p>	<p>Add and subtract within 100 to solve contextual problems involving lengths given with the same units which require composing or decomposing tens, using <u>two</u> different situations. These two situations must include both addition and subtraction. Represent these problems with a mathematical drawing and equation with a symbol for the unknown number.</p>	<p>Add and subtract within 100 to solve contextual problems involving lengths given with the same units which require composing or decomposing tens, using <u>three</u> different situations. These situations must include both addition and subtraction. Represent these problems with a number line model and equation with a symbol for the unknown number.</p>	<p>Add within 100 to solve contextual problems involving three or more lengths given with the same units which require composing or decomposing tens. Represent these problems with a number line model and equation with a symbol for the unknown number.</p> <p>Subtract within 100 to solve contextual problems involving three or more lengths given with the same units which require composing or decomposing tens. Represent these problems with a number line model and equation with a symbol for the unknown number.</p>	<p>Solve contextual problems involving four or more lengths in which the situation requires a mix of addition and subtraction in order to solve the problem. Additionally, the problem involves lengths given with the same units and the mathematics requires both composing and decomposing tens. Represent these problems with a number line model and equation with a symbol for the unknown number.</p>

Instructional Focus Statements

Level 3:

In standard 2.OA.A.1, students develop an understanding of adding and subtracting within 100. The focus of this standard is to specifically integrate lengths that are given in the same units into contextual situations with the same numeric boundaries. Students should continue to use drawings and equations with a symbol for the unknown to represent the problem. Students should use their understanding of addition and subtraction to combine and compare lengths of objects in both one and two step problems. Students should be able to add two lengths to obtain the length of the whole. Students should also be able to subtract one length from another to find out the difference in lengths. In doing so, students should be provided with a variety of opportunities and multiple representations to apply addition and subtraction to solve problems that involve length. For example in the contextual problem, *“Jill built a toy train that is 25 inches long. Sammy built a toy train that is 16 inches long. How much longer is Jill’s train than Sammy’s train, in inches?”* Students should use concrete models and drawings to represent the length of the trains and determine the difference between the two. As with other grade-level standards, this contextual problem can be solved with either addition or subtraction. Students should write an equation that represents the contextual problem such as $25 - 16 = n$ or $16 + n = 25$.” As students use addition and subtraction equations to represent problems, they should be able to explain the relationship between the two representations.

It is also important that students work with the same unit of measurement in contextual problems. As students attend to precision, they should be presented with realistic contextual problems and manipulatives to make a real-world connection. Students may use student-created number lines and drawings that are not true to size. Although this is appropriate, non-precise representations may be confusing to students. That said, this standard is a nice integration of 2.MD.B.6 where students represent whole numbers from 0 on a number line and know that the points corresponding to the numbers on the number line are equally spaced. Using graph paper and folded paper can be useful tools when drawing equally spaced points on a number line.

As students solve contextual problems, they should be able to justify their choice of operation. As mentioned above, both addition and subtraction equations can be used to solve problems and it is imperative that students are able to explain their reasoning. Teaching key words to associate with addition and subtraction should not be an instructional focus. Instruction should focus on developing an understanding of what operation can be used to solve the problem rather than focusing on key words that sometimes, but not always, associate with the operation.

Level 4-7:

As students extend their understanding of adding and subtracting within 100 to solve contextual problems involving lengths, they should be flexible with representing problems using multiple representations such as a number line, equation, and written explanation. Instruction should focus on extending student understanding as they are challenged to work with three and four objects embedded in multi-step problems. These situations should include a mix of addition and subtraction in order to solve the problem. Students should also be able to explain their chosen solution path using precise mathematical vocabulary.

Standard 2.MD.B.6 (Supporting Content)

Represent whole numbers as lengths from 0 on a number line and know that the points corresponding to the numbers on the number line are equally spaced. Use a number line to represent whole number sums and differences of lengths within 100.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:	Students with a level 5 understanding of this standard will most likely be able to:	Students with a level 6 understanding of this standard will most likely be able to:	Students with a level 7 understanding of this standard will most likely be able to:
Locate a whole number and explain that the number represents the length from zero to that number, given a number line model marked with whole number increments.	Represent addition and subtraction of whole number lengths within 20 when given a number line model.	Represent addition and subtraction of whole number lengths within 100 by creating a number line model. Students mark and label the number line model they draw with equal spaces and use the number line to model their strategy and solution.	Represent addition and subtraction of whole number lengths within 100 by creating a number line model. Students mark and label the number line model they draw with equal spaces and use the number line to model their strategy and solution.	Solve contextual problems involving addition and subtraction of lengths within 100. Students model the problem by drawing a number line model and an equation to represent the situation. Students defend their strategies and solutions with a verbal or written explanation.	Write the addition equation modeled and then generate a contextual problem involving lengths that can be solved by the provided model, given a number line model depicting addition within 100.	Write the subtraction equation modeled and then generate a contextual problem involving lengths that can be solved by the provided model, given a number line model depicting subtraction within 100.

Instructional Focus Statements

Level 3:

In previous grades, students may have encountered and worked with number lines as a strategy with counting and cardinality, addition, and subtraction. Standard 2.MD.B.6 will be a student’s first formal introduction to number lines. The instructional focus for this standard is for students to understand and represent whole numbers as lengths from 0, know that the points corresponding to the numbers on the number line are equally spaced, and use the number line for addition and subtraction.

Students should understand that the points corresponding to the numbers on the number line are equally spaced. That said, it is important for students to be given ample opportunity to work equally with pre-generated and student-generated number lines. Instruction should start with pre-generated number lines. It is important for students to understand that the points on the number line are equally spaced. As students begin to transition to student-generated number lines, they should be given opportunities and methods to create number lines with equally spaced points. Drawing equally

spaced points on a number line is difficult for students. Using graph paper and folded paper can be useful tools when attending to precision to create number lines with equally spaced points.

As students use number lines to represent whole number sums and differences, they should model counting on and counting back by drawing “jumps” on the number line to represent the problem. Students should also use previous addition and subtraction strategies of adding to, taking from, putting together, taking apart, and comparing using illustrated “jumps” on the number line. As students utilize number lines to represent addition and subtraction, it is appropriate to work with a variety of increments, including ones, fives, and tens. It is easier for students to visualize the illustration of addition and subtraction by one-unit “jumps” when using a number line in increments of 1’s. However, when working with a larger range of numbers it may be easier for students to work with an open number line. The focus for this part of the standard is on addition and subtraction and it is appropriate to use non-precise, student-generated number lines. For example, when finding the difference of $83 - 29$, students may use an open number line and place a point on 83 and “jump” back 4 increments of 10 units and “jump” back 4 increments of 1 resulting in the difference of 54. This may result in the accurate solution without attention to the equally spaced increments of tens and ones on the same number line, which could be confusing to students. It is more important that students label their “jumps” and understand the difference between the two numbers rather than the spacing accuracy on the number line. In this case, an open number line may be more efficient as students are working with increments of 10’s and 1’s. This is one of many strategies for using a number line for addition and subtraction and students should explore multiple solution paths and be able to explain why they have selected their chosen method. Additionally, students should be able to make connections between solution paths and explain their reasoning using precise mathematical vocabulary.

Level 4-7:

As students extend their understanding, they should solve contextual problems involving addition and subtraction within 100 increasing in rigor over time. Students should be able to model the problem by drawing a number line model and an equation to represent the situation. Students should also be able to defend their strategies and solutions with a verbal or written explanation. Additionally, students should use a given number line model depicting addition or subtraction within 100, write the equation modeled, and then generate a contextual problem that can be solved by the provided model.

Standard 2.MD.C.7 (Supporting Content)

Tell and write time in quarter hours and to the nearest five minutes (in a.m. and p.m.) using analog and digital clocks.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Tell and write time in hours using analog and digital clocks.	Tell and write time in hours and half-hours using analog and digital clocks.	Tell and write time in quarter hours and to the nearest five minutes (in a.m. and p.m.) using analog and digital clocks.	Show the time using analog and digital clocks, Given a time in quarter hours or a time to the nearest five minutes.

Instructional Focus Statements

Level 3:

Students are formally introduced to telling time in grade 1 where they read clocks to the nearest hour and then to the half hour using digital and analog clocks (standard 1.MD.A.2). In grade 2, students learn to read to the quarter hour and 5-minute intervals. Students should have the opportunity to relate telling time to everyday experiences at school such as the time school starts and ends, lunch time, recess time, etc. Students should also link time to their home experiences such as the time they get up, eat dinner, go to sleep, and so on.

Students should have opportunities for telling time embedded throughout the day. They should learn that there are twenty-four hours in each day and should explore the two cycles of twelve hours in a day - a.m. and p.m. For example, students should be able to state that they ate breakfast at 6:30 a.m. but teacher eat dinner at 6:30 p.m. Vocabulary such as noon or midnight are also helpful for students when thinking about a.m. and p.m. As in grade 1, teachers should model for students how to write time using colon notation.

When students are telling time to quarter hours, it is helpful to connect this to partitioning a circle into halves and fourths or quarters (standard 2.G.A.3) as students do not intuitively connect 15 minutes with a quarter hour. To this point, quarters to them are related to the number 25 due to their interaction with monetary amounts in grade 1. Placing a quarter hour at 25 is a very common mistake for grade 2 students and instruction should focus on helping them understand why it is at 15. When students are working on telling time to the nearest five minutes, they should be encouraged to skip-count by fives to help them tell time by five minute intervals. It is important to help students understand first why they are skip counting by fives and second why they stop skip-counting at 55 and what happens at 60.

Level 4:

Students should be able to demonstrate conceptual understanding of time with digital and analog clocks by representing the same time written or spoken in words in both the analog and digital format. Students need to make sure they are using the correct placement of the hour hand when showing time to the hour and half hour. Students can duplicate the time on a clock model or draw the hands on a printed copy of a clock.

Standard 2.MD.C.8 (Supporting Content)

Solve contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and \$ symbols appropriately.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Count the value of a set of like coins less than one dollar using the ¢ symbol only.</p> <p>Count the value of a set of like bills using the dollar sign.</p>	<p>Count the value of a set of mixed coins less than one dollar using the ¢ symbol.</p> <p>Count the value of a collection of mixed bills using the dollar sign.</p>	<p>Solve contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and \$ symbols appropriately.</p> <p>Solve simple two-step contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and \$ symbols appropriately.</p>	<p>Solve complex, two-step contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and \$ symbols appropriately.</p>

Instructional Focus Statements

Level 3:

In grade 2, students solve word problems involving dollars and cents. Since students have not been introduced to decimals, problems should not be notated in decimal notation. An alternative is to notate \$1.25 as \$1 and 25¢. This allows students to interact with more authentic problems without interacting with decimal notation. In grade 1, students counted the value of like sets of coins comprised of pennies, nickels, dimes, and quarters that was restricted to being less than a dollar. In grade 2, students solve contextual problems many of which will require them to work with the value of a mixed set of coins, compare the value of two sets of coins, make and recognize equivalent collections of coins (same amount but different arrangements), select coins for a given amount, and make change. Students should have a wide range of experiences with money in grade 2.

Working with groups of mixed coins can be a challenge for young children as money is a non-proportional model (the value of the coins is not physically related to the value of the other coins). Providing opportunities for students to solve a problem involving the same quantity of like coins such as "Peter has 5 dimes. John has 5 nickels and Steve has 5 pennies. Who has the most money?" can help students make sense of the total value of each set is disproportionate from the quantity. The use of a hundreds chart or number line to count both like and mixed coin values can help students make sense of

the non-proportional coin values. As with finding the value of sets of like coins, skip counting remains a good strategy for students to employ as they work with mixed coin sets. Working with money offers a real-world opportunity for students to use the strategies they have developed for adding and subtracting within 1000 (standards 2.OA.A.1 and 2.NBT.B.7).

Just as students learn that a number (38) can be represented different ways (3 tens and 8 ones; 2 tens and 18 ones) and still remain the same amount (38), students can apply this understanding to money. For example, 25 cents can look like a quarter, two dimes and a nickel, and it can look like 25 pennies, and still all remain 25 cents. This concept of equivalent worth takes time and requires numerous opportunities to create different sets of coins, count sets of coins, and recognize the “purchase power” of coins (a nickel can buy the same things a 5 pennies). This same exploration should be applied to mixed bill combinations. For example, "How many different ways can you make \$12 using \$1, \$5 or \$10 bills?"

As teachers provide students with sufficient opportunities to explore coin values (25 cents) and actual coins (2 dimes, 1 nickel), teachers will help guide students over time to learn how to mentally give each coin in a set a value, place the random set of coins in order, and use mental math, adding on to find differences, and skip counting to determine the final amount.

After success with counting coins from a contextual situation, students should be given the opportunity to solve contextual problems involving all problem types which involve counting sets of mixed coins and mixed dollar amounts. Teachers should ensure students have opportunities to talk and make sense of the problems they are solving. Students should communicate their thinking and justify their answers for the contextual problems.

Level 4:

As students become efficient with solving contextual problems with money, they should be challenged to solve a wide range of more complex, two-step contextual problems. These contextual problems should involve both addition and subtraction and should encompass a wide variety of situations. Challenging problem solving situations offer students the opportunity to further solidify their understanding of working with monetary amounts. Students should be encouraged to attend to precision (MP 6), look for and express regularity in repeated reasoning (MP 8), and make sense of problems and persevere in solving them (MP 1). Students should also be able to explain their reasoning to others.

Standard 2.MD.D.9 (Supporting Content)

Generate measurement data by measuring lengths of several objects to the nearest whole unit. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Show the measurements by making a line plot, when given measurement data pre-sorted into categories and a pre-drawn line plot where the horizontal scale is marked off in whole-number units.	Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units, given measurement data pre-sorted into categories.	Generate measurement data by measuring lengths of several objects to the nearest whole unit. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	Generate measurement data by measuring lengths of several objects to the nearest whole unit. Generate a second set of data by measuring the same objects using a different unit. Show the measurements by making two line plots, one for each set of data, where the horizontal scales are marked off in whole-number units. Compare and contrast the two line plots noticing similarities, differences and patterns within the data.

Instructional Focus Statements

Level 3:

Standard 2.MD.A.1 is a prerequisite for the measurement part of this standard. Students must be able to correctly measure to the nearest unit before they can generate accurate data based on measurements. That said, students can learn the mechanics of drawing a line plot from a provided table of measurement values while they are still solidifying their understanding of measuring.

Students in grade 2 have had no previous experience creating line plots. A line plot is a graph that shows the frequency of data occurring along a portion of a number line diagram and, at this grade, marked in whole number units. To create a line plot, a partial number line diagram is drawn and a mark

(typically an X or a dot) is made above the corresponding value on the line for every corresponding data point. As students are collecting data, they really have a choice of making a record of their data first and then creating the line plot or creating the line plot as they measure data which will require them to reason quantitatively (MP 2) as they set up their partial number line diagram. This is also a good opportunity for students to try both ways and then construct a viable argument for which method is better (MP 3). Another interesting discussion can focus on outliers within data. Students can be challenged to think through if the outlier should be there because an object is simply longer/shorter than all of the rest or if the data point was generated by inaccurate measurement. Ultimately it is important to note that the focus of this standard is more on creating and interpreting line plots than about generating data. Students should experience a balance of generating their own data and data being provided for them as measuring objects is quite time consuming at this grade level.

Level 4:

As students extend their understanding of line plots, they can be challenged to make two line plots to represent the same set of objects where each is representing the objects measured in different units. For example, students could measure a set of objects in inches to create their first line plot and then measure the same objects in feet to create a second line plot. One adaptation could be to provide students the data. Students should be able to compare and contrast the two line plots noticing similarities, differences, and patterns that between the two plots. Additionally, students can be challenged to make conjectures about the patterns that they see and test their conjectures by measuring new sets of objects.

Standard 2.MD.D.10 (Supporting Content)

Measure to determine how much longer one object is than another and express the difference in terms of a standard unit of length.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Organize, represent, and interpret data with up to three categories. Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</p> <p>Complete a pictograph which represents a limited data set with up to three categories. Each picture symbol represents a single element in the data set.</p> <p>Complete a bar graph with intervals of one which represents a limited data set with up to three categories.</p>	<p>Draw a pictograph to represent a sorted data set with up to three categories. Each picture symbol represents a single element in the data set.</p> <p>Draw a bar graph with intervals of one to represent a sorted data set with up to three categories.</p> <p>Solve one-step addition and subtraction problems related to the data presented in a provided pictograph or bar graph.</p>	<p>Draw a pictograph to represent a data set with up to four categories. Solve one and simple two-step addition and subtraction problems related to the data in a graph.</p> <p>Draw a bar graph with intervals of one to represent a data set with up to four categories. Solve one and simple two-step addition and subtraction problems related to the data in a graph.</p>	<p>Draw a pictograph to represent a data set with more than four categories. Create and solve one and two-step addition and subtraction contextual problems related to the data in the graph.</p> <p>Draw a bar graph with intervals of one to represent a data set with more than four categories. Create and solve one and two-step addition and subtraction contextual problems related to the data in the graph.</p> <p>Decide if a pictograph or bar graph better represents a data set and provide justification.</p>

Instructional Focus Statements

Level 3:
 In grade 1, students organized data with up to three categories and asked and answered questions about the data. In grade 2, students expand on this to collect and organize data in four categories and then represent that data specifically in pictographs and/or bar graphs with intervals of one. Additionally, students should be solving one and two-step addition and subtraction problems related to the data in the graph. This supports standard 2.OA.A.1.

Instruction should focus on how to draw a pictograph or bar graph and then on answering questions related to the graphs incorporating a variety of problem types.

Pictographs are one of the most intuitive types of graphs for grade 2 students. Due to experiences with counting and one-to-one correspondence coupled with an understanding of non-proportional representations from working with monetary value of coins, students tend to have an easier time developing a conceptual understanding of and answering questions about data represented in pictographs. That said, as this is a student's first experience reading and drawing pictographs, students will need instruction on how to read a pictograph, how to physically represent data on the graph, what a scale is, how to use a scale, why the scale is important, and when to use a scale of one and when to use a different scale such as two. Students need opportunities to interact both with vertical and horizontal graphs. Students should experience a balance of opportunities both collecting their own data prior to creating pictographs and creating pictographs from provided data.

Bar graphs are less intuitive for students. As with pictographs, students will need instruction on how to read a bar graph, how to physically represent data on the graph, what a scale is, how the scale on bar graphs differs from the scale on pictographs, and why the scale is important. Students should discuss ways in which bar orientation (horizontal or vertical), order, thickness, spacing, shading, colors, and so forth make the bar graphs easier or more difficult to read and interpret. With bar graphs, one of the axes will have numerical meaning and will represent segments of a number line diagram. As this scale represents counts in whole numbers (sometimes called a count scale), when students answer "how many more" or "how many less" questions, they are supporting their understanding of finding differences on a number line diagram (standard 2.MD.B.6). Students should experience a balance of opportunities both collecting their own data prior to creating bar graphs and creating pictographs from provided data.

This standard integrates well with standard 2.G.A.1 as students can make graphs from sorted shapes. Additionally, it provides the opportunity to review and reinforce standard K.MD.B.3 by sorting coins into categories followed by answering addition and subtraction questions about the data (value of various sets of coins) which supports standard 2.MD.C.8.

Level 4:

Students at this level should be challenged to graphically represent a data set with more than four categories. With pictographs, students should thoughtfully choose the scale for their graph and be able to justify their choice. Additionally, students should create their own one and two-step addition and subtraction contextual problems related to the data in their graph and provide solutions for their problems. Students should also be challenged to choose which type of graph best represents their data and provide justification for their choice (MP 3).

Geometry (G)

Standard 2.G.A.1 (Supporting Work)

Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Draw two-dimensional shapes having specified attributes (as determined directly or visually, not by measuring), such as a given number of angles or a given number of sides of equal length.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Correctly name shapes regardless of their orientation or size.</p> <p>Describe similarities and differences between two-dimensional shapes.</p>	<p>Identify triangles, quadrilaterals, pentagons, hexagons, and cubes and identify at least one attribute of each shape.</p> <p>Draw two-dimensional shapes to possess defining attributes such as number of sides and number of vertices.</p>	<p>Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.</p> <p>Draw two-dimensional shapes having specified attributes (as determined directly or visually, not by measuring), such as a given number of angles or a given number of sides of equal length.</p>	<p>Draw two-dimensional shapes having at least two specified attributes (as determined directly or visually, not by measuring), such as a given number of angles and a given number of sides of equal length.</p>

Instructional Focus Statements

Level 3:

In previous grades, students interacted with naming two-dimensional and three-dimensional shapes, drawing two-dimensional shapes, and identifying what constitutes defining attributes of shapes. Students will have previously worked with triangles, a wide variety of different types of quadrilaterals, hexagons and cubes. The terms quadrilateral, rhombus, trapezoid, and pentagon may be unfamiliar to students and instruction may need to focus on helping students develop a conceptual understanding of the uniqueness of each of these shape classifications. As students are developing an understanding of new types of shapes, instruction could present opportunities for students to sort and classify shapes. For example, students could sort two-dimensional shapes by the number of vertices. Additionally, students should be challenged to identify and articulate their own sorting criteria for a set of shapes as this will help reinforce the concept of which attributes are defining and which ones are not. It is important to note that students do not need to know the formal names of triangles but should be able to identify that any three sided closed figure with three vertices is a triangle and recognize that all triangles do not look alike.

As students have had experiences drawing two-dimensional shapes in previous grades, instruction in grade 2 should focus on students drawing shapes with very specific attributes such as a given number of angles or a given number of sides of equal length. This standard builds on work from standard 1.G.A.1 where students drew two-dimensional shapes with defining attributes such as the number of sides or number of vertices. Students should explore and draw shapes in a variety of orientations and configurations as it is important that they have a diversified lens for looking at shapes.

Level 4:

Students at this level should be able to articulate attributes that define a shape and extend this understanding to state attributes that are not present in a shape. To take this concept a step further, students could be asked to draw a non-example of a given shape (e.g., draw a shape that is not a quadrilateral and explain why it is not a quadrilateral). Students can also be challenged to draw two-dimensional shapes having at least two specified attributes (e.g., draw a shape with four angles and with two sides of equal length).

Standard 2.G.A.2 (Supporting Work)

Partition a rectangle into rows and columns of same-sized squares and find the total number of squares.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Construct a wide variety of different rectangles when given squares and count to determine how many squares compose each rectangle.	Identify how many squares are in a given rectangle that has been pre-partitioned into rows and columns of same-sized squares by counting. Partition a rectangle into two or four equal shares.	Partition a rectangle into rows and columns of same-sized squares and find the total number of squares.	Partition a simple, rectilinear shape into rows and columns of same-sized squares and find the total number of squares.

Instructional Focus Statements

Level 3:

The instructional focus for this standard should be centered on students partitioning a rectangle into squares (or square-like regions) and then determining the total number of squares. It is more important that the regions be the same basic size than that they be perfect squares. This standard supports standard 2.OA.C.4 where students are finding the number of objects arranged in rectangular arrays using repeated addition. Both standards play an important part in building a necessary foundation prior to formally defining multiplication in grade 3.

This standard explicitly connects equal sharing and partitioning with rectangles of various sizes supporting students' necessary foundational understanding of topics precursory to multiplication. Teachers should plan activities with square tiles providing students the opportunity to discover how many tiles it will take to cover an entire rectangle followed by a discussion of why all rows have the same number of tiles, why all columns have the same number of tiles, and the most efficient way to calculate the number of tiles needed. Initially it is helpful to intentionally have students work with rectangles where either the rows or columns contain a number of tiles that fit common, familiar skip-counting patterns. Also supporting standard 2.OA.C.4, students can write an equation to express the total number of tiles as a sum of equal addends. Once students are comfortable building rectangles when given square tiles, instruction should shift to where students are using drawings and partitioning rectangles into rows and columns. Graph paper can be a particularly helpful tool for students struggling with the "same-sized squares" portion of this standard.

It is important to note that initially some students may not be able to distinguish between a row and a column. Precise vocabulary should be modeled during instruction and encouraged in class discussions so all students are able to make a distinction between rows and columns.

Level 4:

Instruction at this level extends students understanding of partitioning rectangles in rows and columns to partitioning simple, rectilinear shapes into rows and columns of same-sized squares and then finding the total number of squares. This is pre-cursory to student formally finding the area of rectilinear shapes in grade 3. Students should be able to explain how finding the total number of squares it takes to cover a rectilinear shape differs from a simple rectangle and provide justification for the methods they use that exceed simple counting by ones.

Standard 2.G.A.3 (Supporting Work)

Partition circles and rectangles into two, three, and four equal shares, describe the shares using the words *halves*, *thirds*, *fourths*, *half of*, *a third of*, and *a fourth of*, and describe the whole as *two halves*, *three thirds*, *four fourths*. Recognize that equal shares of identical wholes need not have the same shape.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Explain that circles and rectangles can be partitioned into smaller pieces.</p> <p>Partition circles and rectangles in 2 equal pieces and identify them as halves.</p> <p>Compose a circle from half or quarter circles.</p> <p>Compose a rectangle from 2 smaller, congruent rectangles.</p>	<p>Partition circles and rectangles into two equal shares, describe the shares using the words <i>halves</i>, and use the phrases <i>half of</i> to describe the relationship between a share and the whole.</p> <p>Partition circles and rectangles into four equal shares, describe the shares using the words <i>fourths</i> and <i>quarters</i>, and use the phrases <i>fourth of</i> or <i>quarter of</i> to describe the relationship between a share and the whole.</p> <p>Describe the whole as two of, or four of the shares.</p> <p>Explain that partitioning circles or rectangles into more equal shares creates smaller shares.</p>	<p>Partition circles and rectangles into two, three, and four equal shares, describe the shares using the words halves, thirds, fourths, half of, a third of, and a fourth of.</p> <p>Describe the whole as two halves, three thirds, and four fourths.</p> <p>Recognize that equal shares of identical wholes need not have the same shape.</p>	<p>Partition the same rectangle into halves in more than one way and explain why all of the halves are equal despite them not having the same shape.</p> <p>Partition the same rectangle into thirds in more than one way and explain why all of the thirds are equal despite them not having the same shape.</p> <p>Partition the same rectangle into fourths in more than one way and explain why all of the fourths are equal despite them not having the same shape.</p> <p>Recognize and articulate that two halves, three thirds, and four fourths each referencing the same whole are equivalent.</p>

Instructional Focus Statements

Level 3:

In grade 1, students partitioned circles and rectangles into halves and fourths. In grade 2, students extend their understanding of partitioning to include thirds. Students also increase their academic vocabulary with respect to fractional amounts using the words, halves, thirds, fourths, and quarters, and the phrases half of, third of, fourth of, and quarter of to describe their thinking and solutions. Additionally, students recognize that a whole is made up of two halves, three thirds, and four fourths.

Creating experiences where students explore shapes that are cut into two, three, or four pieces that are not equal and facilitating discourse around why they are not halves, thirds, or fourths can help reinforce student understanding from grade 1 that fractional pieces must be equal shares of the whole.

Students continue to explore and divide shapes to reinforce their understanding developed in grade 1 that as they create more shares within a whole, the shares get smaller (e.g. fourths are smaller than halves). Students add to this understanding as they explore decomposing the same rectangle into fractional regions in different ways. For example, a rectangle may be split first into four equal pieces that are smaller rectangles and then second into equal pieces along the diagonals that are small triangles. The rectangular fourth and the triangular fourth each represent an equal share of the whole rectangle even though the shares are different shapes.

Level 4:

As students solidify their understanding that a rectangle can be split into the same fractional amount in multiple ways and that equal shares of identical wholes need not have the same shape, they should be able to explain why this occurs. Students should also realize that this phenomenon is not true of circles and provide an explanation as to why. Students should also be challenge with thinking about if equal shares of identical circles can have the same shape. This leads to the discovery that due to the fact that circles have much different properties than rectangles (e.g., circles do not have sides) that all halves, thirds, or fourths of the same circle are congruent (i.e., the same shape and size).