Geometry

Instructional Focus Documents

Introduction:
The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:
The evidence of learning statements are guidance to help teachers connect the Tennessee mathematics standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee mathematics standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade/course level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade/course level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade/course level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at this level demonstrates that the student has an extensive understanding and expert ability to apply the grade/course level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in this same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting.

Instructional Focus Statements:
Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.
**SIMILARITY, RIGHT TRIANGLES, and TRIGONOMETRY (G.SRT)**

**Standard G.SRT.A.1**

*Major Work of the Grade*

Verify informally the properties of dilations given by a center and a scale factor.

**Scope and Clarification:**
Properties include but are not limited to: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center of the dilation unchanged; the dilation of a line segment is longer or shorter in the ratio given by the scale factor.

**Evidence of Learning Statements**

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguish between a dilation and a translation, reflection, or rotation</td>
<td>Perform a dilation in the coordinate plane centered at the origin.</td>
<td>Determine the properties of a dilation given by a center and a scale factor.</td>
<td>Given a pre-image, center, and scale factor illustrate a dilation and explain the properties between the images using precise mathematical language.</td>
</tr>
<tr>
<td>Choose a dilation given a center and a scale factor.</td>
<td>Choose properties of a dilation given by a center and a scale factor.</td>
<td>Perform a dilation in the coordinate plane given a scale factor and a center.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Choose a scale factor given a visual representation of a dilation and a center.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Instructional Focus Statements**

**Level 3:**

In grade 8, students are introduced to the foundation of dilations. They describe the effect of dilations on two-dimensional figures using coordinates. In high school geometry, students should extend their understanding of dilations by identifying and verifying properties that are formed between a pre-image and a dilated image. Students should use precise mathematical justification to explain that dilations preserve angle measure, betweenness, and collinearity.

Students should also be able to understand and demonstrate that dilations map a line segment (the pre-image) to another one segment whose length is the product of the scale factor and the length of the pre-image. Additionally, students should be able to understand and demonstrate that dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged.
As students solidify their understanding of dilations they should be using a range of scale factors, including scale factors less and greater than 1 to gain a full understanding of how dilations can shrink as well as expand figures.

**Level 4:**
As students verify informally the properties of dilations, they should be able to work with a range of dilations where the scale factor is greater and less than 1 and a variety of centers of dilation. Students should be able to illustrate dilations using a variety of tools including but not limited to paper and pencil, dynamic geometry software, and the coordinate plane.

As students make connections to relate the sides of the image and pre-image of a dilation, they should be able to justify their reasoning using precise mathematical vocabulary.
**Standard G.SRT.A.2**  
**Major Work of the Grade**

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Scope and Clarification:**  
There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose two congruent triangles given all corresponding congruent angles and sides.</td>
<td>Choose two similar triangles given all congruent corresponding angles and the proportionality of all corresponding pairs of sides.</td>
<td>Determine if two figures are similar using the definition of similarity in terms of similarity transformations.</td>
<td>Explain the relationship between similarity transformations and the similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</td>
</tr>
<tr>
<td>Choose corresponding congruent sides and angles, given a triangle congruence statement.</td>
<td>Given a triangle similarity statement, choose congruent corresponding angles and the proportionality or all corresponding pairs of sides.</td>
<td>Identify which transformations preserve similarity for two triangles.</td>
<td>Explain the definition of similarity both in terms of transformations and in terms of measurements of corresponding sides and angles.</td>
</tr>
<tr>
<td>Identify that two triangles are similar given that the triangles have been transformed by a single rigid transformation followed by a dilation.</td>
<td>Given 2 triangles, use similarity transformations to verify that all corresponding pairs of angles are congruent and verify the proportionality or all corresponding pairs of sides to show that the triangles are similar.</td>
<td>Given two figures, determine the sequence of transformations that have occurred in order to determine when the two figures are similar.</td>
<td>Create a pair of similar figures with a sequence of dilations and rigid motions and mathematically justify that the figures are similar.</td>
</tr>
<tr>
<td>Choose a scale factor given a simple dilation.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Identify the ratio of proportionality for similar figures.

**Instructional Focus**

**Level 3:**
In grade 8, students began to develop a conceptual understanding of the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles. In the high school geometry standards, students use their prior understandings of transformations to discover the definition of similarity and further to discover which transformations preserve similarity in figures. In doing so, students should be given ample opportunity to explore sequences of transformations to visually conceptualize the relationship between the sides and the angles of similar figures. This should be done using tools that include but are not limited to patty paper, graph paper, and technology. Through this exploration, students should discover the definition of similarity in terms of similarity transformations and understand that similar figures have congruent pairs of corresponding angles and all pairs of corresponding sides have the same constant of proportionality. As students solidify their understanding of similar figures, they should understand that the scale factor increases or decreases the lengths proportionally while the angles remain congruent.

**Level 4:**
As students extend their understanding, they should be able work with a variety of sequences of dilations and rigid motions that determine if the image and pre-image are similar. Students should be able to explain the definition of similarity both in terms of transformations and in terms of measurements of corresponding sides and angles and explain the relationship using precise mathematical symbols, illustrations, and language. Additionally, students should be able to create a sequence of dilations and rigid motions to demonstrate a pair of similar figures and justify the similarity and the relationship that exists between the corresponding sides and angles.
Standard G.SRT.A.3
Major Work of the Grade

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course.

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify that two triangles congruent, given that two pairs of angles are congruent.</td>
<td>Given two pairs of independently drawn triangles, determine if they are similar using the AA criterion.</td>
<td>Determine if two triangles are similar or not similar by AA criterion using properties of similarity transformations.</td>
<td>Derive the AA criterion for similarity of triangles and explain the justification using precise mathematical vocabulary.</td>
</tr>
<tr>
<td>Given two angles in a triangle, determine the third angle.</td>
<td></td>
<td></td>
<td>Determine if two triangles are similar or not similar by AA, SSS, and SAS criteria, using properties of similarity transformations.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Create similar triangles and prove that they are similar using the AA criterion using precise mathematical justification and vocabulary.</td>
</tr>
</tbody>
</table>

Instructional Focus

Level 3:
Students began to develop a conceptual understanding of similar triangles in grade 8. In G.SRT.A.2, students gain an understanding that by using similarity transformations you can determine if two triangles are similar. Students grasped an understanding that the meaning of similarity for triangles is that all corresponding pairs of angles are congruent and all corresponding pairs of sides have the same constant of proportionality. This understanding, coupled with the angle sum theorem learned in the middle grades, results in students recognizing when pairs of corresponding angles are congruent, then the triangles are similar without verifying the proportionality of the side lengths, culminating in establishing the AA criterion.
Level 4:
As student gain an in-depth understanding of using the properties of similarity transformations to establish the AA similarity criterion for triangles, including SSS and SAS, they should be able to generalize and explain what measurements are needed to ensure that two triangles are similar. Students should also be able to provide illustrations with mathematical justifications, explaining which criteria is appropriate to prove similarity in which situations. Additionally, while the standards only explicitly states the AA similarity criterion, students should also discover other similarity criteria that work such as SSS (side-side-side) and SAS (side-angle-side), where the lengths of the corresponding pairs of sides are proportional.
**Standard G.SRT.B.4**  
**Major Work of the Grade**

Prove theorems about similar triangles.

**Scope and Clarification:**
Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

<table>
<thead>
<tr>
<th>Evidence of Learning Statements</th>
</tr>
</thead>
</table>
| **Students with a level 1 understanding of this standard will most likely be able to:**  
Given two similar triangles that are formed by a line parallel to one side of the triangle, complete a proportionality statement by choosing a missing side length.  
Given a partial two-column proof, choose the missing reason(s) that completes the proof. |
| **Students with a level 2 understanding of this standard will most likely be able to:**  
Given two similar triangles that are formed by a line parallel to one side of the triangle choose a proportionality statement.  
Given a partial two-column proof, choose the statement(s) and/or reason(s) that completes the proof.  
Given a deconstructed paragraph proof, choose the order in which the statements should appear. |
| **Students with a level 3 understanding of this standard will most likely be able to:**  
Prove theorems about similar triangles by completing two-column and paragraph proofs.  
Use triangle similarity to prove the Pythagorean Theorem and its converse. |
| **Students with a level 4 understanding of this standard will most likely be able to:**  
Given a two-column and paragraph proof with errors, determine and fix the error and explain the reasoning using precise mathematical justification.  
Prove theorems about similar triangles to discover that the geometric mean is the length of two parts of a right triangle when a perpendicular is dropped from the right angle to the hypotenuse. |

**Instructional Focus**

**Level 3:**
In this standard, students will apply their knowledge of similarity to prove various theorems about triangles. Students should have opportunities to explore situations and form conjectures that they can prove. In doing so, students should discover theorems, such as a line parallel to one side of a triangle divides the other two sides proportionally. Students should recognize that the line cuts the two sides proportionally and make the connection that the parallel lines form congruent corresponding angels and by the AA criteria the triangles are similar resulting in congruent angles and proportional side lengths. Also, students should use triangle similarity to prove the Pythagorean Theorem and its converse. These are explicitly stated in the scope and clarification of the standard but are not an exhaustive list.
Level 4:
As students extend their understanding of proving theorems about similar triangles, they should be able to work with increasingly challenging proofs and explain their reasoning in written and verbal formats. The standard's scope and clarification explicitly gives a few examples of theorems to prove but is not an exhaustive list. An additional example could include proving theorems about similar triangles to discover that the geometric mean is the length of two parts of a right triangle when a perpendicular is dropped from the right angle to the hypotenuse. To solidify understanding, students should be able to explain their reasoning by using appropriate mathematical symbols, vocabulary, and justifications.
| Standard G.SRT.B.5  
Major Work of the Grade |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures.</td>
</tr>
</tbody>
</table>

**Scope and Clarification:**
There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose transformation relationships in simple geometric figures in cases where an image is not provided.</td>
<td>Choose transformation relationships in simple geometric figures in cases where an image is not provided.</td>
<td>Use transformations to determine relationships among simple geometric figures and to solve problems.</td>
<td>Apply congruence or similarity criteria to solve complex problems involving multiple concepts, and explain the geometric reasoning involved.</td>
</tr>
</tbody>
</table>

### Instructional Focus

**Level 3:**
Students expand their learning with proofs to a broader set of situations that may include triangle similarity or triangle congruence. Students should be able to work with a broad range of situations to provide arguments about their observations using triangle similarity and congruence to explain geometric relationships. As students form connections between pairs of similar and congruent triangles they should refer back to geometric transformations to explain how the triangles are related and establish the correspondence between the triangles. Students should also make prior course work connections and use triangle congruency criteria to determine properties of parallelograms and special parallelograms.

**Level 4:**
As students solidify their understanding of congruence and similarity criteria for triangles and how they are used to solve problems, they should extend their understanding by determining properties of parallelograms, special parallelograms, and other geometric figures. Students should be able to provide mathematical justifications explicitly linking similarity and congruence of triangles to other geometric figures. Additionally, students should be able to examine and critique justifications presented by others. This will help them develop more precise arguments of their own when solving real world problems.
Solve triangles.

**G.SRT.B.8a**
Know and use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

**G.SRT.B.8b**
Know and use the Law of Sines and Law of Cosines to solve problems in real life situations. Recognize when it is appropriate to use each.

**Scope and Clarification:**
Ambiguous cases will not be included in assessment.

<table>
<thead>
<tr>
<th>Evidence of Learning Statements</th>
</tr>
</thead>
</table>
| **Students with a level 1 understanding of this standard will most likely be able to:**  
Sketch and label the sides of right triangles.  
Use trigonometric ratios and the Pythagorean Theorem to choose the unknown side lengths of a right triangle. |
| **Students with a level 2 understanding of this standard will most likely be able to:**  
Sketch, label, and identify the trigonometric ratios of a right triangle.  
Choose which trigonometric ratio is appropriate to use in solving a right triangle.  
Use trigonometric ratios and the Pythagorean Theorem to choose the unknown side lengths and angle measurements of a right triangle. |
| **Students with a level 3 understanding of this standard will most likely be able to:**  
Use the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Law of Cosines to solve mathematical and real life problems and recognize when it is appropriate to use each. |
| **Students with a level 4 understanding of this standard will most likely be able to:**  
Use the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Law of Cosines to solve complex mathematical and real life problems and recognize and explain when it is appropriate to use each. |
Instructional Focus

Level 3:
In grade 8, students develop an understanding of solving applied problems using the Pythagorean Theorem. As students encounter more complex applied problems and triangles that cannot be solved using the Pythagorean Theorem, they should develop an understanding of when it is appropriate to use the Pythagorean Theorem and when it is appropriate to use trigonometry.

Students work with three trigonometric ratios in this course: tangent, sine, and cosine. It is imperative that students develop a conceptual understanding of the trigonometric ratios and the underpinning of trigonometry. Students should begin by solidifying the procedural fluency of labeling and accurately setting up trigonometric ratios to solve applied problems.

Students should progress from using trigonometric ratios to using the Law of Sines and Law of Cosines. Students should encounter multiple examples where they develop a conceptual understanding of when it is appropriate to use the Pythagorean Theorem, right triangle trigonometry, and the Law of Sines and Law of Cosines. For any triangle, students should be able to solve for a specified angle or side and also be able to solve the triangle.

Level 4:
Students should solidify their procedural fluency around solving triangles using the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Cosines. They should be able to distinguish when it is appropriate to use each strategy and apply the strategy to solve applied problems. As students solidify their understanding, they should be able to explain their solution path and reasoning for their calculations, as well as their reasoning for selecting the appropriate strategy. Students should use precise mathematical vocabulary in their reasoning in written and verbal forms.
**MODELING with GEOMETRY (MG)**

**Standard G.MG.A.1**  
Major Work of the Grade  
Modeling Standard

Use geometric shapes, their measures, and their properties to describe objects.

**Scope and Clarification:**  
For example, modeling a tree trunk or a human torso as a cylinder. There are no assessment limits for this standard. The entire standard is assessed in this course.

<table>
<thead>
<tr>
<th>Evidence of Learning Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Students with a level 1 understanding of this standard will most likely be able to:</strong></td>
</tr>
<tr>
<td>Given a geometric shape, list real-world objects that can also be described by the shape.</td>
</tr>
<tr>
<td><strong>Students with a level 2 understanding of this standard will most likely be able to:</strong></td>
</tr>
<tr>
<td>Choose a geometric shape that can describe an object.</td>
</tr>
<tr>
<td><strong>Students with a level 3 understanding of this standard will most likely be able to:</strong></td>
</tr>
<tr>
<td>Use geometric shapes, their measures, and their properties to describe objects.</td>
</tr>
<tr>
<td><strong>Students with a level 4 understanding of this standard will most likely be able to:</strong></td>
</tr>
<tr>
<td>Determine a geometric shape or combination of geometric shapes that can describe a real-world object.</td>
</tr>
</tbody>
</table>

**Instructional Focus**

**Level 3:**  
As students solve real-world problems that involve objects, they should make the connection that geometric shapes can be used to model real-world objects. This is imperative for students to understand and apply as they solve real-world problems in future course work in G.MG.A.2. As students make this connection, they should be able to assign geometric shapes to describe objects, such as understanding that a cylinder’s measure and properties can be used to model a tree truck or a rocket ship.

**Level 4:**  
As students solidify the understanding of using geometric shapes to describe real-world objects, they should also be able to translate this understanding to using a combination of shapes to model real-world situations. As a natural integration of G.MG.A.1 and G.MG.A.2, students solidify their understanding by applying geometric methods to solve real-world problems.
**Standard G.MG.A.2**  
Major Work of the Grade  
Modeling Standard

Apply geometric methods to solve real-world problems.

**Scope and Clarification:**  
Geometric methods may include but are not limited to using geometric shapes, the probability of a shaded region, density, and design problems. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose which geometric attribute(s) need(s) to be calculated in order to solve a real-world geometric problem.</td>
<td>Identify which geometric attribute(s) need(s) to be calculated in order to solve a real-world geometric problem.</td>
<td>Apply geometric methods to solve real-world problems.</td>
<td>Create a variety of real-world problems whose solutions require the application of geometric methods.</td>
</tr>
<tr>
<td>Solve mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms when a visual representation is provided.</td>
<td>Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
<td>Solve real-world and mathematical problems involving surface area of cones, cylinders, and spheres.</td>
<td></td>
</tr>
<tr>
<td>Solve mathematical problems involving volume of cones, cylinders, and spheres when a visual representation is provided.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Instructional Focus

**Level 3:**
Students are applying geometric concepts learned in previous grades in order to solve real-world geometric application problems. Students should have familiarity with not only how to calculate area, volume, and surface area, but also the hallmark attributes of each. Counter to its predecessor standards, geometric modeling is considered major work of the grade in geometry.

Students should be formulating a strategy to solve the problem based on a mathematical understanding of the situation, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems.

**Level 4:**
Students should be formulating a strategy to solve the problem based on a mathematical understanding of the situation, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems geometric problems with increased rigor over the course.
**GEOMETRIC MEASUREMENT and DIMENSION (GMD)**

**Standard G.GMD.A.1**

**Supporting Work**

Give an informal argument for the formulas for the circumference of a circle and the volume and surface area of a cylinder, cone, prism, and pyramid.

**Scope and Clarification:**

Informal arguments may include but are not limited to using the dissection argument, applying Cavalieri’s principle, and constructing informal limit arguments.

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose and apply the formula for the circumference of a circle.</td>
<td>Apply the formula for the circumference of a circle.</td>
<td>Write an informal argument for the formulas for the circumference and area of a circle.</td>
<td>Create a visual representation and write an informal argument for the formulas for the circumference and area of a circle.</td>
</tr>
<tr>
<td>Given a partial informal argument for the formula for the circumference of a circle, choose a statement that completes the argument.</td>
<td>Apply the formula for the volume of a cylinder, cone, prism, and pyramid given the formula.</td>
<td>Write an informal argument for the formulas for the volumes and surface areas of a cylinder, cone, prism, and pyramid.</td>
<td>Create a visual representation and write an informal argument for formulas for volumes and surface areas of a cylinder, cone, prism, and pyramid.</td>
</tr>
<tr>
<td>Apply the formula for the volume of a cylinder, cone, prism, and pyramid given the formula and a visual model.</td>
<td>Given a partial informal argument for the formula for the volume or surface area of a cylinder, cone, prism, or pyramid, choose a statement that completes the argument.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Instructional Focus**

**Level 3:**

Students are applying geometric concepts learned in the middle grades in order to discuss why certain formulas work using informal arguments. The difference in this standard is that students should be able to justify why these formulas hold true.
Students began working with area in grade 3 and continued developing their conceptual understanding throughout the middle grades. This foundational understanding is the building block for students to develop a conceptual understanding of the surface area of geometric figures.

Students also began exploring circles in the early grades. The formula for the circumference of a circle can be seen when considering that, since all circles are similar, the ratios of their circumference to their diameter will be constant, defined as π. This is \( \frac{C}{d} = \pi \) or \( C = d \pi \), where \( C \) is the circumference and \( d \) is the diameter of the circle.

Students should also explore the formula for the area of a circle which is foundational for understanding the volume formula for a cylinder. Students can use the Cavalier’s principle to extend their observation about the area of a circle to consider the volume of a cylinder. Students should also make the connection that the volume of a pyramid is 1/3 the volume of a rectangular prism with the same base and height resulting in \( V = \frac{1}{3}Bh \), where \( B \) is the area of the base and \( h \) is the height of the prism. The same argument can be applied to a cone such that the volume of a cone is 1/3 of a cylinder with the same base and height resulting in the volume of a cone formula of \( V = \frac{1}{3} \pi r^2h \), where \( r \) is the radius of the cylinder, and \( h \) is the height of the cylinder.

As students make these connections, they should be able to explain why the formulas work using drawings and models in written and verbal explanations. Central to this standard is students developing an understanding of why formulas look the way they do and an understanding of the origin of the formula as opposed to simply memorizing a formula for the sake of memorizing it.

**Level 4:**
As students solidify their understanding of why these formulas work using informal arguments, they should be able to explain the connections between the formulas. Students should be able to sketch drawings of geometric figures and explain the relationship between the different figures and how and why the volume formulas work. These justifications should be accompanied with the use of precise mathematical vocabulary.
**Standard G.GMD.A.2**

**Supporting Work**

**Modeling Standard**

Know and use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems.

**Scope and Clarification:**
There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<table>
<thead>
<tr>
<th>Students with a level 1 understanding of this standard will most likely be able to:</th>
<th>Students with a level 2 understanding of this standard will most likely be able to:</th>
<th>Students with a level 3 understanding of this standard will most likely be able to:</th>
<th>Students with a level 4 understanding of this standard will most likely be able to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apply the formula for the volume of a rectangular prism with integer dimensions given a visual model.</td>
<td>Apply the formula for the volume of cylinders, cones, and spheres given the formula.</td>
<td>Apply volume formulas to solve mathematical and real-world problems.</td>
<td>Create real world and mathematical problems and apply volume formulas to solve the problems.</td>
</tr>
</tbody>
</table>

**Instructional Focus**

**Level 3:**
Students should use prior knowledge in order to know and use the volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems. Students should develop a conceptual understanding of applying the volume and surface area formula by making connections to the visual model and prior learnings. In G.GMD.A.1, students give informal arguments of volume and surface area formulas and their conceptual understandings. In this standard, students should use that developed conceptual understanding as a foundation to know the formulas and apply them to solve mathematical and real world problems.

**Level 4:**
As students solidify their understanding of surface area and volume formulas and their applications, they should begin the display procedural fluency in efficiently and accurately know formulas and using them in problems.

This standard directly addresses mathematical modeling. Given that geometric solids can be used to approximate many real life objects, the volume formulas can be used to address a broad range of contexts. Students should extend their learning to connect that not all real world scenarios can be perfectly modeled by geometric solids but it can provide approximation that yields useful information about the situation. Students should be able to explain this connection using precise mathematical vocabulary.