Integrated Math I
Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at this level demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in this same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

TN Department of Education

Quantities (N.Q)

Standard M1.N.Q.A.1 (Supporting Content)
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose units appropriately when solving a simple problem. Choose a graphical representation to represent a real-world problem. Choose a data display that describes the values and units in a problem.	Choose and interpret units appropriately when solving a simple problem. Identify when units need to be converted to the same unit within a contextual problem. Connect values to the units to represent given information. Choose appropriate units in order to evaluate a formula, given an input value. Choose an interpretation of the graph that represent a real-world problem. Choose a data display with	Use units as a way to understand problems and to guide the solution path for a multi-step problem. Choose and interpret units appropriately when solving a multi-step problem, including problems that contain real-world formulas. Recognize the relationship between the units for all variables in a formula. Choose and interpret the scale and the origin in graphs and data displays. Determine the most appropriate data display based on the units given in a problem.	Explain if the information is represented appropriately using mathematical justification, given a numerical and/or a graphical representation of a real-world problem. Create a real-world problem involving formulas and data represented either in a table or graph in which the data must be analyzed for appropriate units and scale. Explain the interpretation of the units, scale, and origin with respect to the contextual situation using precise mathematical vocabulary.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	appropriate units that represent a real-world problem.		

Instructional Focus Statements

Level 3:

Instruction for this standard should focus on the importance of correctly interpreting units and representing contextual problems using appropriate data displays. Students should explore the importance of correctly interpreting units and designing appropriate displays of data to most appropriately represent a contextual problem. Instruction should be centered around tasks that provide real-world, multi-step problems where interpreting units appropriately is critical. Students should be engaged in classroom discourse (MP 8) that promotes explaining their reasoning and justifying quantities as a result of a solution pathway. Students should be given ample opportunity to work with data representations where students have to think critically to set an appropriate scale that displays the data to show key features. This standards is important throughout Algebra I, Geometry, and Algebra II as modeling real-world problems should be a prevalent of all 3 courses. It is imperative that students attend to precision in using, interpreting, and reporting units. This is a standard that students will continue to utilize throughout high school. Students should understand that a key relationship exists between units and appropriate representation of units and this understanding is beneficial to develop a conceptual understanding of units in contextual problems. This standard should be integrated within classroom instruction throughout the year, and students should apply it in descriptive modeling.

Level 4:

One extension of this standard is for students to differentiate between data that is appropriately displayed versus data that is not appropriately displayed. Students can then analyze the difference between both displays providing a critique of the representation. This should be done using logical arguments, including explaining how to improve the representation so that all key features and units are displayed appropriately. Data should be represented graphically, numerically, algebraically, and verbally described. Students should be able to interpret, solve, and represent contextual information using appropriate units. Students at this level should attend to precision when interpreting and using units. Furthermore, students should be given opportunities to synthesize information from multiple sources and produce a descriptive model that represents the contextual situation.

Standard M1.N.Q.A.2 (Supporting Content)
Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

Scope and Clarifications: (Modeling Standard)
Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the units in a problem. Connect the units to the values in a real-world problem.	Identify individual quantities in context of the real-world problem and label them with appropriate units. Determine if quantities are labeled with the correct units in the context of a real-world problem. Recognize extraneous information in a real-world problem.	Identify and interpret necessary information in order to select or create a quantity that models a real-world problem. Explain the meaning of individual quantities in the context of the real-world problem. Attend to precision when defining quantities and their units embedded in context. Explain and justify the relationship between solutions to contextual problems and the values used to compute the solutions. Appropriately interpret, explain the meaning of, and draw conclusions about the quantities in a real-world problems.	Identify, interpret, and justify complex information embedded in a real-word problem containing a variety of descriptors or units in order to solve contextual problems for the purpose of descriptive modeling. Represent quantities in descriptive modeling situations and explain their relationship using numeric, algebraic, and graphical representations.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Make observations about quantities given a graph or model. Explain why information is extraneous in a real-world problem.	

Instructional Focus Statements

Level 3:

In grades K-8, students developed an understanding of measuring, labeling values, and understanding how the value of a number relates to the described quantity. In the high school Numbers and Quantity (NQ) domain, students develop an understanding of reasoning quantitatively and solving problems requiring the evaluation of the appropriateness of the form in which quantities are provided. Instruction for this standard should be integrated with a wide variety of standards throughout the course. Students should extend their understanding of using appropriate quantities in descriptive modeling situations where they can make comparisons between two distinct quantities and justify the quantities appropriately in order to describe or to solve a contextual problem. Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc. Instruction should focus on providing opportunities for students to select appropriate quantities embedded in real-world contextual problems and attend to precision by describing the quantities in descriptive modeling situations. The study of dimensional analysis is an excellent avenue to help students understand how critical values, units, and quantities are used in interpreting information and modeling a real-world problem. Furthermore, students must be given opportunities to write and create appropriate labels for quantities and explain the meaning of the quantities in a context. Being able to identify, interpret, and justify quantities is a skill that will serve students well to have mastered during this course as this standard lays the foundation for using units as a way to understand problems.

Level 4:

Instruction should focus on providing opportunities for students to work with problems that have a variety of descriptors and units embedded in the context. Students should be asked to extend their knowledge of quantities by representing them in multiple formats such as a graphical representation of the given information, algebraic representation of the quantities, and multiple representations to predict or draw conclusions about the solution of the real-world problem. Instruction should provide opportunities for students to analyze and critique the interpretation of quantities in a descriptive modeling problem. Additionally, students should be given ample opportunities to design their own contextual problem in which they would have to use quantities appropriately in order to describe the modeled contextual situation.

Standard M1.N.Q.A.3 (Supporting Content)
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the difference between precision and accuracy.	Choose a solution that is both accurate and reasonable with respect to the contextual situation. Describe how using inaccurate measurements when reporting quantities can affect the solution.	Report a quantity with precision and accuracy. Choose an appropriate level of accuracy that reflects the limitations on measurement. Explain the reasonableness of answers with respect to the context of the problem when reporting quantities as a result of solving the contextual problem. Describe the most common causes of inaccuracies in contextual problems (e.g., when using measurement tools).	Describe the accuracy of a measurement embedded in a real-world context by stating the possible error when appropriate. Explain why it is important to choose an appropriate level of accuracy and what limitations exist on measurement when reporting quantities in contextual problems.

Instructional Focus Statements

Level 3:

This standard builds upon the opportunities students have been given to explain values of numbers in terms of units in previous grades. In middle school, students focused on ratios and proportional relationships and also using quantities to describe data from a statistical lens. Both fields help prepare students for choosing a level of accuracy when reporting quantities. Additionally, students have had experience interpreting and reporting quantities that involve area, volume, and rates.

As this is a modeling standard, students should solve contextual problems and be able to choose a level of accuracy of measurement quantities that is reasonable and makes sense to the contextual situation. For example, when solving a multi-step problem or using graphing technology, students should determine when it is appropriate and not appropriate to use precise values (values that are not rounded or truncated), rounded values, or truncated values. Students should be able to justify their reasoning for using certain values and explain why their choice is important with respect to the context. Additionally, students should experience different solution paths that involve using different forms of values and explain how accuracy does or does not have an impact on the solution within the context of the problem.

Instruction should focus on providing students a plethora of opportunities to use a variety of measurements including measuring tools and graphing technology. Students should have ample time to explore traditional, physical tools as well as electronic, and digital tools. During this exploration, class discussion should focus on helping students ascertain the difference between precision and accuracy and when it is appropriate to apply each of them in certain problems. Furthermore, instruction should be infused with a broad spectrum of different types of units that describe tiny to very large quantities. This is a modeling standard and students should make connections to other disciplines such as science.

Level 4:

Students have a great opportunity to support their understanding of this standard through the lens of a wide variety of other disciplines. Instruction should focus on providing students with experiences involving problem situations that interest them. Include inquiry with this standard and allow students ample time to explore repeated measurement in order to determine an acceptable level of accuracy when reporting quantities. Also, instruction should provide the opportunity for students to analyze and critique the level of accuracy chosen by others to report quantities.

This modeling standard is a great way to make connections to other disciplines, specifically science. An extension of this standard can include applying the concept of significant figures, especially in science related contexts. Additionally, in science contexts, students can apply their knowledge of significant digits and scientific notation to explore tasks and report quantities appropriately.

SEEING STRUCTURE in EXPRESSIONS (A.SSE)

Standard M1.A.SSE.A.1 (Major Work of the Grade)
Interpret expressions that represent a quantity in terms of its context.
M1.A.SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.
M1.A.SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

Scope and Clarifications: (Modeling Standard)
For example, interpret P (1 + r) n as the product of P and a factor not depending on P.
Tasks are limited to linear and exponential expressions, including related numerical expressions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify parts of an expression (i.e., factor, coefficient, term). Define the formal definition of the terms: factor, coefficient, and term. Define the formal definition of the term expression. Label the single entities in an expression.	Recognize arithmetic operations in an expression in order to see the structure of the expression. Understand and use the definitions of terms, factors, coefficients, and like terms in order to describe the structure of the individual parts of the expression. Identify parts of an expression as a single entity. Recognize that individual parts of an expression affect the whole expression. State arithmetic operations performed within an expression.	Interpret parts of an expression (i.e., term, factor, coefficient) embedded in a real-world situation and explain each part in terms of the context. Interpret parts of an expression (i.e,. term, factor, and coefficient) and explain each part in terms of the function the expression defines. Explain the structure of an expression and how each term is related to the other terms by interpreting the arithmetic meaning of each term in the expression and recognizing when combining like terms is appropriate.	Interpret expressions in a variety of forms by explaining the relationship between the terms and the structure of the expression. Interpret parts of complex expressions with varying combinations of arithmetic operations and exponents by viewing one or more of their parts as a single entity. Write and interpret expressions that represent a real-world context and use the expressions to solve contextual problems. Write expressions in a wide variety of formats and then for each

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Interpret an expression by describing each individual term as a single entity and the relationship to the expression.	describe the effects each term has considering them first individually and then considering them as a part of the expression. Identify and explain structure in patterns represented pictorially or graphically and write an algebraic expression to represent the pattern

Instructional Focus Statements

Level 3:

Seeing structure in expressions is the connecting bridge between arithmetic operations in grades K-8 and algebraic thinking in high school. Instruction should build on students understanding of the relationship between arithmetic operations in expressions and equations. Students should explore a variety of expressions in equivalent forms to see and evaluate the structure in each form. Students should be exposed to exponents of varying degree. This allows them to recognize the attributes of a term in order to combine it appropriately with other like terms. Instruction should expose students to a variety of multiple representations and require students to interpret and explain the relationship between the representations. Students should be challenged with complex, multi-variable expressions to interpret.

Furthermore, students must be able to explain individual terms and interpret that term as a single entity and as a whole expression. Instruction should focus on using the structure of the expression to uncover the attributes of the function it defines. Students should also be able to use precise language to explain the relationship between a verbal description and an algebraic representation. Particular focus needs to be placed on translating words into mathematical expressions and vice versa.

This standard appears in both Integrated Mathematics I and Integrated Mathematics II. Tasks are limited to linear and exponential in Integrated Mathematics I. In Integrated Mathematics II, students will extend this understanding with quadratics. In future courses, students will experience this standard with radical and trigonometric expressions, solidifying students' comprehension of the structure of expressions and interpreting the meaning of terms as single entities is imperative.

Level 4:

Students need to be presented with complex expressions that include a combination of different arithmetic operations and interpret in terms of a real-world context. The pinnacle of level 4 understanding is being able understand, interpret, and explain the relationship between equivalent representations of an expression. Students should be able to explain not only the expression in terms of a contextual situation, but also how each term within the expression connects back to the contextual situation.

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Additionally, instruction should focus on relating expressions to real world contexts. For example, students should be given problems that describe contextual situations from multiple perspectives. Students should interpret the contextual situation for each individual perspective and write an expression that represents the context for each. Students should be challenged to interpret the meaning of the expressions created and use them to predict outcomes and solve problems. Instruction should expose students to multiple representations of the expressions by making connections between the equivalent expressions, which will in turn help students recognize the most useful form of an expression depending on context. Students should be challenged to justify why other formats are equivalent and which format is most relevant given the context of the problem.

Standard M1.A.SSE.B.2 (Major Work of the Grade)
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A1.A.SSE.B.2a Use the properties of exponents to rewrite exponential expressions.

Scope and Clarifications: (Modeling Standard)
For example, the growth of bacteria can be modeled by either $f(t) = 3^{(t+2)}$ or $g(t) = 9^{(3t)}$ because the expression $3^{(t+2)}$ can be rewritten as $(3^t)(3^2) = 9(3^t)$.
Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize an exponential expression. Recognize properties of exponents. Without a context, choose an equivalent form of an exponential expression.	Choose an equivalent form of an exponential expression and choose the properties used to transform the expression, from a real-world context.	Generate an equivalent form of an exponential expression and identify the properties of exponents used to generate the expression, From a real-world context.	Generate equivalent forms of an exponential expression, justify each transformation with a property, and explain the benefits of the equivalent expression, from a real-world context.

Instructional Focus Statements

Level 3:

The introduction of rational exponents and practice with the properties of exponents in high school further widens the field of operations students will be manipulating. It is important to note that this is a modeling standard and that the exponential expressions should be embedded in real-world situations. This provides a context for seeing structure in the expression and allows students to see when and why it is beneficial to view them in different forms.

Additionally, it's important to note that the focus is not on writing expressions in simplest form as there really is no simplest form. The form that expressions are written in should be driven by what is being done with the expression in the first place.

Level 4:

Students should continue to demonstrate an understanding of seeing structure in expressions by not only being able to rewrite exponential expressions in various forms, but also in mathematically justifying the steps to reach the desired rewritten form and describing when and why the rewritten form would be beneficial. Students should encounter exponential expressions of increasing difficulty in increasingly more complex real-world situational problems.

CREATING EQUATIONS* (A.CED)

Standard M1.A.CED.A.1 (Major Work of the Grade)
Create equations and inequalities in one variable and use them to solve problems.

Scope and Clarifications: (Modeling Standard)

i. Tasks are limited to linear or exponential equations with integer exponents.
ii. Tasks have a real-world context.
iii. In the linear case, tasks have more of the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear equation in one variable that represents a simple, real-world situation. Solve a one variable linear equation. Solve a one variable linear inequality. Identify if a real-world situation can be represented by a linear or exponential equation. Determine if the solution to a real-world situation requires a one-variable or two variable equation or inequality.	Create and solve a one variable linear equation that represents a simple, real-world situation. Create and solve a one variable linear inequality that represents a simple, real-world situation. Choose an exponential equation to represent a simple, real-world situation. Choose an exponential inequality to represents a simple, real-world situation.	Create and solve a one variable linear, or exponential equation that represents a real-world situation. Create and solve a one-variable linear inequality that represents a real-world situation. Create and solve a one-variable exponential inequality that represents a simple real-world situation.	Create a real-world situational problem to represent a given linear or exponential equation or inequality. Create and solve a one-variable exponential inequality that represents a real-world situation.

Instructional Focus Statements

Level 3:

In Integrated Math I, the variety of function types that students encounter allows students to create more complex equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations and inequalities that evolve from real-world situations. Students should be formulating equations and inequalities, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems. Real-world situations should elicit equations and inequalities from situations which are linear and exponential in nature. It is imperative that students have the opportunity to work with each of these function types equally.

Level 4:

When given an equation or inequality, students can generate a real-world situation that could be solved by a provided equation or inequality demonstrating a deep understanding of the interplay that exists between the situation and the equation or inequality used to solve the problem. Additionally, students should continue to encounter real-world problems that are increasingly more complex. Students should be using the modeling cycle to solve real-world problems.

Standard M1.A.CED.A.2 (Major Work of the Grade)
Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.

Scope and Clarifications: (Modeling Standard)

i. Tasks are limited to linear equations
ii. Tasks have a real-world context.
iii. Tasks have the hallmarks of modeling as a mathematical practice(less defined tasks, more of the modeling cycle, etc.).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear equation that represents a simple real-world or mathematical situation. Choose a linear graph to represent a simple real world or mathematical situation. Determine if the solution to a real-world or mathematical situation requires a one-variable or two variable equation.	Graph a two variable linear equation that represents a simple, real-world or mathematical situation.	Create and graph a two variable linear equation that represents a real-world or mathematical situation.	Create a real-world situational problem to represent a given two-variable linear equation or graph.

Instructional Focus Statements

Level 3:

In Integrated Math I, students need to encounter situations that allow students to create more complex linear equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations that evolve from both mathematical and real-world situations. Students should be formulating equations, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to mathematical and real-world problems.
Mathematical situations should elicit equations from situations which are exclusively linear in nature.

Level 4:

One of the most natural situations for students to create an equation or graph from is a real-world situation. Students need to be exposed to variety of real world situations that illicit varying linear functions. Students should encounter real-world problems that are increasingly more complex over time. They should be using the modeling cycle in order to develop and provide justification for their solutions.

Additionally, students should be posed with an equation and then asked to generate a real-world situation that could be solved by a provided equation. Students with this capability are demonstrating a deep understanding of the interplay that exists between the situation and the equation used to solve the problem.

Standard M1.A.CED.A.3 (Major Work of the Grade)
Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Scope and Clarifications: (Modeling Standard)
For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define variables that represent unknown values in a real-world problem. Describe the difference of a viable solution and a non-viable solution.	Choose an equation or inequality that models the constraint on a variable given a contextual problem. Determine when a solution would viable or non-viable solution, given an equation or inequality that represents a real-world problem. Determine the viability of each solution, given an equation or inequality that represents a contextual situation and a set of possible solutions.	Write an equation or inequality that models the constraint on a variable given a contextual problem. Write a system of equations or inequalities that models the constraint on a variable given a contextual problem. Explain constraints on a variable in context of a real-world problem and interpret solutions to determine the viability by using a graph, table, and equation. Justify the solution that models a real-world problem where there is a limitation on a variable. Interpret solutions as viable or nonviable options in a modeling	Create and provide a solution to a real-world problem that has natural limitations on variables. Explain the solution and its viability using multiple representations (i.e. table, graph, equation) and precise mathematical language. Explain examples of both viable and nonviable solutions in context of a real-world problem. Use multiple representations to justify a solution's viability and explain when one representation elicits a more efficient justification.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		context using multiple representations (i.e. table, graph, equation).	

Instructional Focus Statements

Level 3:

Students begin to develop a conceptual understanding of creating equations that represent contextual problems beginning as early as kindergarten. In the middle grades, students extend this understanding to linear equations. In high school, students expand their knowledge of variables and equations in this standard by experiencing real-world problems that produce limitations on the variables that define the unknown values. Through this modeling standard, students should explore the impact of constraints on variables and see how the constraints effect the table, graph, and equation that represents the real-world problem. Students should also be able to define constraints and write an equation, inequality, or system of equations or inequalities that represents the constraint. It is likely for students to experience trouble creating equations that represent constraints. Using multiple representations of the contextual situation can help students see how the constraint effects the problem. For example, students should have the opportunity to explore the impact of the constraint algebraically, graphically, and numerically.

Instruction should provide a variety of contexts with variable limitations and allow students to explain the solutions in context of the real-world problem. Students should experience both viable and non-viable solutions and make sense of them with respect to the contextual problem. Students should experience problems where they have to decide the best way to report the solution. For example, when a student obtains a solution to a problem involving the amount of animals and the answer includes a fractional part, the student should make sense of the fractional part and determine the best way to report the amount of animals. Instruction should require students to report the solution in context of the problem so that they can make sense and justify the viability of the solution. Since this is a modeling standard, students should have ample opportunity working with applications of equations and inequalities with real-world constraints (i.e. volume and linear programming).

Level 4:

Instruction should provide opportunities for students to explore in-depth the impact of constraints and natural limitations on variables for a real-world problem. Students at this level of understanding should be given the opportunity to design their own real-world problems that would have constraints on the unknown variables. Furthermore, students should be provided the opportunity to critique the solutions of others, and instruction should require students to justify and develop logical arguments for the viability of solutions.

Standard M1.A.CED.A.4 (Major Work of the Grade)
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Scope and Clarifications: (Modeling Standard)

i. Tasks are limited to linear equations.
ii. Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose equivalent forms of a given linear real-world formula.	Rearrange simple real-world linear formulas to highlight a quantity of interest.	Rearrange complex real-world linear formulas to highlight a quantity of interest.	Rearrange real-world linear formulas and explain the benefit of solving the formula for the various variables.

Instructional Focus Statements

Level 3:

In previous grades, students have focused on rearranging simple linear formulas to highlight a quantity of interest. In Integrated Math I, the linear formulas student work with should be fairly complex.

As this is a modeling standard, student should be encountering formulas that come from real-world situations. Additionally, students need to be developing a conceptual understanding of why they might need to write formulas in different ways and what the benefit would be to these various representations of the same real-world formula.

Level 4:

Students need to be exposed to a wide variety of real-world formulas increasing in complexity over time. Additionally, it is imperative that they are able to explain why formulas might need to be expressed in different ways and the benefit that each form provides.

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REASONING with EQUATIONS and INEQUALITIES (A.REI)

Standard M1.A.REI.A.1 (Major Work of the Grade)
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Determine if a linear equation in one variable has one solution, infinitely many solutions, or no solutions. Solving linear equations in the form $x + p = q$ and $px = q$.	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Solve linear inequalities in the form $px + q > r$ or $px + q < r$.	Solve linear equations in one variable with coefficients represented by letters. Solve multi-step linear equations. Solve multi-step linear inequalities in one variable.	Create a linear equation in one variable that has infinitely many solutions and provide mathematical justification as to why. Create a linear equation in one variable that has one solution and provide mathematical justification as to why. Create a linear equation in one variable that has no solution and provide mathematical justification as to why. Explain the difference between a linear equation and a linear inequality and provide examples when each would be used.

Instructional Focus Statements

Level 3:

It is important that students begin developing an understanding that solving any equation is a process. With this understanding, students can organize the various techniques for solving equations into a coherent picture instead of viewing each part in isolation. For example, solving linear equations involves only steps that are reversible (adding a constant to both sides, multiplying both sides by a non-zero constant, transforming an expression on one side into an equivalent expression). Thus, students can deduce that solving linear equations does not produce extraneous solutions.

As linear equations and inequalities have been addressed in previous grades, this is the opportunity for students to interact with more complex equations and really practice looking at the big picture for solving linear equations and inequalities.

Level 4:

Students should be pushed to work beyond simply solving equations and inequalities to create their own that fall within certain parameters. Students with the ability to create equations and inequalities from a provided solution have a deep understanding of the process for solving linear equations and inequalities.

Standard M1.A.REI.B.2 (Supporting Content)
Write and solve a system of linear equations in context.

Scope and Clarifications:
Solve systems both algebraically and graphically.
Systems are limited to at most two equations in two variables.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the solution to a system of equations in two variables from a graph. Identify the solution to a system of equations in two variables from a table. Use substitution to determine if a given solution is correct for the system of equations.	Graph two linear equations and find the solution. Determine if two linear equations create parallel lines, the same line, or intersecting lines.	Solve a system of equations in two variables algebraically through substitution. Solve a system of equations in two variables algebraically through elimination. From a real world situation, write a system of equations in two variables. Interpret the solution of a system of equations in two variables in context. Justify why a system of two linear equations may have one solution, no solutions, or infinitely many solutions.	Create a real world scenario to represent a system of equations in two variables. Determine if the solution to a system of equations is reasonable within the context of the problem. Justify whether substitution or elimination would be more efficient for a system of equations.

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Instructional Focus Statements

Level 3:

Students should understand that the solution to a system of linear equations is the point at which the two graphs intersect. Instruction should use multiple representations including graphs and tables to help students visualize the solution and support their ability to solve algebraically. When focusing on the graphical representation, students should be encouraged to approximate this intersection with both graphs created by hand and with technology. When creating a graph by hand, students will encounter solutions that do not lie on integer points on the graph and should discuss approximate solutions that could exist. The calculation of the intersection on a graphing calculator may be used to allow students to continue this exploration of approximate points of intersection. Encouraging students to substitute their approximated solutions back into the equations can foster the understanding of the importance of accuracy in the solutions.

Algebraically, students should have experience with multiple methods of solving a system. Students can be introduced to the substitution method through a conversation about the property of substitution and how to substitute the value of a variable in place of the variable. Building on this idea can help solidify the understanding of solving for a variable and then substituting the equivalent expression in for that variable in the other equation. The elimination method can be introduced based on the concept that we cannot solve for more than one unknown value at a time. Beginning with equations that have variables that already equal zero when added together will demonstrate to students how to "eliminate" a variable so the other can be found. This will help students understand why multiplying the equations would be necessary to create an inverse pair of terms that would equal zero. Emphasis will need to be placed on the importance of finding both variables since the solution is an ordered pair, and that point where the two equations intersect on the graph. Student learning will be solidified as they understand that the solution they find will be the value of the variables that satisfy both equations. In integrated math II, students will expand their work to include a system of three equations with three variables as well as a system containing a linear and a quadratic function.

Opportunities are provided for students to engage in real-world problems in which they must determine if an approximation of a solution using a graph or an exact solution using other methods is most appropriate for the problem. Students should differentiate among problems where there is one solution, no solutions, or infinitely many solutions and justify the meaning or reason for these results.

Level 4:

Given a system of linear equations, students with a deep understanding of this standard should be able to write a real-world scenario to represent the system and then make a graphical model either by hand or using technology. As students produce their answer, they should be able to determine the reasonableness of the solution and then justify their thinking.

Instruction should also allow students to demonstrate a deeper level of understanding by choosing the most appropriate algebraic method for solving a system. Students should be expected to justify their reasoning and explain the steps in their chosen solution path.

Standard M1.A.REI.C.3 (Major Work of the Grade)
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Plot the ordered pairs from a table on a coordinate plane. Determine if points on a graph form a line. Organize ordered pairs into a table. Use technology to produce the graph of a given equation.	Find corresponding values of y, given an equation and sample values of x. Graph ordered pairs to identify whether they represent a linear or non-linear function.	Find a set of solutions that can be used to create the graph, given an equation. Interpret graphs as the solution set to an equation with two variables. Explain why the points on a curve (or line) would be continuous. Illustrate the relationship between the graphical representation and the solutions to the equation, given a real-world situation.	Analyze the pattern of points to determine the type of function represented by the graph, given a set of ordered pairs. Determine if the graphical representation for a real-world situation is continuous or discrete and justify the reasoning. Use the regression feature on a graphing calculator to determine the curve of best fit given a real-world problem.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

As students increase their understanding of graphs, they begin to connect the solutions of an equation to the graphical representation. Students have some understanding of graphs of linear equations from grade 8 and build on that knowledge to include quadratic and other non-linear equations. It is imperative that instruction focus on the multiple representations of an equation and make connections between the equation, the table, and the graph throughout the learning process. From an equation, students should be asked to find the ordered pairs that are solutions to the equation and generate a table and a graph representing those solutions. Allowing students to choose their own input values can create a variety of different tables that all represent the same equation. This can foster positive discussion to help develop the understanding of the infinite amount of points that lie along a line or curve. Often times, students will interpret solutions at integer values only, so teachers must provide opportunities for discussion of solution sets and ways to represent these values on the coordinate plane. As students evaluate their graph, they determine if all possible solutions are represented to discover the meaning of a continuous line or curve.

As students become proficient in graphing, they will extend their learning to include real-world problems. When solutions are graphed, they will determine if the graphical representation of the real-world problem is continuous or discrete and justify their reasoning. In addition, students should be able to identify the domain and range of the function based on their multiple representations.

Level 4:

Instruction can extend students learning by making connections to real-world problems. When given a table that represents real-world data, discussion could be extended beyond possible solutions to solutions that fit the context. Students should have experience with recognizing limits of an equation within context and making connections to the context effecting whether there solutions would be continuous or discrete. For example, if the original input represents time, it should be understood that time cannot be negative so the equation could not be used for negative x-values.

This standard can be connected with M1.S.ID.B.4 by having students use the regression feature on a graphing calculator to determine the curve of best fit. As students experiment with multiple function types, they will use their knowledge to determine which regression equation will produce the curve of best fit for a given real-world situation and justify their reasoning. In addition, equations in appropriate context can be used for making predictions.

Standard M1.A.REI.C.4 (Major Work of the Grade)
Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the approximate solutions using technology.

Scope and Clarifications: (Modeling Standard)
Include cases where f(x) and/or g(x) are linear, absolute value, and exponential functions. For example, f(x) = 3x + 5 and g(x)=x +1.
Exponential functions are limited to domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the solution of the equation *f(x) = g(x),* given two linear equations *f(x)* and *g(x)*.	Identify the solution(s) for *f(x) = g(x)* when *f(x)* and *g(x)* are linear, absolute value or exponential, given graphs of two equations *f(x) and g(x)*. Choose the solution(s) for *f(x)=g(x)* when *f(x)* and *g(x)* are linear, absolute value or exponential, given two equations *f(x) and g(x)*,	Approximate the solution(s) for *f(x) = g(x)* using technology when *f(x)* and *g(x)* are linear, absolute value or exponential, given two equations *f(x)* and *g(x)* embedded in a real-world situation. Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x).*	Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)* and explain the meaning of the solution in terms of a real-world context.

Instructional Focus Statements

Level 3:

In developing an understanding of what it means to find the solution to two equations using graphing, it is important that just as we did not want algebraically solving equations to become a series of steps unsupported by reasoning, we want to make sure that graphically solving them the reasoning piece is not left out either. The simple idea that an equation can be solved (approximately) by graphing can often lead to a rote series of steps involving simply finding the intersection point(s) without employing the reasoning of what is actually occurring. Explaining why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)* involves a rather sophisticated series of thinking as students must connect the idea of two equations in two variables and how that relates to a single equation in one variable and then understand how both connect to a point(s) on a coordinate plane which is built around two variables. Thus, it is imperative that students reason through this process without being given a truncated set of meaningless steps to follow.

As this is a modeling standard, students should be formulating equations, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions built out of real-world situations.

In Integrated Math I, students are focusing on linear and absolute value functions. Students need the opportunity to interact with both of these function types. Additionally, they need to encounter situations where *f(x)* and *g(x)* are different function types. These should increase in difficulty over time.

Level 4:

Students should continue to be exposed to a wide variety of linear, absolute value, and exponential functions with increasing difficulty embedded in real-world situations. Additionally, they need to explain the meaning of the solution in terms of the real-world context.

Standard M1.A.REI.C.5 (Major Work of the Grade)
Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify if a point is a solution to a linear inequality in two variables. Identify if a point is a solution to a system of linear equalities in two variables.	Choose the graphical representation of the solution to a linear inequality in two variables. Choose the graphical representation of the solution set to a system of linear inequalities in two variables.	Graph the solutions to a linear inequality in two variables as a half-plane. Graph the solution set to a system of two linear inequalities in two variables as the intersection of the corresponding half-planes.	Graph the solution set to a system of more than two linear inequalities in two variables.

Instructional Focus Statements

Level 3:

Instruction should focus on extending a student's understanding of graphing linear inequalities in one variable on a number line to graphing linear inequalities in two variables on a coordinate plane. Students need to make the connection as to why inequalities have multiple solutions. It is important that students understand why they are shading not simply following a set of steps without conceptual understanding.

Additionally, this understanding should then be extended to a system of linear inequalities in two variables.

Level 4:

As students develop a strong command of systems of linear inequalities in two variables, they need to experience a wide variety of systems increasing in difficulty including those with more than two inequalities including those having no solution.

INTERPRETING FUNCTIONS (F.IF)

Standard M1.F.IF.A.1 (Major Work of the Grade)
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if a given table of values represents a function. Identify the domain is the set of input values. Identify the range is the set of output values.	Determine if a given graph represents a function. Find f(a) where a is a real number when given a function f.	Create an example of a function using a set of ordered pairs, a graph, and a table of values to show the correspondence between one input value (domain) and one output value (range). Explain the meaning of a function using correct vocabulary. Interpret the meaning of an expression written in the form of f(a+6), for example.	Identify the domain and the range and determine if the relationship represents a function, given a real-world situation. Determine if the domain and range are continuous or discrete and explain your reasoning, given real-world situations. Create a real-world situation that does not represent a function and explain your reasoning.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

In grade 8, students have used semi-formal notation for functions and refer to the values used in those functions as input and output values. In integrated math I, students build on that knowledge and begin using function notation and the mathematical language that describes a function. Instead of using input and output, students use the mathematical vocabulary of domain and range. Students will understand that a function is a special relationship that assigns to one input value (domain), exactly one output value (range) in a pair of elements. They will recognize that each element of the domain is different, however elements of the range may repeat themselves. Students often confuse this concept with the idea that each value of the range can only be paired with one domain value.

Students determine if a relation is a function by examining sets of ordered pairs, graphs, or tables of values. As students begin to examine graphs to determine if they represent functions, using the vertical line test may be helpful. This does not provide a robust definition of a function that transfers well into other situations in future courses, but provides a concrete tool for students to use as they begin their study of graphs of functions. As students further develop their understanding of functions, they will know that f(x) = y and can construct a viable argument using correct vocabulary to explain the meaning of a function. This standard works well with M1.F.IF.A.2 which includes function notation in a real-world context.

Level 4:

Students with deep level of understanding know that the domain of a function represents the input values and the range represents the output values. They will identify the domain and the range in a real-world situation and understand that each element in the domain is unique and different and is paired with an element in the range. Students will then construct a viable argument explaining why the real-world situation is a function. As students solidify their understanding of what kind of real-world situations create functions, they begin to describe real-world situations that are not functions and explain their reasoning.

Standard M1.F.IF.A.2 (Major Work of the Grade)
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Match input-output tables with a collection of equations. Write an ordered pair given the function notation, for example $f(2) = 10$ is the coordinate (2,10).	Substitute input values into a given equation to reveal output values. Construct an input-output table with ordered pairs.	Determine what each variable represents, given a function that represents a real-world problem. Interpret the meaning of output values when given input values and vice versa, given a function that represents a real-world problem. Use multiple representations to model a function in a real world situation: equation, graph, and table of values.	Explain situations when it is imperative to use function notation. Construct a viable argument to explain the solution of a function in a real world situation. Identify and explain possible restrictions on the domain and range, given a real-world situation.

Instructional Focus Statements

Level 3:

As students begin to use function notation, they understand that f(x) is another name for y at a given x and is read as “the value of f at x” or “f of x”. They also realize that a functions can have a different name such as g(x), h(x) or d(t), for example. Instruction can begin by giving students values of x and an equation written in function notation. Students will use substitution to determine the value of the function at the given x value. For example: given $f(x) = 5x - 8$, find f(-4). This is helpful to build students’ understanding that function notation is not an arithmetic operation. Graphing would be a natural progression to help students understand that a table of values can be constructed from an equation written in function notation and a graph can then be produced from the table. In addition, students should be given a function embedded in a real-world situation where they are to explain what the variables represent and what a solution means in the given context. An example of this might be given that a consultant earns a flat fee of $25 plus $40 per hour for a contracted job, how much will she earn in 16 hours? Students can write the function in function notation, construct a table of values, graph the function, and determine the meaning of the key features of the graph in context. When a question related to the problem is asked, students solve for the numerical answer and provide an explanation as to what the answer means and why it is or is not viable. They should also identify the domain and the range that is represented in the situation and explain why this is correct for the context of the problem. This standard pairs nicely with standard M1.F.IF.A.1.

Level 4:

With a deeper understanding, students can use functions and function notation in a real-world context. Real-world problems should be represented multiple ways. Students can write a function in function notation, construct a table of values, graph the function, and determine the meaning of the key features of the graph in context. When a question related to the problem is asked, students solve for the numerical answer and provide an explanation as to what the answer means and why it is or is not viable. They should also identify the domain and the range that is represented in the situation and explain why this is correct for the context of the problem.

Students can apply their knowledge of functions to a real-world problem such as showing trends across time with a scatter plot, and show functional reasoning between two sets of information such as a name and social security or a name and a cell phone number.

Standard M1.F.IF.B.3 (Major Work of the Grade)
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Scope and Clarifications: (Modeling Standard)
Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

i. Tasks have a real-world context.
ii. Tasks are limited to linear functions, absolute value functions, and exponential functions with domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify intercepts, maximums and minimums when provided a graphical representation of the function.	Identify intervals where a given function is increasing, decreasing, positive or negative when provided a graphical representation of the function. Identify key features of the graph or table of values, and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a linear function embedded in a real-world context. Identify evident intercepts, maximums and minimums when provided a table of values representing a linear or absolute function.	Identify all evident intercepts, maximums and minimums when provided a table of values representing an exponential function with domain in the integers. Identify all evident key features when provided a table of values representing a linear or absolute value equation. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing an absolute value function embedded in a real-world context.	Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given an absolute value function embedded in a real-world context, graph the function, Create a real-world context that would generate a function with the provided attributes, given key features of a linear or absolute value function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Sketch a graph of the function, given a verbal description of the key features of a linear function.	Identify key features of the graph and interpret the meaning of the key features in relationship to the context of the problem, given a graph of an exponential function with domain in the integers embedded in a real-world context. Sketch a graph of the function, given a verbal description of the key features of an absolute value function,	

Instructional Focus Statements

Level 3:

Functions are often described and understood in terms of their key features and behaviors. Instruction for this standard should, in part, focus on helping students develop an understanding of how to identify key features and behaviors from both graphs and tables. That said, instruction should extend beyond simple identification from isolated graphs and tables. As this is a modeling standard, students need opportunities to develop an understanding of the relationship between key features/behaviors and the real-world situation that the function models. The focus should be on developing an understanding of what key features/behaviors are while also developing a strong understanding of their relationship and meaning to real-world situations. Additionally, instruction should provide students with an opportunity to develop an understanding of not only how to identify key features/behavior in graphs and tables, but also on how to generate a graph when provided the key features/behaviors.

Instruction can be very nicely paired with standard M1.F.IF.C.6 where students generate linear graphs from real-world situations. This pairing allows students the opportunity to generate a graph from a real-world situation, identify key features/behaviors, and then discuss their meaning as related to the real-world situation. That said, it is not a requirement of this course that students generate graphs of exponential equations with domain in the integers or graphs of absolute value functions. Thus, discussions around these particular function families will need to be carefully planned out.

Level 4:

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate functions with a pre-determined list of key features/behaviors. Additionally, students with a deep understanding of this standard can interpret key features/behaviors from non-traditional linear, absolute value, and exponential functions embedded in real-world situations.

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Standard M1.F.IF.B.4 (Major Work of the Grade)
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Scope and Clarifications: (Modeling Standard)
For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

i) Tasks have a real-world context.
ii) Tasks are limited to linear functions, piecewise functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the difference between domain and range. Identify the domain and range from a table of values. Identify the domain and range from a discrete graph. Identify the domain and range, given a mapping. Identify the domain and range from a set of ordered pairs. Explain the difference between a continuous function and a discrete function.	Explain how domain relates to the definition of a function. Identify the domain, given the continuous graph of the function. Explain why a function is continuous or discrete given its graph.	Explain how the domain relates to the graph of a function. Explain why a function is continuous or discrete given an equation. Describe how a function's domain is affected when situated within a context. Explain if a function is continuous or discrete, given a context.	Determine an appropriate domain and range, given a function in context, Create a contextual situation to describe a function with a given domain and range. Using the definition of discrete and continuous, compare and contrast sequences and the functions used to model them.

Instructional Focus Statements

Level 3:

As students begin their work with this modeling standard, instruction should start by making sure the students have a good understanding of domain and range. Providing tables, graphs, ordered pairs and mappings for students to identify the domain and range will lead to rich classroom discussion and the multiple representations will give students a deep understanding of the definitions of domain and range. Teachers should have students explain how and why they have identified the domain and range in these varied forms. After students have a clear understanding of how to identify the domain and range, they should have experience relating the domain to the definition of a function. That is, every input must correspond to exactly one output value. They should also identify the domain given the graph of a function. Discussion should include comparing and contrasting discrete and continuous functions and identifying them from a graph and real-world situations.

An example that could be used to help students apply this standard might include small packs of skittles in the check-out line at a convenient store. If one bag contains 10 skittles and 2 bags contain 20 skittles, students create a table of values and then determine if the graph is continuous of discrete. In this particular case, they should relate the number of packs to the domain and the number of skittles to the range. Students may ask if there has to be at least 1 bag at the check-out line and/or is there a limit of bags that could be placed there? After students have determined whether the domain is continuous or discrete, they should graph the function. In addition, careful attention must be paid to real-world problems where the domain might be continuous, but also restricted because of the context. An example might be the cost of a cell phone with respect to its value over time (t). The domain would be continuous, but given the context, t must be greater than or equal to 0. As students become proficient with different function families, they begin to realize that the domain and range do not both have to be continuous or discrete. This is a good place to introduced step functions and explain how the domain might be is continuous while the range is discrete.

Level 4:

As students are exposed to a variety of real-world problems, they begin to realize how unique and different every problem can be, but that every modeling situation will have a domain and range. At a level 4 understanding, students can determine the domain and range of some real-world problems, but should have practice with a variety of scenarios. They should also have practice with identifying whether the function is continuous or discrete and explain why. As they develop a good understanding of domain and range, opportunities should be provided for students to create their own scenarios when given a domain and range. In addition, students should construct an argument explaining why their scenario represents the given domain and range. Instruction should also focus on connecting arithmetic and geometric sequences with the functions that model them. Students should justify why these sequences are discrete, while linear and exponential functions are continuous.

Examples which might be used to support understanding of this standard would include a scenario representing the cost of movie tickets per person. This is a discrete function and the domain and range are restricted to positive integer values. A continuous example might be a punter kicking a football over time (x) compared to height (y). This example also restricts the domain and range to positive values. As students become proficient in determining domain and range and graphing functions, they should be able to create their own scenarios with both discrete domain and range and continuous domain and range.

Standard M1.F.IF.B.5 (Major Work of the Grade)
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Scope and Clarifications: (Modeling Standard)

i. Tasks have a real-world context.
ii. Tasks are limited to linear functions, piecewise functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the average rate of change for a linear function when given a symbolic representation, table, or graph. Choose the estimated rate of change when given a graph of a linear function.	Calculate the average rate of change of a linear function when given a graph. Interpret the rate of change for a linear function in terms of a real-world context. Choose the estimated rate of change for a specific interval when given a graph.	Calculate average rate of change when given an equation or table of a linear, absolute value, piece-wise, or exponential functions, where exponential functions are limited to domains in the integers. Interpret the average rate of change of a linear, absolute value, piece-wise, or exponential functions, where exponential functions are limited to domains in the integers. Estimate the average rate of change for a specific interval of a linear, absolute value, piece-wise, or exponential function, where exponential functions are limited to domains in the integers, when given a graph.	Identify the average rate of change for specific intervals of a function as being greater or less than other intervals of the same function. Compare the average rate of change of multiple intervals of the same function and make connections to the real-world situation. Create a contextual situation and identify and interpret the average rate of change with a specific interval.

Instructional Focus Statements

Level 3:

In grades 6 and 7, students began developing the understanding of ratios and proportional relationships. Their understanding of rate of change involved both ratios and proportions using similar triangles to show the additive and multiplicative conceptual underpinnings of the concept. In grade 8, students extended this understanding to functions by examining rate of change in linear functions. In high school, students should solidify this understanding for linear functions and generalize this concept to applying to additional function types. Students should make the connection that the rate of change is the ratio of the change between the dependent and independent variable. For linear functions, students have discovered that this ratio of change is constant between any two points on the line. Students should now make the connection that, for non-linear functions, the ratio of change is not constant due to the functions curvature. This results in the ability to calculate the average rate of change over a specified interval. For example, for the exponential function $f(x) = 2^x$, the average rate of change from x = 1 to x = 4 is $\frac{f(4)-f(1)}{4-1} = \frac{16-2}{4-1} = \frac{14}{3}$ This is the slope of the line from (1,2) to (4,16) on the graph *f*. This calculation means that over this interval it has an average rate of change of 14/3 units.

It is imperative that students gain a conceptual understanding of the average rate of change for a specified interval for non-linear functions. To grasp this idea, students should draw illustrations of the graph and the secant line connecting the intended endpoints. Students should not only be able to calculate the average rate of change, but they should also be able to generate a visual representation and use the visual representation to estimate the average rate of change over a specified interval. Students will gain a deeper conceptual understanding when they compare their estimations to the actual average rate of change for a non-linear function. As students solidify their understanding, they should be able to explain what the average rate of change means in the context of a problem when given symbolic representations, tables, graphs, or contextual situations. As students use multiple representations to evaluate the average rate of change, they should be able to explain the relationship between the multiple representations using both appropriate mathematical language and appropriate justifications.

Level 4:

Students should extend their understanding of average rate of change by comparing the average rate of change of one interval to another interval of the same function. Students should also further their understanding by creating their own contextual situations and interpreting the average rate of change for a significant interval. Students should be intentional in determining which interval or intervals they select and explain the importance of the interval(s) with respect to the context using both precise mathematical vocabulary and precise justifications.

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Standard M1.F.IF.C.6 (Supporting Content)
Graph functions expressed symbolically and show key features of the graph, by hand and using technology.
M1.F.IF.C.6a Graph linear functions and show its intercepts.

Scope and Clarifications:
Tasks are limited to linear functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent a constant rate of change between two variables as slope of a line. Use characteristics of the symbolic representation of a linear function to determine the behavior of the graph. Identify y = x as the parent function for linear function Explain the effects of slope & intercepts on a linear function.	Graph a linear function by hand and using technology. Identify the slope and y-intercept, given a linear function expressed symbolically in slope-intercept form. Locate the x-intercept and y-intercept of a linear function by substituting y with 0 and x with 0, respectively. Graph a line by plotting the x- and y-intercepts. Give and explain the domain and range of a linear function.	Graph a linear function by hand and using technology and identify the slope and intercepts. Explain the effects of slope and intercepts of a linear function. Attend to precision when illustrating intercepts and understand the domain and range for all linear functions.	Explain the relationship that exists between a contextual problem and the key features of a graph for linear function. Explain the relationship that exists between the graph, key features, and table of values that represent a linear function. Critique graphs drawn by others to ensure intercepts are shown efficiently and appropriately. Write the linear function symbolically, given a graph.

Instructional Focus Statements

Level 3:

In grade 8, students used functions to model relationships between quantities and construct a function to model a linear relationship. Students also determine if a function is linear or nonlinear, and they had experience interpreting and representing functions algebraically, numerically, graphically, and verbally. In Integrated Math I, students should build on this foundational knowledge and provide students the opportunity to work with functions that vary in their symbolic representation. For example, students should experience point-slope, slope-intercept, and standard form of a linear function. This will help students have access to the problem regardless of the symbolic representation. Students should be able to use a variety of graphing methods and be flexible graphing multiple symbolic representations of the function.

Students should be able to graph functions by hand and with the use of technology. It is imperative to model how to graph with a graphing calculator or other graphing device. Furthermore, ample time must be given for students to explore how a table of value can be helpful in identifying key features, domain, and range from a graph. The use of technology allows students to explore functions whose key features are irrational values, which can be located with the use of a device.

This standard appears in Integrated Math II as well. Students will be required to graph and interpret key features of additional function types. Therefore, instruction should introduce to the concept of parent functions. This will help students make the connection of how transformations affect the graph, equation, and table of a function. This standard can be integrated in instruction as students are presented with problem types whose symbolic representation varies and asked to identify the parent function and describe the transformation from its original, non-transformed graph. Students should explore through linear transformations that a horizontal translation can yield the same results as a vertical translation. For example, consider shifting $f(x) = 3x - 2$ to the right 4 units. Ask students to explore a vertical translation that would produce the same horizontal translation. Providing students with the opportunity to conjecture and test hypothesis will help them internalize the impact of transformations symbolically, graphically, and numerically. Students should be asked to identify the parent function as the linear function and describe the transformation from its original, non-transformed graph of the line $y = x$. This will help students attend to precision as they graph functions of many types and use their understanding of transformations to support the reasonableness of their graph. Discourse should allow students to discuss the effects of transformations on domain, range, slope, and intercepts. Instruction should provide ample opportunity for students to compare and contrast the graphs of functions, and it should help them efficiently recognize a parent function when expressed symbolically and graph it fluently.

Level 4:

Students should extend their conceptual understanding of key features of graphs by connecting key features to the relationships that exist in contextual problems. Using their knowledge of multiple representations built in standard M1.F.IF.C.7, students should be able to provide a graph, table, equation, and verbal representations of a contextual situation. Instruction should include posing purposeful questions asking them to show and describe key features from their created problem in context. Students should be given the opportunity to look at graphs drawn by others so they can analyze and critique their peers work. Through the analysis of many graphs, students should develop an understanding of when key features are efficiently and effectively represented, and, if not, provide a suggestion for representing them more appropriately.

Standard M1.F.IF.C.7 (Supporting Content)
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Scope and Clarifications:

i) Tasks have a real-world context.
ii) Tasks are limited to linear functions, piecewise functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the y-intercept of a function from multiple representations. Identify the slope of a linear function from multiple representations. Describe connections among multiple representations of a linear function. Compare properties of two linear functions each represented in a different way.	Identify the zeros of a function from multiple representations. Identify the percent rate of change of an exponential function from multiple representations. Describe connections among multiple representations of an exponential function. Describe connections among multiple representations of a piecewise-defined function. Move fluently among multiple representations of a function.	Compare properties of two exponential functions each represented in a different way. Compare properties of two piecewise-defined functions each represented in a different way. Compare properties of two functions from different function families each represented in a different way.	Compare properties of two functions within a context. Use precise mathematical vocabulary to explain the relationships of the various representations of a function.

Instructional Focus Statements

Level 3:

Prior to comparing properties of two functions represented in different ways, students need to first identify properties of functions and make connections between different representations of the same function. This is an important standard with respect to achieving access and equity for all students. Teachers should represent a function in multiple ways, especially for English language learners, learners with special needs, or struggling learners, because math drawings and other visuals allow more students to participate meaningfully in the mathematical discourse in the classroom. As students move fluently between representations they must consider relationships among quantities and how each representation provides a unique perspective of the function. Teachers can foster this way of seeing mathematics by having students discuss the similarities among representations that reveal the key features of a function that persist regardless of the form. Through these discussions students can determine which representations are most appropriate for revealing certain key features of the function.

In grade 8, students compare properties of two linear functions each represented in a different way. Once students have a strong understanding of the various representations of linear, piecewise-defined, and exponential functions, they can begin to compare properties of two functions represented in different ways. For example, given a graph of one exponential function and a table of another, a student should be able to compare their y-intercepts. One strategy that can sometimes be useful is to convert one or both to a different form so that both functions are represented the same way. As student begin to grasp this concept, it is important that teachers provide students with examples that include each function type, with some situated within a context. Therefore, comparing properties in different representations further supports students' understanding of each function type, which means this standard can be paired nicely with other standards that focus on properties and graphs of linear, piecewise-defined, and exponential functions, such as M1.F.IF.B.3 and M1.F.IF.C.6. As students recognize various function types in multiple representations, discussion should lead to the comparison of functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing an exponential function and a verbal description of a linear function. Instruction should support students in first recognizing the function family prior to comparing properties.

Level 4:

Students with a deep understanding of the various function types and representations should also be able to compare functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a linear function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties. Once conclusions are formed, teachers can ask further questions related to the context. For example, given a graph of a linear function and an algebraic representation of a piecewise-defined function each describing the cost of a cell phone plan, decide which plan is better. Students should be given the opportunity to describe how to identify function types and compare the properties of functions in various forms. At this level, teachers should expect students to use precise mathematical vocabulary to describe and justify these relationships and qualities.

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Building Functions (F.BF)

Standard M1.F.BF.A.1 (Supporting Content)
Write a function that describes a relationship between two quantities.
M1.F.BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

Scope and Clarifications: (Modeling Standard)
i) Tasks have a real-world context.
ii) Tasks are limited to linear functions and exponential functions with domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Write a function defined by an expression to model a linear relationship, given a table or graph. Identify the independent and dependent variable in a real-world context. Identify the first term and rate of change of a linear function, given a real-world context. Identify the first term and common ratio of an exponential function, given a real-world context.	Determine whether a real-world context would be represented by a linear or non-linear function. Write a function defined by a recursive process or steps for calculation to model a linear relationship, given a real-world context. Write a function defined by a recursive process or steps for calculation to model an exponential relationship, given a real-world context.	Write a function defined by an expression to model a linear relationship, given a real-world context. Write a function defined by an expression to model an exponential relationship with domain in the integers, given a real-world context. Compare key characteristics of real-world contexts that can be described by various types of functions.	Create a real-world context that would generate the given function, given a function defined by an expression, a recursive process, or steps for calculation. Explain the various ways a function can be defined and in what real-world contexts they would be appropriate. Justify why a specific type of function should be used to describe a given real-world context.

Instructional Focus Statements

Level 3:

In grade 8, students construct a function defined by an expression to model a linear relationship. In integrated math I, students create linear functions given a real-world context defined by a recursive process or steps for calculation in addition to an explicit expression. In many situations it is natural to use a function defined recursively, which generates values by applying operations on previous terms. For example, mortgage payments and drug dosages can be described with a recursive process. Students also create exponential functions defined in the same three ways. Instruction should focus on creating functions given a real-world context, while recognizing appropriate ways to define functions.

If given a table of values, students should first recognize which type of function the table of values represents. Teachers should focus students' attention on the relationship between consecutive points to see if there is a common first difference or constant additive change (linear function), a common second difference (quadratic function), or a common ratio or constant multiplicative change (exponential function). Once students identify the function type, teachers can then help students begin to write the function given the common first difference, second difference, or ratio and other information from the table.

To build coherence, it is important that teachers make connections between linear functions and arithmetic sequences and between exponential functions and geometric sequences. Thus, instruction can be nicely paired with standards M1.F.BF.A.2 and M1.F.LE.A.2, where students generate arithmetic and geometric explicit formulas to model situations. Both linear functions ($y = ax + b$) and arithmetic sequences ($a_n = a_1 + d(n - 1)$) describe additive changes, and students should make connections between the two. For example, b is equivalent to a_0 and a is equivalent to d. Similarly, exponential functions ($y = ab^x$) and geometric sequences ($a_n = a_1 r^{n-1}$) both describe multiplicative changes and a is equivalent to a_0 and b is equivalent to r. Students should understand the similarities, but instruction should also help students realize an important difference: arithmetic and geometric sequences are discrete while linear and exponential functions are continuous. This can be done by comparing the graphs of an arithmetic sequence and a linear function, for example.

Level 4:

As students develop a deep understanding of this standard, they should be able to create a real-world scenario given a function or combination of functions. Moreover, they should be able to describe which characteristics of their scenario correspond to each part of the given function. For example, given $y = 10 + 5X$ a student might create a scenario similar to the following: A ski lodge charges $10 to rent a snowboard and $5 for each hour it is used. In this function, x represents time in hours and y represents the final cost. The student should also be able to explain how the $5X$ relates to $5 per hour and how the 10 in the function relates to the initial charge of $10. Similar senarios can be created to describe exponential functions (e.g., sharing news or half-life).

Standard M1.F.BF.A.2 (Supporting Content)
Write arithmetic and geometric sequences with an explicit formula and use them to model situations.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if a sequence of numbers is an arithmetic sequence, geometric sequence, or neither. Identify the common difference in an arithmetic sequence. Identify the common ratio in a geometric sequence.	Write an arithmetic explicit formula to represent a relationship given by a sequence of numbers. Write a geometric explicit formula to represent a relationship given by a sequence of numbers.	Write an arithmetic explicit formula to model situations, given a real-world context. Write a geometric explicit formula to model situations, given a real-world context.	Create a real-world context that would generate the given function, given a function defined by an arithmetic or geometric explicit formula. Justify why specific real-world contexts should be represented by arithmetic or geometric sequences.

Instructional Focus Statements

Level 3:

In grade 8, students construct a function defined by an expression to model a linear relationship. In integrated math I, students build on their knowledge of linear functions to form arithmetic explicit formulas from a real-world context. Instruction should provide students with choice on whether to use the zeroth term ($y = mx + b$), the first term ($a_n = a_1 + d(n-1)$) or others in developing their function. Students should recognize that these equations are equivalent and connect the concepts of slope and common difference.

Students will make connections between geometric sequences and exponential functions created by explicit formulas in a real world context. Instruction should focus on making connections between the table of values and an explicit formula. Students should notice that each output value in the table can be rewritten as the initial value times a power of the common ratio. Then, relating each input value to the power of the common ratio will help students develop a function. Students should also be given choice as to which term to use as the coefficient in their formula. For example, if students are given the fourth term and the ratio, then $a_n = a_4 r^{n-4}$ might be a more accessible formula.

To build coherence, it is important that students make connections between linear functions and arithmetic sequences and between exponential functions and geometric sequences. Thus, instruction can be nicely paired with M1.F.BF.A.1, where students write a function defined by an expression, a recursive process, or steps for calculation to model a linear, quadratic, or exponential relationship.

Level 4:

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate a given function and justify why their scenario represents that function type. Additionally, students with a deep understanding of this standard can create a function from a geometric pattern and describe how each component of their function relates to characteristics of figures in the pattern. For example, students should be able to build a function to represent the number of line segments used to form shapes in a series of shapes following a particular pattern.

Linear and Exponential Models (F.LE)

M1.F.LE.A.1 (Supporting Content)
Distinguish between situations that can be modeled with linear functions and with exponential functions.
M1.F.LE.A.1a Recognize that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
M1.F.LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
M1.F.LE.A.1c Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize a function as linear, both from a graph and an equation. Recognize a function as exponential, both from a graph and an equation.	Find slope of a line from a graph, table of values, or given two coordinate points. Recognize that a line has a constant rate of change. Recognize that an exponential function does not have a constant rate of change.	Recognize that linear functions have a constant rate of change, while exponential functions do not. Informally show or explain that linear functions grow by adding the same number per unit. This should be done algebraically, graphically, and using words in context of a real-world application. Informally show or explain that exponential functions grow by multiplying the same factor per unit. This should be done algebraically, graphically, and using words in context of a real-world application.	Prove using precise mathematical language that a linear function grows by adding the same number per unit, while an exponential function grows by multiplying the same factor per unit. Create a real-world example of a situation that can be modeled by a linear function and explain why it is linear. Create a real-world example of a situation that can be modeled by an exponential function and explain why it is exponential, including why it is growth or decay.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Determine if a given real-world situation has a constant rate of change and can be modeled by a linear function. Determine if a given real-world situation can be modeled by an exponential function. Determine if a given real-world situation that can be modeled by an exponential function represents growth or decay.	

Instructional Focus Statements

Level 3:

Students first learn how to find slope of a line in grade 7. In grade 8, they begin exploring non-linear functions. In integrated math I, students compare the behavior of linear and exponential functions in terms of rate of change and its effect on end behavior.

As this is a modeling standard, students should be working with linear and exponential situations embedded in real-world applications. Allow students to explore the rate of change of linear and exponential functions from graphs, tables of values, equations, and real-world examples to determine if the rate of change remains constant. Slope should be calculated over equal intervals and compared. This may be easier if students organize the information in a table.

It is important for students to be able to identify when a function is linear vs. exponential, therefore, students should be provided with mixed examples and not just one or the other in isolation.

Students should be expected to use mathematical structure and repeated reasoning through multiple representations to see that the rate of change for a linear function remains constant.

Students may struggle with finding the percent growth or decay in an exponential function. For example, the amount of money earned in an investment is represented by the function $f(x) = 1500(1.07)^t$. By completing a table of values, students can see that the function represents growth, but they may be confused by the 1. It may be easier for them to understand where the 1 comes from in an example of buying a pair of blue jeans at a store. Show students that they can calculate the cost of the blue jeans and add sales tax in one step by taking 100% of the cost of the blue jeans plus the percent of sales tax. For a $20 pair of blue jeans at 8% sales tax, they can multiply 20 by 1.08 (100% + 8%) to get the total out of pocket cost. Likewise, if the blue jeans are on sale at 15% off, they can multiply 20 by .85 (100% - 15%) to get the sale price.

Level 4:

The focus of this standard is on the comparison of the rates of change between linear and exponential functions. Students should be challenged to prove that functions that grow by adding the same number are linear as compared to functions that grow by multiplying the same factor are exponential. Students should attend to precision and use appropriate mathematical language in their argument.

To ensure students are making connections between real-world situations and linear functions, have them create their own real-world examples that can be represented by linear functions. They should also explain why the situation is linear.

To ensure students are making connections between real-world situations and exponential functions, have them create their own real-world examples that can be represented by exponential functions. Students should represent their examples in words, algebraically, in a table, and graphically. They should include in their explanation why the situation represents exponential growth or decay.

Standard M1.F.LE.A.2 (Supporting Content)
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize a graph of a function as linear. Recognize a graph of a function as exponential.	Recognize a function as linear from a table, a description, or a set of ordered pairs and justify it using rates of change. Recognize a function as exponential from a table, a description, or a set of ordered pairs and justify it using rates of change.	Write a linear function given a graph. Write a linear function given a table of values. Write a linear function given a description of a simple real-world relationship. Write a linear function given a set of input-output pairs (ordered pairs). Write an exponential function given a graph. Write an exponential function given a table of values. Write an exponential function given a description of a simple real-world relationship.	Analyze functions created by others to determine accuracy and explain and correct any errors. Create a real-world situation that may be modeled by a linear function and write the function. Create a real-world situation that may be modeled by an exponential function and write the function. Collect data for a real-world situation that can be represented by a linear or exponential function and write the function that models it. Define the variables and explain in context why the function models the situation.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Write an exponential function given a set of input-output pairs (ordered pairs). Write a function given an arithmetic or geometric sequence or a description of one.	

Instructional Focus Statements

Level 3:

This standard aligns with several integrated math I standards, including the CED, IF, BF, and SSE clusters, and should be incorporated with those rather than as a stand-alone lesson. Once students have been exposed to both linear and exponential, instruction should include examples of both function types in multiple representations including graphs, tables, and descriptions to allow students time to determine whether the function is linear or exponential as well as write the function. This is a good opportunity to relate input and output values with independent and dependent variables.

As this is a modeling standard, real-world examples should be provided and students should be required to explain how the function models the context.

This standard also allows for students to connect their learning about arithmetic and geometric sequences to creating the functions that model them. These should also come from multiple representations including graphs, tables, and descriptions of real-world situations.

Level 4:

To increase the level of understanding, students should critique examples of functions created by others. One way to do this is to have students work in pairs with one students constructing a function and the other student checking it. For example, student A constructs a function from a graph while student B constructs a function from a table of values. Then they swap papers and student B graphs student A's function while student A creates a table of values from student B's function. Then each compares their work with the originals. If they do not match, they must determine where the mistake was made and correct it.

Once students have a good understanding of constructing both linear and exponential functions, they can create their own real-world examples. This can also involve students predicting a situation that would provide data that could be modeled by a linear or exponential function, collecting that data, and writing the function based on that data to test their prediction. Students should show and explain why the real-world example represents a linear or an exponential function, including representing it in multiple ways.

Standard M1.F.LE.A.3 (Major Work of the Grade)
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.

Scope and Clarifications: (Modeling Standard)
Tasks are limited linear and exponential functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Understand what is meant by an increasing function. Describe numerically why an increasing graph visually rises. Identify the interval(s) where a function is increasing and decreasing given a graph or a table of values.	Calculate the average rate of change of linear, quadratic, polynomial, and exponential functions over a given interval. Describe interval(s) where a function is increasing and decreasing using interval notation or inequality notation given a graph or table of values.	Compare the end behavior of graphs of lines and exponentials to determine which increases faster. Find and compare the average rate of change of lines and exponentials over equal intervals and make conclusions. Defend why a quantity increasing exponentially will eventually exceed a linear function and justify their conclusion by testing values.	Verify and explain why a quantity in one function type will eventually exceed a quantity in another function type. Observe graphs that model real-world scenarios and explain in context the reasonableness of why one graph increases faster than the other. Find the exact quantity where an exponential function exceeds another using technology and explain what it means in context of the real-world situation.

Instructional Focus Statements

Level 3:

Students should be provided examples of different types of functions in multiple representations to compare which type function will increase faster. It is important that students understand what is meant by "increasing faster". This will typically be easier for students to see in a table of values with the independent variable values listed in numerical order.

When analyzing graphs, students should be instructed to read the graph left to right and can consider end behavior to help them draw a conclusion. As this is a modeling standard, it is important that students interpret the graphs in context to understand the relationship between the variables.

Students have more experience with linear functions and should be able to tell if it is increasing or decreasing since the rate of change remains constant. To help students understand the rate at which a function is increasing for functions other than linear, they should compare rates of change over equal intervals for the functions being compared. Therefore, standard M1.F.IF.B.5 is a pre-requisite standard for this one. Organizing this data in a table will help students draw conclusions.

Comparing graphs of functions on the same coordinate plane will also help students see what "increasing faster" looks like. If students struggle with getting the two graphs confused when they are on the same coordinate plane, have them graph them with different colors, or put one on tracing paper so that it can be placed over the other one. This can also be alleviated by having students compare the graphs using technology.

Level 4:

At this level of understanding, students should be able to explain precisely how they know a quantity that increases exponentially will eventually exceed that of a quantity that increases linearly.

Explain in context the reasonableness of why a graph that models one situation would increase faster than a graph that models another situation.

Allowing students to work in pairs or small groups when analyzing pairs of functions will provide the opportunity to experience new approaches to their thinking and a deeper understanding of the concepts.

Standard M1.F.LE.B.4 (Supporting Content)
Interpret the parameters in a linear or exponential function in terms of a context.

Scope and Clarifications: (Modeling Standard)
For example, the total cost of an electrician who charges 35 dollars for a house call and 50 dollars per hour would be expressed as the function y = 50x + 35. If the rate were raised to 65 dollars per hour, describe how the function would change.
Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define slope as a rate of change. Identify the slope and the y-intercept in a linear function written in slope-intercept form. Identify the coefficient, base, and exponent in an exponential function.	Determine the slope and y-intercept of a linear function in a graph. Calculate the slope of a line that passes through two given points. Calculate the y-intercept of a linear function algebraically. Identify the initial value of an exponential function in a graph. Calculate the initial value of an exponential function algebraically. Calculate the growth rate of an exponential function by finding the ratio of successive terms.	Explain the meaning of the slope and y-intercept in context of the real-world situation, given a linear function. Explain the meaning of the coefficient, the base, and the exponent in context of the real-world situation, given an exponential function with a domain in the integers. Predict and determine how a linear function is affected by a change in the slope or y-intercept. Explain this change in context. Predict and determine how an exponential function is affected by a change in the coefficient, base, or exponent. Explain this change in context.	Reflect and respond to the explanations given by others. Create a real-world scenario that can be modeled by it, given a linear function. Create a real-world scenario that can be modeled by it, given an exponential function.

Instructional Focus Statements

Level 3:

This standard is an extension of M1.A.SSE.A.1, in which students interpret parts of expressions. This focus on this standard is on how the different components affect each other.

As this standard is a modeling standard, examples should connect to a real-world context. Use questions that ask students to interpret the slope and y-intercept of linear functions in the context of a real-world situation. Likewise, ask students to interpret the coefficient, base, and exponent of exponential functions in context of a real-world situation. Then extend their learning by asking them to determine the effect of changes to the parameters on the function. The scope provides an example of a linear function question. An exponential example might be: given an account that is modeled by $A(t) = 200(1.005)^{12t}$, determine the initial amount invested, the interest rate, and how often the money is compounded. Then describe what the effect would be if the initial investment is increased by $100.

Have students make a prediction of the effect of a change in a parameter and then verify if their prediction was correct by applying the change and comparing the results. Repetition of this activity will help students develop a better understanding of the properties of the operations within the function. Connecting the function to the context will help them justify their reasoning for their predictions. This repetition will also help students see the structure of the function and make a connection to its use as a general formula for the given real-world situation.

It is important for students to attend to precision in their interpretations and explanations should include units to ensure they are interpreting completely and correctly.

Level 4:

Teachers should provide students examples of linear and exponential functions and ask them to create a real world scenario that could be modeled by them.

Have students critique others' interpretations of the parameters and correct any mistakes. Students should look for correct and precise mathematical language in others' explanations, explain any mistakes made or lack of precision, and provide accurate corrections.

This standard can be easily integrated with M1.F.IF.B.3 and interpreting key features of a graph in connection with the function and the context.

Congruence (G.CO)

Standard M1.G.CO.A.1 (Supporting Content)

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, plane, distance along a line, and distance around a circular arc.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a point, line, line segment, ray, plane, angle, circle, and parallel and perpendicular lines from a simple two-dimensional figure. Differentiate between a line, line segment, and ray from a two-dimensional and three-dimensional figure.	Define an angle, circle, perpendicular line, parallel line, and line segment in simple terms. Identify a point, line, line segment, ray, plane, angle, circle, and parallel and perpendicular lines from two-dimensional composite figures and three-dimensional figures.	Generate a precise definition of an angle, circle, perpendicular line, parallel line and line segment based on the undefined notions of points, lines, planes, and the distance along a line and around an arc.	Compare and contrast different versions of precise definitions of an angle, circle, perpendicular line, parallel line, and line segment. Provide mathematical justification for which definition is the most precise.

Instructional Focus Statements

Level 3:

Students begin to develop an understanding of points, lines, and planes in elementary grades. In high school, students should extend this understanding in order to self-generate more precise definitions of angle, circle, perpendicular line, parallel line, and line segment. This should be cultivated through experiences with rigid motions that provide insights on how the terms might be defined rather than merely memorizing definitions. Students should be encouraged to develop their own definitions. Discussing student-generated definitions and challenging the evidence used to form the definitions will lead to a deeper conceptual understanding of the terms. Students should understand that a point, line, and plane are undefined terms with simplistic definitions. A point is a location and has no size. A line is a one-dimensional object that extends infinitely in either direction but has no width. A plane is a

two-dimensional object that extends infinitely with no edges or height. These three undefined terms are the cornerstone on which students define all other terms in geometry.
As students understand the basic undefined terms, they should be allowed time to explore angles, circles, line segments, and parallel and perpendicular lines to develop their own precise definitions. In this exploration, students should compare images of what an object is and is not, eventually creating their own images and counterexamples. A Frayer model can be a helpful tool for students to organize their thinking to develop student-generated definitions. It is beneficial to do this after students have worked with transformations, since these definitions are derived from transformations. For example, a line parallel to another is the translation of the original line.

A common misconception with angles is that students often think of an angle in a polygon as the vertex (a point) instead of the relationship between the consecutive sides or the degrees of the opening. Students should be asked questions to guide them to see an angle as a ray rotated about its endpoint by a specific degree. It is imperative that students develop an in-depth conceptual understanding of geometric definitions through hands-on discovery learning.

Level 4:

Students at this level should be challenged to critiquing others' precise definitions providing counterexamples when appropriate, adding to the definition as needed, or correcting misconceptions evident in the definition. This can be achieved by students critiquing other students' definitions or by looking at teacher created definitions that were intentionally created to guide students to look for specific misconceptions.

Standard M1.G.CO.A.2 (Supporting Content)
Represent transformations in the plane in multiple ways, including technology. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify when a figure has been rotated, reflected, or translated in a picture or on a graph. Identify when a figure has been dilated, stretched, or compressed in a picture or on a graph.	Generate a single rigid transformation on or off a coordinate plane. Generate a dilation of a simple figure on or off a coordinate plane. Determine equal distances and congruent angle measures in given figures.	Use multiple representations to represent transformations: in words, algebraically, graphically, and in a table of values. Use technology to perform a given transformation and identify the effects of transformation(s) on a figure. Describe transformations as functions in which points in the pre-image are input values that result in points in the image as output values.	Critique transformations drawn by others based on a given description, providing mathematical justification for their thinking. Critique others' description of a transformation represented in words, algebraically, graphically, or in a table of values, providing mathematical justification for their thinking. Represent a sequence of transformations that includes a combination of a rigid motion and a dilation in words, algebraically, graphically, and in a table of values.

Instructional Focus Statements

Level 3:

Students begin exploring rigid motion in grade 8. In this course, they further explore how a rigid motion affects the graph of a function. By writing out effects of the transformations on each coordinate algebraically, they will begin to connect that when the coordinate values of the pre-image are used as input values, the coordinates of the output will result in the image after the transformation. In Integrated math II, they further explore this concept by using function notation to represent transformations in M2.F.BF.B.2 by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ in linear, quadratic, and absolute value functions. This is explored further in Integrated Math III standard M3.F.BF.A.1 for polynomial, exponential, and logarithmic functions.

Students should be exposed to multiple ways to represent all transformations including words, coordinate notation, function notation, graphs, and technology. Students should also discover that rigid motions preserve distance and angle measures, while other transformations, such as dilations, do not. They should be able to distinguish between a transformation that is rigid motion and one that is not. Allowing students to explore transformations using dynamic geometry software is beneficial in discovering these differences.

Level 4:

As students extend their understanding of representing, describing, and comparing transformations, they should be able to analyze others' transformations or descriptions of transformations. This will help develop a deep understanding of the connections between the different representations. These samples need to include all representations of transformations including drawings, words, algebraic and graphical representations, and in a table of values.

Extending this standard to include a sequence of transformations will help students bridge the Integrated Math II standard M2.F. BF.B.2 and Integrated Math III standard M3.F.BF.A.1 which requires them to work a larger set of function family problems involving transformations.

Standard M1.G.CO.A.3 (Supporting Content)
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry the shape onto itself.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a rectangle, parallelogram, trapezoid, or regular polygon that indicates all possible lines of symmetry. Determine if a rectangle, parallelogram, trapezoid, or regular polygon has rotational symmetry. Draw a line of symmetry when given a rectangle, parallelogram, trapezoid, or regular polygon.	Explain symmetry in terms of transformations. Draw multiple lines of symmetry when given a rectangle, parallelogram, trapezoid, or regular polygon. Determine if a rectangle, parallelogram, trapezoid, or regular polygon has rotational symmetry, limited to 90 or 180 degrees.	Determine a line of symmetry and/or the degree of rotational symmetry that exist in a rectangle, parallelogram, trapezoid, or regular polygon. Describe the rotations and/or reflections that carry a rectangle, parallelogram, trapezoid, or regular polygon onto itself. Determine the attributes of a figure based on its symmetries.	Create figures based on a given set of transformations that allows any two-dimensional figure to be carried onto itself. Synthesize and explain the relationships that exists between the symmetries of a figure and its geometric attributes.

Instructional Focus Statements

Level 3:
Initially, instruction should focus on having students explore transformations with rectangles, parallelograms, trapezoids, and regular polygons enabling them to discover which ones cause the image to exactly overlap the pre-image. This often confuses students at first but is essential in their conceptual understanding of this standard. Students should be provided with appropriate tools to aide with their explorations such as tracing paper, mirrors, and dynamic geometry software. These explorations should lead to students defining a set of transformations in which the image ends up exactly the matching the pre-image, ultimately leading students to recognize the mathematical connection between transformations and the symmetries within a figure. For example, while learning about rectangles, students should build on their understanding by connecting symmetry with the attributes of the

figure. Students should be able to compare the symmetries of a rectangle with quadrilaterals that are not rectangles to better understand the similarities and differences of each polygons attributes. This standard lays the groundwork for standard M1.G.CO.A.4.

Level 4:

Students should have ample opportunities to compare figures that have and do not have symmetry. They should be encouraged to use these comparisons to explain the connection between the symmetries and the geometric attributes of a figure. To solidify this understanding, students should be able to create their own examples of figures with one or more types of symmetry. Additionally, students should be able to explain their reasoning with precise mathematical vocabulary.

Standard M1.G.CO.A.4 (Supporting Content)
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify rotations, reflections, and translations in a picture or graph.	Describe rotations, reflections, and translations in general terms. Measure and construct angles, line segments, and circles both on and off a coordinate plane.	Develop and give a student-generated definition of a rotation in terms of distances, angles, and arcs. Develop and give a student-generated definition of a reflection in terms of distance, and parallel and perpendicular lines. Develop and give a student-generated definition of a translation in terms of distance and parallel lines. Generate precise definitions of rotations, reflections, and translations such that they are unique to the given transformation.	Critique others' definitions of rotations, reflections, and translations, paying particular attention to the level of precision and providing counter-examples when they exist.

Instructional Focus Statements

Level 3:

This standard is horizontally aligned with the other standards within the congruence cluster. One of the most difficult aspects of this standard for students is the level of precision required to generate definitions for rotations, reflections, and translations. Students should already have experience with transformations prior to independently developing these definitions. They should also have experience with appropriate tools such as protractors, compasses, and/or dynamic geometry software allowing them to precisely measure the angles and construct the arcs which occur during the transformations. It is important that students be allowed adequate time to explore transformations so that they have the appropriate conceptual understanding that allows them to develop the definitions on their own. Students may be encouraged to use the properties of each transformation and focus on the relationships that exist between the lines and angles in the pre-image and the lines and angles in the original image. Students should be provided examples of transformed figures and asked to explain why or why not a relationship can be described as a specific rigid motion. It is vital that students attend to precision when writing their definitions to ensure they are unique to the given transformation.

Level 4:

Critiquing others' definitions will help students discover the necessary level of precision needed to accurately define these transformations. Providing students with both examples and non-examples of rigid motions is vital to this process. Likewise, students who can self-generate a counter-example demonstrate a much deeper understanding of the properties of these transformations. As students create their own student-developed definitions and critique others' definitions, they should use examples, counter-examples, and precise mathematical vocabulary.

Standard M1.G.CO.A.5 (Supporting Content)
Given a geometric figure and a rigid motion, draw the image of the figure in multiple ways, including technology. Specify a sequence of rigid motions that will carry a given figure onto another.

Scope and Clarifications:
Rigid motions include rotations, reflections, and translations.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the correct term to describe the rigid transformation from a picture.	Sketch the image of a geometric figure when given a description of the figure in words. Determine a single rigid motion that carries a given figure onto another.	Draw the image of a geometric figure and a rigid motion (rotation, reflection, or translation) in multiple ways, including by hand, on or off a coordinate plane, or by using technology such as dynamic geometry software. Draw the image of a geometric figure in multiple ways when given a sequence of rigid motions. Describe a sequence of rigid motions that will carry a given figure onto another. Recognize and explain that there can be more than one correct sequence that will map a given pre-image onto an image.	Describe multiple sequences that will map one to the other when given a pre-image and image. Critique others' drawings or descriptions, recognizing mistakes and correcting them by explaining their justifications using precise mathematical language. Compare the relationship between a sequence of transformations and a single transformation and understand that one transformation can more efficiently result in the same image. For example, a sequence of two reflections across parallel lines results in a translation.

Instructional Focus Statements

Level 3:

Students began exploring rigid motions in grade 8 both on and off the coordinate plane. In this course, students should explore rigid motions further with appropriate tools to help them visualize each transformation including tracing paper, mirrors, and dynamic geometry software. They should generate transformations both on and off a coordinate plane. When working on a coordinate plane, students should explore the effects on the coordinates and begin to see transformations as functions with input values and output values. Instructions for this standard integrates nicely with standard M1.G.CO.A.2. Once students have had ample experience exploring each transformation, students need to explore what happens when a sequence of transformations are applied. This will lead to students being able to determine the sequence applied to a given pre-image and image. This standard is the foundation to all the rigid motion standards in this cluster.

Level 4:

Students should continue with their exploration of the effects on a figure when a sequence of rigid motions are applied. Finding alternate lists of sequences that will map a pre-image onto an image and recognizing that the order of the transformations may change the result will deepen their understanding of their properties. Additionally, students will solidify their understanding by critiquing others' applications of a specified sequence or description of a sequence provided a pre-image and image, analyzing their mistakes, if any, and providing corrections as needed.

Standard M1.G.CO.B.6 (Major Work of the Grade)
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define rigid motion. Choose the correct term to describe the rigid transformation given a picture.	Measure distances and angles of figures. Understand that congruent polygons have corresponding side lengths and angle measures that are equal or circles have radii that are equal in length.	Use definitions of rigid motions to perform a given transformation. Predict the effect of a given rigid motion on a figure. Determine what rigid motion(s) will map one figure on to another. Verify that two figures are congruent by measuring lengths and angle measures to ensure they are equal, given the pre-image and its image after a rigid motion, Make the connection that two figures are congruent because one is the resulting image of a rigid motion on the other.	Construct a viable argument on why a rigid motion results in congruent figures using the definition of rigid motions and the definition of congruence. Critique others' predictions of the effect of a given transformation on a figure.

Instructional Focus Statements

Level 3:

In standard G.CO.A.4, students developed definitions of rigid motions. In this standard, students should be able to use those definitions to create the transformation. It is important that they perform the transformation on individual points in the figure rather than on the whole figure at once. For example, to perform a reflection of a given polygon in the y-axis, students need to draw a line perpendicular to the y-axis that passes through a vertex of the polygon and draw the reflection of the vertex at the same distance along that line on the other side of the y-axis.

Students also need to recognize that rigid motion results in a congruent figure. Therefore, two figures can be determined congruent if a rigid motion exists that maps one figure onto the other. This will require students to have already been exposed to standard G.CO.A.5. It is best for students to discover this concept by comparing side and angle measurements of figures that have been transformed by a rigid motion with those that have not. Students must practice precision in their measurements to verify these results.

This standard develops the understanding of the concept of using rigid motion to prove congruent figures which leads to other standards in this cluster where students will be using this skill.

Level 4:

Students will deepen their understanding of this standard by describing why the image of a rigid motion results in a figure congruent to its pre-image. Providing students with examples of transformations that are and are not rigid motions and having them explain why the figures are congruent or not using the definitions will help students build their arguments. These examples can come from other student work or teacher created examples.

Standard M1.G.CO.B.7 (Major Work of the Grade)
Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define rigid motion. Define congruence. Choose the correct term to describe the rigid transformation of one triangle to another from a picture.	Identify corresponding pairs of sides and angles given two figures that are marked or a congruence statement describing two figures. Perform a rigid motion of a given triangle both on a blank space and on a coordinate plane using appropriate tools such as a mirror, tracing paper, or dynamic geometry software. Find the measures of sides and angles of a triangle both on a blank space and on a coordinate plane using appropriate tools such as a ruler, protractor, distance formula, or dynamic geometry software. Understand that congruent figures have equal lengths and angle measurements.	Identify and name corresponding parts of triangles. Make the connection that two triangles are congruent because one is the resulting image of a rigid motion on the other. Verify that corresponding sides have equal lengths and corresponding angles have equal measures, given congruent triangles, Explain why the triangles are congruent using the definition of congruence in terms of rigid motion, given that the corresponding sides and angles of two triangles are congruent. Read and use correct notation that shows corresponding parts are	Explore corresponding parts of other shapes to determine if the figures are congruent. Create examples of non-congruent triangles that have some congruent corresponding parts and explain why they are not congruent using the definition of congruence in terms of rigid motion.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		congruent and triangles are congruent. Write a triangle congruence statement using correct notation.	

Instructional Focus Statements

Level 3:

This standard extends G.CO.B.6 to verify that in order for two figures to be congruent, their corresponding parts must also be congruent. This concept is not a difficult one, but the converse is often more challenging. If only some of the corresponding parts of two triangles are congruent does that mean that the triangles are congruent? Students can explore examples of when this is and is not the case.

It is important that students also know how to read and correctly notate congruent triangles (ex.: $\Delta ABC \cong \Delta DEF$) and their corresponding parts. They must attend to precision when setting up these congruence statements and understand that the statement itself will clearly indicate the congruent corresponding parts when written correctly.

This discovery will lead nicely into standard G.CO.B.8 in which students will learn that, with triangles, they only need to know specific groups of congruent corresponding parts to know that all corresponding parts are congruent and this the triangles are congruent.

Level 4:

Allowing students to explore shapes other than triangles will help them understand that this concept extends to all figures. Students must use their critical thinking skills to create their own examples of triangles that have some congruent corresponding parts but do not result in congruent triangles. To extend this further, students need to use their communication skills to explain why this happens using the definition of congruence in terms of rigid motion.

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Standard M1.G.CO.B.8 (Major Work of the Grade)
Explain how the criteria for triangle congruence (ASA, SAS, AAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Identify corresponding parts in two triangles. Write the congruence statement for each corresponding part (ie.: $< A \cong < D, AB \cong DE$)	Determine if two triangles are congruent in terms of rigid motion using appropriate tools such as mirrors or tracing paper Determine if two triangles are congruent in terms of congruent corresponding parts using tools such as rulers, protractors, distance formula, etc.	Determine which combinations of congruent corresponding parts must be known to verify that two triangles are congruent. Explain how knowing SSS, SAS, ASA, or AAS is enough to say that two triangles are congruent using the definition of congruence in terms of rigid motions. Read and use correct notation that shows corresponding parts are congruent and triangles are congruent. Provide instructions to another student giving only three measurements of triangle sides and/or angles so that they can accurately draw a triangle congruent to their own drawing.	Explain and provide examples of why two combinations of three congruent corresponding parts (AAA and SAA) do not prove triangles congruent. Critique the work of others in determining congruent triangles based on SSS, SAS, ASA, or AAS and explain why that work is correct or incorrect. Explain the effects on these properties when the triangles are right triangles.

Instructional Focus Statements

Level 3:

This standard builds on G.CO.B.7, where students have to verify that all corresponding parts of two triangles must be congruent to say the triangles are congruent. In this standard, students explore the properties of triangles to lead to a discovery that if certain groups of corresponding parts in two triangles are congruent, then the rest of them will be, too. Therefore, students only need to determine specific criteria to prove that two triangles are congruent (ASA, SAS, AAS, and SSS).

Instruction should focus on allowing students to explore combinations of congruent sides and/or angles to determine which combinations work and which ones do not work. Physical manipulatives allow students to create triangles based on given congruent parts and then compare them to determine if they are congruent. For example, a pair of students can each cut three straws to given lengths, assemble them into a triangle, and then hold the triangles next to each other to see if they are congruent by SSS. Students can also measure the angle between two straws and tape them to hold that angle measure along with other given information to compare triangles using SAS, AAS, or ASA. Guide students to see that by holding the triangles they are comparing next to each other, they are physically performing the rigid motion. This will help them see that the congruence criteria (ASA, SAS, AAS, and SSS) can be verified by the definition of congruence in terms of rigid motion. Dynamic geometry software is also a useful tool to help students see how this works.

Students often struggle to understand the difference between ASA and AAS or SAS and SSA (which does not work). Guide students to see that ASA and SAS indicate the included corresponding side or angle must be congruent, whereas AAS and SSA indicate a non-included side or angle. Allowing students to explore the combinations using physical manipulatives or dynamic geometry software can help. To solidify their understanding, challenge students to draw a triangle on paper and then give a partner a combination of 3 side lengths and/or angle measures such that they can draw a triangle that is congruent to their own. (Ex. Draw a 3 inch segment with a 40 degree angle on one end and a 60 degree angle on the other end.)

Level 4:

Challenge students to demonstrate using precise drawings or dynamic geometry software examples that show why two combinations of congruent corresponding parts (AAA and SAA) do not prove triangles congruent. Provide examples of others' work for students to critique. This can be student work or teacher created examples. Some should be correct and some incorrect. Have students explain why the conclusion is correct or incorrect and verify using the definition of congruence in terms of rigid motions. Have students further explore this standard using right triangles. Ask them to determine if the pre-determined combinations still work and if the combinations that don't work for non-right triangles work for right triangles, leading them to discover the combination SAA actually works if and only if it is a right triangle (HL).

Standard G.CO.C.9 (Major Work of the Grade)
Prove theorems about lines and angles

Scope and Clarifications:
Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify special lines, such as parallel, perpendicular, and bisectors. Identify special pairs of angles, such as supplementary, complementary, vertical, and those formed by lines crossed by a transversal.	Understand the relationships of lines, such as perpendicular bisectors. Understand the relationships of special pairs of angles, such as complementary angles have a sum of 90 degrees. Recognize pairs of lines or angles as a rigid motion that results in congruence. Give an informal explanation of the relationship of pairs of lines and/or angles. Complete a partial proof by filling in the blanks when given either a statement or a reason.	Make conjectures about the relationships between lines and/or angles. Prove those conjectures are true using precise mathematical language and a logical order of statements. Use rigid motions to prove the relationship between the figures in the conjectures. Construct a two-column proof or paragraph proof. Compare their proof with other students' proofs or teacher created proof examples.	Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument. Improve their own proofs based on what they have seen from others' proofs.

Instructional Focus Statements

Level 3:

Proofs, both formal and informal, should be addressed multiple times throughout this course. Constructing a proof is challenging and requires practice. Requiring students to informally explain properties and relationships of lines and angles to justify answers to problems will lead students into constructing more formal two-column and paragraph proofs. Having students make conjectures about those relationships will help students recognize the need for the proof and learn to think more strategically. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logical order before writing it down.

Depending on the selected approach to the proof, students should be encouraged to draw on their experience with transformations and congruence as well as other prior knowledge to prove conjectures about relationships of lines and angles.

Students need to be familiar with different types of proofs including two-column proofs and paragraph proofs. Having students fill in a partially completed proof to start with can be helpful before creating proofs on their own. Comparing their proofs with others can help them better understand the expectations of the reasoning, order, and precise language required in a formal proof.

Level 4:

Students need to be exposed to other proofs to analyze and critique. These should be a combination of accurate proofs and some with mistakes. Students should provide counterexamples with their explanation when disproving these proofs. These critiques will help students develop a deeper understanding of the logic of proofs and the need for clear and precise language that they can then apply to their own proofs.

Standard G.CO.C.10 (Major Work of the Grade)
Prove theorems about triangles.

Scope and Clarifications:
Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define congruent. Identify congruent figures in a picture using symbols or given notation. Define complementary and supplementary angles. Define parallel lines. Identify parallel lines in a picture using symbols or given notation. Identify the angle opposite a specified side in a triangle and the side opposite a specified angle in a triangle.	Know that the sum of the angles in a triangle is 180 degrees. Write a statement relating two figures using correct mathematical notation. Such as $< A \cong < B$ or $\overline{CD} \| \overline{EF}$ Define congruence in terms of rigid motion. Recognize when rigid motions can be used to prove congruent corresponding parts in a given figure.	Define median of a triangle, identify or draw it in a picture. Explore the properties of triangles and make conjectures. Formally prove the conjectures using precise mathematical language. Use rigid motions to prove the conjectures. Construct a two-column proof or paragraph proof. Compare their proof with other students' proofs or teacher created proof examples.	Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument. Collaborate and critique proofs, given proofs written by others. .

Instructional Focus Statements

Level 3:

This standard is an extension to G.CO.C.9, since those skills will be required for this standard as well. Students should be allowed to explore and make their own conjectures about triangles and their properties using appropriate tools such as a compass, protractor, tracing paper, or dynamic geometry software.

Note that this is the first time students will be introduced to a median of a triangle. This is a great place to review constructions and allow students to use appropriate tools to construct the medians in a triangle. Students should be encouraged to start with different types of triangles (acute, obtuse, right, etc.), attend to precision in their construction, and make observations to lead them to the conjecture that the medians will always meet at a single point.

Proofs, both formal and informal, should be addressed multiple times throughout this course. Constructing a proof is challenging and requires practice. Requiring students to informally explain the properties of triangles will lead students into constructing more formal two-column and paragraph proofs. Having students make conjectures about those relationships will help students recognize the need for the proof and learn to think more strategically. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logical order before writing it down. Classroom discussions can help students broaden their thinking by making them consider different approaches to a proof. It will also help students develop more precise mathematical language. Having students restate other student's informal explanations in a more formal and precise way will help them focus on the language. Having students find and correct mistakes in an incorrect proof will help them better understand the organization and flow of a proof.

Students need to apply their prior knowledge to their proofs. Therefore, depending on the approach taken, students may be encouraged to draw on their experience with transformations, symmetry, and congruence to prove conjectures about triangles and their properties.

Students need to be familiar with different types of proofs including two-column proofs and paragraph proofs. Having students fill in a partially completed proof to start with can be helpful before creating proofs on their own. Comparing their proofs with others can help them better understand the expectations of the reasoning, order, and precise language required in a formal proof.

Level 4:

Students need to be exposed to other's proofs to analyze and critique. These should be a combination of accurate proofs and some with mistakes. Students should provide counterexamples with their explanation when disproving these proofs. These critiques will help students develop a deeper understanding of the logic of proofs and the need for clear and precise language that they can then apply to their own proofs.

Standard G.CO.C.11 (Major Work of the Grade)
Prove theorems about parallelograms.

Scope and Clarifications:
Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify opposite and consecutive sides or angles in a figure. Define congruent. Identify congruent figures in a picture using symbols or given notation. Define parallel lines. Identify parallel lines in a picture using symbols or given notation. Define perpendicular. Identify perpendicular lines in a picture using symbols or given notation. Define supplementary angles. Define diagonals of a polygon and identify or draw them in a given figure. Define bisect in terms of a line	Define a parallelogram as a quadrilateral with two pairs of opposite parallel sides. Write a statement relating two figures using correct mathematical notation. Such as $< A \cong < B$ or $\overline{CD} \parallel \overline{EF}$ Define congruence in terms of rigid motion. Recognize when rigid motions can be used to prove congruent corresponding parts in a given figure. Prove two triangles are congruent.	Explore the relationships that exist between the sides, angles, and diagonals of parallelograms, including rectangles, rhombuses, and squares. Make conjectures about the properties of parallelograms, rectangles, rhombuses, and squares. Formally prove the properties of parallelograms using precise mathematical language. Use rigid motions to prove the conjectures. Construct a two-column proof or paragraph proof. Compare their proof with other	Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument. Improve their own proofs based on what they have seen from others' proofs.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
segment or angle.		students' proofs or teacher created proof examples. Explain the relationships between parallelograms, rectangles, rhombuses, and squares.	

Instructional Focus Statements

Level 3:

This standard is an extension to G.CO.C.9 and G.CO.C.10, since those skills will be applied here as well. Students should be allowed to explore and make their own conjectures about the relationships between the sides, angles, and diagonals in a parallelogram using appropriate tools such as a compass, protractor, tracing paper, or dynamic geometry software.

Proofs, both formal and informal, should be addressed multiple times throughout this course. Constructing a proof is challenging and requires practice. Requiring students to informally explain properties and relationships of lines and angles in parallelograms, rectangles, rhombuses, and squares will lead students into constructing more formal two-column and paragraph proofs. Having students make conjectures about those relationships will help students recognize the need for the proof and learn to think more strategically. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logical order before writing it down. Classroom discussions can help students broaden their thinking by realizing how they are limiting themselves. It will also help students develop more precise mathematical language. By having students restate other student's informal explanations in a more formal and precise way, will help them focus on the language.

Students need to apply their prior knowledge to their proofs. Therefore, depending on the approach taken, students may be encouraged to draw on their experience with transformations and congruence to prove conjectures about relationships of lines and angles.

Students need to be familiar with different types of proofs including two-column proofs and paragraph proofs. Having students fill in a partially completed proof to start with can be helpful before creating proofs on their own. Comparing their proofs with others can help them better understand the expectations of the reasoning, order, and precise language required in a formal proof.

This standard should also lead to a comparison of the properties of a parallelogram, rectangle, rhombus, and square to help students develop an understanding of the relationship between these figures. For example, a rectangle is a parallelogram because it holds all the same properties of a parallelogram, but is special because it also has four congruent angles. Likewise, a square holds all the properties of a rectangle, but it is even more special because it also holds all the properties of a rhombus.

Level 4:

Students need to be exposed to other's proofs to analyze and critique. These should be a combination of accurate proofs and some with mistakes. Students should provide counterexamples with their explanation when disproving these proofs. These critiques will help students develop a deeper understanding of the logic of proofs and the need for clear and precise language that they can then apply to their own proofs.

TN Department of Education

INTERPRETING CATEGORICAL and QUANTITATIVE DATA (S.ID)

M1.S.ID.A.1 (Supporting Content)
Represent single or multiple data sets with dot plots, histograms, stem plots (stem and leaf), and box plots.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Calculate the median of a data set. Organize given data values onto an already created stem and leaf plot. Represent data on a histogram with given scale and axes. Apply correct labels for components and/or axes when representing data graphically.	Represent a single data set with a dot plot. Represent a single data set with a histogram. Represent a single data set with a box plot. Represent a single data set with a stem and leaf plot. Choose an appropriate scale for the graph when representing data graphically.	Create parallel or side-by-side box plots or histograms with the same scale. Determine which type of data plot would be most appropriate for a set of data. Use real-world data (represented in a table) to create dot plots, histograms, stem plots, or box plots. Use technology to represent single or multiple data sets with dot plots, histograms, stem plots (stem and leaf), and box plots.	Explain advantages and disadvantages of displaying data using different representations. Recognize misrepresentation of data in various types of plots. Explain what changes could be made to improve the representation of the data.

Instructional Focus Statements

Level 3:

In grade 7, students interpret dot plots and box plots and use them to informally compare two data sets. In integrated math I, students use real-world data to create dot plots, histograms, or box plots, apply correct labels for components and/or axes, and choose an appropriate scale for the graph. Teachers should emphasize attending to precision as students label their axes, choose an appropriate scale, and specify the units of measurement so that others can easily understand their plots and histograms. Given two data sets, students should create parallel box plots or histograms with the same scale to, along with M1.S.ID.A.2, compare the center and spread of two or more data sets. Teachers should provide examples of parallel box plots or histograms that do not have the same scale to reveal how these comparisons can be misleading.

As students become comfortable creating each type of data plot, teachers should help students determine which type of plot is most appropriate for a given set of data. Teachers should have students consider how large the data set is, the format of the given data (values or frequencies), and the purpose of plotting the data. In M1.S.ID.A.2 and later in integrated math III, students will use their knowledge of histograms to understand the normal distribution curve, a continuous probability distribution.

Level 4:

As students develop a deep understanding of this standard, they should realize the advantages and disadvantages to each type of representation and be able to explain them using precise mathematical vocabulary. For example, stem and leaf plots make it easy to find the median of a data set. However, stem and leaf plots are not very informative for small data sets. Box plots are great for identifying outliers and provide a nice summary of the data, but the exact data values are not retained.

As mentioned in level 3, teachers should provide examples of comparisons that are misleading due to different scales. To deepen this understanding, instruction should focus students' attention on recognizing misrepresented data and explaining how the visual representation of the data can be altered. Various components should be considered such as, scale, labels, units of measurement, breaks in the axes, or when a less appropriate plot or histogram is used to represent the data.

Standard M1.S.ID.A.2 (Supporting Content)
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the mean, median, and interquartile range of a single set of data from a table. Define center and spread, when related to a data set.	Interpret and compare the mean, median, and interquartile range in the context of a data set. Calculate the mean, median, interquartile range, and standard deviation of a data set using technology.	Explain similarities and differences using specific measures of central tendency and measures of dispersion, given two or more data sets. Determine within how many standard deviations above or below the mean a data value is.	Compare and contrast multiple data sets using measures of center and spread. Explain how standard deviations can be used to describe how much of the data is around the mean.

Instructional Focus Statements

Level 3:

One of the major goals of statistics is to summarize, compare, and predict. In grade 7, students informally compare the center and spread of two data sets represented in a dot plot or box plot. In integrated math I, students formalize their understanding by using interquartile range and standard deviation to compare the spreads of multiple data sets represented visually or in a table. Students should be able to discuss which data set has a greater average or typical value and which data set has greater variability, when using appropriate measures of center and spread. When data is approximately normal or is intended to represent the population mean, students should use mean and standard deviation. However, the median and interquartile range better represent data that is strongly skewed. Using statistics appropriate to the shape of the data will allow students to represent their data sets accurately and make stronger comparisons between multiple data sets within given contexts.

To aid students in making valid comparisons, teachers should provide multiple data sets with equal centers but different measures of spread and vice versa. In addition, teachers should provide data sets in which appropriate measures of center and dispersion vary so that students have opportunities to justify parameters based on the shape of the given distributions. Thus, instruction can be nicely paired with M1.S.ID.A.3, in which students explain the advantages and disadvantages to using each parameter and how outliers impact the mean and median. In this course, students informally making inferences between data sets, while in future courses, they will use statistical tests (i.e., t-test) to determine if there is a significant difference between the means of two data sets.

Level 4:

Students should be able to compare and contrast data sets based on real-world situations. Therefore, students should think about how the differences in center and spread relate to the real-world context and how the information can inform decision making. Once the conclusions are made, teachers should help students write their results using precise mathematical vocabulary.

In addition, students at this level should use the empirical rule of approximately normal distributions to tell what percent of data values fall within whole-numbered standard deviations from the mean. For example, given wait times at a particular restaurant have a mean of 15 minutes with a standard deviation of 5 minutes, what percent of the wait times are between 10 and 20 minutes? In this case, 68% of the wait times would be between 10 and 20 minutes. In integrated math III, students will continue to use the empirical rule to investigate more complex problems such as: what percent of wait times are below 30, above 5, or between 10 and 25?

Standard M1.S.ID.A.3 (Supporting Content)
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify unusual data points in a data set. Know that standard deviation uses the mean and that interquartile range uses the median.	Recognize and name different distribution shapes and describe their center, shape, and spread. Identify outliers in a data set.	Choose which measure(s) are most appropriate for comparison based on the shape of the distribution. Describe the impact of an outlier on the center and spread of a data set.	Justify which measure(s) are most appropriate for comparison based on the shape of the distribution. Explain advantages and disadvantages of using each measure of center and spread. Explain how an outlier affects the mean and median differently.

Instructional Focus Statements

Level 3:

Students should be expected to recognize the advantages and disadvantages of using each measure of center and spread and how outliers impact each. A common misconception can occur regarding the labeling of outliers. Some students identify data points that are not typical and assume they are outliers. However, identifying these points as outliers prior to performing the appropriate calculations is misguided. Teachers should address this misconception carefully and consider using the term unusual or extreme data points in these situations prior to conducting the calculations.

As students are asked to discuss data values, instruction should focus on the relationship of the outlier to the skew of the data. Discussion should lead students to discover that the mean and standard deviation are less useful with strongly skewed data sets and the importance of using the median and interquartile range because they are not impacted by extreme data values. As students use the interquartile range to quickly identify outliers (data values more than one-and-a-half times the IQR distance below the first quartile or above the third quartile), they should then be asked to compare the median to the quartiles and come to conclusions about the set of data based on these relationships.

This standard can be taught together with M1.S.ID.A.2, in which students compare multiple data sets, as the parameters used in the comparisons are largely determined by the impact outliers has on each measure of center. The focus of M1.S.ID.A.3 is to develop a deep conceptual understanding of why median and interquartile range are more appropriate for skewed data sets and why the mean and standard deviation are best used with approximately bell-shaped distributions, which occur naturally in many situations (e.g., height, weight, or strength of adults).

Level 4:

Students at this level should be able to explain how the mean and median are affected by a skew or an extreme value. Teachers could show students a distribution with its mean labeled with and without an extreme value to see how the mean is impacted by including this value. Teachers should expect students to explain how the mean shifts towards a skew or extreme value and explain why this tendency is the reason mean is not an appropriate in these situations.

To better understand this idea, students can be asked to compare the median and mean visually. The median of a density curve is the value that splits the area of the distribution in half, while the mean is the balance point (the point at which a fulcrum can be placed). A skew or extreme value affects the balance point (mean) much more than the halfway point of the area.

Standard M1.S.ID.B.4 (Supporting Content)
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
M1.S.ID.B.4a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.
M1.S.ID.B.4b Fit a linear function for a scatter plot that suggests a linear association

Scope and Clarifications:
Tasks have a real-world context.
Tasks are limited to linear functions and exponential functions with domains in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear function to fit a given data set. Choose if a given scatter plot is best represented by a linear or exponential function.	Choose an exponential function that fits a given data set, where exponential functions are limited with domains in the integers. Use a given linear function to solve a problems in the context of the data. Fit a linear function to a given set of data. Create a line of best fit and discuss reasons for choosing the line, given a scatter plot.	Fit an exponential function to a given set of data, where exponential functions are limited to domains in the integers. Solve problems using a linear or exponential function in the context of the data, where exponential function are limited to domains in the integers. Describe the similarities and differences between their chosen line of best fit and the line of best fit created using technology, given a scatter plot.	Create a contextual situation with an embedded data set derived from a given function. Explain the relationship between the function, data set, and the contextual situation using precise mathematical language and justifications. Use a given function to explain the relationship between two quantities in a created context. Explain the difference between association and causation, given a set of data within context that suggests a linear relationship.

Instructional Focus Statements

Level 3:

In grade 8, students developed an understanding of how to create a scatterplot, evaluate the scatterplot in order to describe any pattern associations between the two quantities, and informally fit a straight line to data when it visually resembled a straight line. In high school, students should extend this understanding to summarize, represent, and interpret data on two categorical and quantitative variables. This allows students to use mathematical models to capture key elements of the relationship between the two variables and explain what the model tells about the relationship. Students should gain a conceptual understanding of how to draw conclusions in addition to finding the equation for the line of best fit. As students' progress through algebra it should become apparent to them that many real-world situations produce data that can be modeled using functions that are not linear. The exposure to quadratic and exponential functions broadens the options students have for modeling data sets, where data sets can be represented in tabular, graphical, or as a discrete set of points.

Students should be exposed to real-world situations where it is apparent that the scatter plot suggests a pattern that is more curved than linear in its visual depiction. Thus leading the student to realize that a linear function does not provide the closest fit to the data causing the student to consider other function types. It is imperative that students discover that sometimes obvious patterns may not tell the whole story. Students should develop an understanding that sometimes curves fit better than lines. Students should not only discover this algebraically but also develop an understanding of the connection that exists between the model and the contextual situation that it represents and understand that this connection is essential in identifying and building appropriate models. As students solidify their understanding, they should be able to describe how the variables are related within the context of the situation. Students should also use various forms of technology to explore and represent scatterplots as this will enhance their ability to see the relationship that exits between the variables.

Level 4:

As students extend their understanding, they should be able to create a contextual situation with an embedded data set derived from a given function. Students should also be able to explain and provide justifications for the relationships that exist between the function, data set, and the contextual situation using precise mathematical language. Particular attention should be put on creating situations that differentiate between linear, quadratic, and exponential functions. Students should be able to explain why one function is more appropriate than another function for the contextual situation.

Standard M1.S.ID.C.5 (Major Work of the Grade)
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the slope and y-intercept, given a linear function in slope-intercept form. Identify two points on a scatterplot that could be used to build the line of best.	Estimate the slope and y-intercept of a linear model that would best fit data on a given scatterplot. Identify the slope of a linear model. Identify the y-intercept of a linear model.	Interpret the slope of a linear model in the context of the data. Interpret the y-intercept of a linear model in the context of the data.	Justify the appropriateness of the slope and y-intercept of a linear model in the context of the data. Explain why a linear model may only represent data in context within a certain domain. Explain why extrapolation may be unreliable for making predictions.

Instructional Focus Statements

Level 3:

In grade 8, students developed an understanding of how to build a scatterplot, describe any patterns, and informally fit a straight line to data when it resembled a straight line. In high school, students should be expected to summarize, represent, and interpret data on two categorical and quantitative variables. Instruction should expose students to mathematical models and ask them to capture key elements of the relationship between the two variables and explain what the model tells about the relationship. Specifically, students should be interpreting key features of linear models such as slope and y-intercept. Instruction should focus on slope as the rate of change of the function, specifically identifying for every increase of one by the independent variable (i.e., x), the dependent variable (i.e., y) increases or decreases by the given slope. Discussion and questioning should be focused on interpretation of the y-intercept as the value of the dependent variable when the independent variable is zero and that when the independent variable is a unit of time, the y-intercept can be interpreted as the initial value of the dependent variable. For example, given a linear model of $f(t) = 13.04t + 6.79$ where t represents years since 2009 and $f(t)$ represents height of a tree in inches students should interpret 13.04 as the number of inches the tree grows per year and 6.79 inches as the initial height of the tree in 2009. Students should be expected to interpret values such as $f(12)$, $f(0)$, or solve equations like $f(t) = 0$. Overall, students should practice modeling with mathematics by using linear models to describe how one real-world quantity of interest depends on another, which will then be expanded to include other function types in future courses.

Level 4:

As students develop a deep understanding of linear models, they should be expected to justify the reasonableness of their model to determine if it makes sense in the given context. Given the example in level 3, students would consider if $13.04x + 6.79$ is reasonable for describing tree growth, but students should be challenged to relate life experiences and number sense or research on the internet to make and explain this determination. Teachers should provide examples of scatterplots that do not fit the given context so that students have opportunities to justify why the model does not appropriately fit the context. For example, if the tree model was $y = 275.23x - 5.47$ instead, students should realize that the slope is much larger than expected and that the tree had a negative height in 2009. Although some students might say that the tree was in the ground at that point, the height of the tree would still be positive.

As students apply linear models to make predictions about unknown situations, they should justify what domain is appropriate based on the given context. For example, if time is the independent variable, students should recognize that the domain should be restricted to values greater than or equal to zero. Teachers should also expect students to justify why extrapolation is not reliable in some situations and that it cannot be assumed that the existing trend will continue to unknown values beyond the given data

Standard M1.S.ID.C.6 (Major Work of the Grade)
Compute (using technology) and interpret the correlation coefficient of a linear fit.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe the correlation of the data set that it represents as strong or weak, given a correlation coefficient. Describe the correlation of the data set as having a positive or negative direction, given a graph.	Choose a correlation coefficient that represents a data set given a graph suggesting a linear fit. Choose the strength and direction that describes the relationship, given graph that represents a data set.	Using technology, calculate the correlation coefficient of a linear fit in mathematical problems. Interpret the correlation coefficient of a linear fit in mathematical problems.	Use technology to calculate and interpret the correlation coefficient of a linear fit in real-world problems. Create a real-world linear situation and calculate and interpret the correlation coefficient. Explain what the correlation coefficient represents with respect to the contextual problem, using mathematical precise vocabulary and justifications. Determine a situation in which it is predicted that there is a strong linear relationship between two varying amounts. Create an experiment in order to collect relevant data. Test the hypothesis by generating a line of best fit and calculating and interpreting the correlation coefficient.

Instructional Focus Statements

Level 3:

Students developed a conceptual understanding of what it means for a set of data to have a linear association in grade 8. In Algebra I, students should extend their understanding to determine how strong the relationship is between the variables. Additionally, they should develop an understanding of the relationship between the direction of the fitted line and the data collected. Students will have had experience determining a linear model for a set of data. Here, they develop an understanding of the correlation coefficient as the relative closeness of the points as a set to the line of best fit. Students should understand that the correlation coefficient, denoted by r, measures the "tightness" of the data points about a line fitted to data, with a range of $-1 \leq r \leq 1$. Students should interpret this as the closer the $|r|$ is to 1 the stronger the correlation of the points and the closer the $|r|$ is to 0 the weaker the correlation of the points on the line. Students should also understand what this means with respect to the graphical nature of the scatter plot and the line of best fit. Students should be able to calculate the correlation coefficient, using technology, and understand that this value indicates that the data set has a correlation that can be described as strong, weak, positive or negative. This standard specifically states to calculate the correlation using technology rather than using a formula. The focus should be on interpreting the correlation coefficient with respect to the data set and contextual situation. Students should develop a conceptual understanding by analyzing graphical representations that illustrate what the correlation coefficient represents with respect to the graph as the distance each point is from the line of best fit. Students should also understand what this means with respect to the graphical nature on the scatter plot and the line of best fit. Additionally, students should explain in written and verbal form the connection between the correlation coefficient and contextual situation that it represents.

Level 4:

As students extend their understanding of what a correlation coefficient means and tells about the data set, they should be able to illustrate and explain the relationship between the points in the scatter plot and the line of best fit. Students should also be able to create their own contextual situations and compute the correlation coefficient showing and comparing various correlations and explain the reasoning for the nature of the correlation based on the context. The emphasis on technology is specifically called out in the standard and students should use technology to not only compute the correlation coefficient but also use technology as a means to compare different models and their respective features. Additionally, students should solidify their understanding by reasoning and making sense of different correlation coefficients and their relationship to their contextual situations.
At this level, students' understanding of correlation coefficient is such that given a real-world context, and a set of data, students can first calculate the correlation coefficient and then use it to qualitatively describe the relationship between the two variables in the context.

Standard M1.S.ID.C.7 (Major Work of the Grade)
Distinguish between correlation and causation.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Given a correlation coefficient, describe the correlation of the data set that it represents as strong or weak. Given a scatter plot, describe the correlation of the data set as strong or weak.	Define the correlation between two variables as the association and provide an example. Define causation between two variables as a cause and effect relationship and provide an example.	Explain why a strong correlation does not imply causation. Distinguish variables that are correlated because one is a cause of another and justify their reasoning.	Critique and recognize misinterpretation of correlation as causation in worked examples and explain why the reasoning is incorrect. Create real-world data points that suggest a strong linear correlation between two variables, but they are obviously not linked.

Instructional Focus Statements

Level 3:

One of the most common misconceptions when learning statistics is assuming that a strong correlation implies causation. As teachers teach for deep understanding with standard M1.S.ID.C.6, an emphasis should be placed on interpreting a strong correlation appropriately (standard M1.S.ID.C.7). Therefore, these two standards can be paired together nicely within instruction. Students should understand correlation as the strength of association of two variables, which does not mean that changes in one variable causes changes in the other. Teachers should provide examples of contextual situations in which there exists a strong correlation, but the implication of causation is obviously incorrect. For example, teachers can show students a scatterplot of children's shoe sizes versus their vocabulary level, indicating a strong correlation between the two. Students should realize that this does not indicate that shoe size influences a child's vocabulary. In fact, students should list other factors or lurking variables that may have been affecting vocabulary instead. Until all other factors can be eliminated, we cannot imply that shoe size causes changes in vocabulary

Although students should not imply that correlation means causation, there are situations in which variables are correlated because one is the cause of another. For example, there is a correlation between number of hours studied for a test and the grade you receive on a test because there is a cause and effect relationship between the two. So, a causal relationship between two variables implies the variables will be correlated, but a strong correlation does not imply causation.

Level 4:

As students develop a deep understanding of the relationship between correlation and causation, they should be given opportunities to critique the work of others and justify why their reasoning is correct or incorrect. Students should recognize when a worked example is implying correlation from causation or causation from correlation. Therefore, teachers should provide students with both types of situations. The worked examples can come from other students as they create real-world data points that suggest a strong linear correlation between two variables, but they are obviously not linked. Experiencing these situations solidifies the idea that correlation does not imply causation.