Algebra II
Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at this level demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in this same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

The Real Number System (N.RN)

Standard A2.N.RN.A.1 (Major Work of the Grade)
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Scope and Clarifications:
For example, we define 51/3 to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify expressions with rational exponents. Match equivalent forms of expressions involving integer exponents that include the product of powers or power of powers. Match equivalent forms of expressions involving integer exponents that include the quotient of powers..	Identify patterns that arise from properties of integer exponents and connect them to rational exponents. Given an expression with a rational exponent, rewrite the expression in radical form. Recognize the relationship between square root and an exponent of 1/2 and a cubed root and an exponent of 1/3.	Explain the relationship between the rational exponent, the index of the radical, and the power of the expression. Explain the difference between rewriting equivalent expressions by taking the square root of a number and solving an equation which includes a square root, using the principal square root function. Compare properties of integer exponents with properties of rational exponents.	Construct a viable argument explaining the relationship between rational exponents and radical notation. Critique the reasoning of others by finding errors and justify changes that could be made to correct mistakes.

Instructional Focus Statements

Level 3:

In Algebra I, standard A1.A.SSE.B.3c, students experienced using the power of a power, power of a product, and quotient of powers properties with integer exponents. In Algebra II, students extend their knowledge of these to include rational exponents. Instruction should include problems where students see a connection between the inverse operations of multiplication and division and how these inverse operations are expanded to radical and exponential forms of numbers. Instruction should begin by giving students the opportunity to explore examples such as $\left(\sqrt{16}\right)^2$to make the connection that this is the same thing as 4^2. By exploring this concept, students will understand how the square of a number is the inverse operation of taking the square root. Instruction should also lead students to the understanding that the ½ power represents a square root. Students should be expected to rewrite expression using factorizations and exponent rules to apply the meaning of these rational exponents. For example, given $\left(\sqrt{16}\right)^3$, students will rewrite the example as $16^{2/3}$. This would help students recognize the square root of 16 is equal to 4, and then cube that number to reveal the answer of 64. Multiple solution methods could be explored to help students find what patterns hold true to reinforce the order of operations and various exponent properties.

A common misconception for students is the confusion of the meaning of an exponent -2 with the meaning of ½ power. Students may confuse $(9^{-2})^2$ as the same as $\left(9^{1/2}\right)^2$. In addition, students often confuse rewriting expression in a simpler form by taking the square root of a number with finding the solution set of an equation. Discussion should address the difference between the $\sqrt{16}$ and $x^2 = 4$, using the principal square root function. With full understanding of this standard, students should be able to explain why $\left(4^{1/2}\right)^2 = 4$.

Level 4:

To deepen the level of understanding, instruction should provide the opportunity for students to engage in more complex problems involving rational exponents and the notation for radicals in terms of rational exponents. Students will apply properties of rational exponents in order to simplify an expression and change parts written in radical form into exponential form. This process might make it easier for students to apply properties of exponents. An example would be to simplify $\sqrt[3]{x^2} * \left(\sqrt[6]{x^4}\right)^{-3}$ or $\frac{\sqrt[3]{x} * \sqrt{x^5}}{\sqrt{25x^{16}}}$. As students simplify these expressions, they should justify their steps and explain their reasoning.

Additionally, focus should be placed on critiquing the reasoning of others. Instructions should include providing students with expressions that include rational exponents and radicals which have been simplified. Students determine if the expressions are simplified correctly and if not, they should correct the mistakes and explain why it was incorrect.

Standard A2.N.RN.A.2 (Major Work of the Grade)
Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the index and power of an expression written in exponential form. Identify the index and power of an expression written in radical form.	Match expressions written in radical form with the equivalent expression written in exponential form. Match expressions written in exponential form with the equivalent expression written in radical form.	Write an equivalent expression using rational exponents, given an expression in radical form. Write an equivalent expression using radicals, given an expression written exponential form. Move fluently between radical and exponential form of expressions. Explain the process of changing an expression from radial form to exponential form. Explain the process of changing an expression from exponential form to radical form. Write an equivalent expression using the properties of exponents, given an expression with rational exponents.	Simplify complex fractions which contain radicals in the denominator. Critique the reasoning of others, given expression which contains rational exponents and/or radicals and the steps to simplify it. Construct an argument explaining why or why not a chosen strategy to simplify expressions best suits the problem.

Instructional Focus Statements

Level 3:

Students should be able to simplify fluently between radical and exponential form. Instruction should expose students to a variety of problems that include radicals and exponents so that they realize that some problems are easier to work a certain way. Students should be provided expressions that can be simplified in either radical or exponential form. By having students rely on their knowledge of exponent rules and meaning of radicals, by asking students to solve without direct steps, students may present multiple ways to simplify these expressions. Discussion can then take place on different methods that can work on the same type of problem and allow teachers the chance to explain there is not only one correct method. After students have practiced with multiple approaches, they should be able to choose which approach they prefer and defend their reasoning for choosing a particular method.

Students should be given both symbolic and contextual problems to solidify their understanding of radicals and rational exponents. Symbolic examples could be problems such as $c^2 * \sqrt{c} = c^2 * c^{1/2}$. This will help students understand they must find a common denominator to add 2 and ½ to get a new power of 5/2. Students could also be given problems involving the volume of a sphere to apply these skills in contextual situations.

Level 4:

Exposing students to more complex expressions, such as ones that include fractions with radicals in the denominator, will help deepen students understanding of this standard. Algebra II is the first math course where students are asked to rationalize denominators. Instruction should include providing students with simple problems where they are given a radical in the denominator. Challenging students to find another expression that, when multiplied, will produce a value that has an integer root will lead to discussions about repeated factors and various types of roots. For example, students should understand that when the $\sqrt[5]{4} * \sqrt[5]{8}$ = $\sqrt[5]{32} = 2$ and would eliminate the radical.

As students become familiar with these types of expressions, instruction will extend to fractions and eventually, complex fractions. As students become proficient with rationalizing the denominator, they should understand why this process is important for eliminating the radical. Instruction should also include expressions where variables occur under the radical sign. Students should be introduced to the Absolute-Value-Square Root Theorem and in simple terms understand that if an odd power is removed from an even root, an absolute value sign will ensure variables have a positive value. In addition, students should be provided with radical and exponential expressions which have been simplified.

Quantities (N.Q)

Standard A2.N.Q.A.1 (Supporting Content)
Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

Scope and Clarifications: (Modeling Standard)
Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the units in a problem. Connect the units to the values in a real-world problem.	Identify individual quantities in context of the real-world problem and label them with appropriate units. Recognize irrelevant or extraneous information in a real-world problem.	Identify and interpret information to select or create a quantity to model a real-world problem. Describe individual quantities in context of the real-world problem. Attend to precision when defining quantities and their units in context. Explain and justify the relationship between a solution to a contextual problem and the values used to compute the solution. Appropriately interpret, explain the meaning of, and draw conclusions about the quantities in a real-world problems.	Identify, interpret, and justify complex information with a variety of descriptors or units to solve contextual problems for the purpose of descriptive modeling. Represent quantities in descriptive modeling situations and explain their relationship using multiple formats such as numeric, algebraic, and graphic representations.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Make observations about quantities given a graph or model. Interpret and explain irrelevant or extraneous information in a real-world problem.	

Instructional Focus Statements

Level 3:

In grades K-8, students developed an understanding of measuring, labeling values, and understanding how the value of the number relates to the described quantity. In the high school NQ domain, students develop an understanding of reasoning quantitatively and using units to solve problems. This standard should be taught within integration with other standards throughout the course. Students should extend this understanding by applying their knowledge to modeling situations where they can make comparisons between two distinct quantities and justify the quantities appropriately in order to describe a contextual problem. Instruction should focus on providing opportunities of real-world problems where students have to select appropriate quantities and attend to precision in describing the quantities in descriptive modeling situations. Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc. The study of dimensional analysis is an excellent avenue to help students understand how critical values, units, and quantities are used in interpreting information and modeling a real-world problem. Furthermore, students must be given opportunities to write and create appropriate labels for quantities and explain the meaning of the quantities in a context. Being able to identify, interpret, and justify quantities is a skill that will serve students well to have mastered during this course as this standard lays the foundation for using units as a way to understand problems.

Level 4:

Instruction should focus on providing opportunities for students to work with problems that have a variety of descriptors and units within the context. Students should be asked to extend their knowledge of quantities by representing them in multiple formats such as a graphical representation of the given information, algebraic representation of the quantities, and multiple representations to predict or draw conclusions about the solution of the real-world problem. Instruction should provide opportunities for students to analyze and critique the interpretation of quantities in a descriptive modeling problem. Additionally, students should be given ample opportunities that promote inquiry to design their own contextual problem in which they would have to use quantities appropriately in order to describe the modeled contextual situation.

The Complex Number System (N.CN)

Standard A2.N.CN.A.1 (Supporting Content)
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify when a square root represents an imaginary number. Rewrite the square root of a negative perfect square number as a pure imaginary number.	Rewrite a number containing only the square root of a negative number in $a + bi$ form. Differentiate between the real and imaginary terms in a complex number.	State that there is a complex number i such that $i^2 = -1$. Distinguish between a real number, a pure imaginary number and a complex number. Express complex numbers in the written form $a + bi$, where a and b represent real numbers.	Recognize that solutions to problems may be complex numbers and identify whether or not the solutions are viable within a mathematical or real-world context. Plot complex numbers on a complex plane. Find the absolute value of a complex number graphed on a complex plane.

Instructional Focus Statements

Level 3:

Algebra II is a student's first interaction with the complex number system. Students learn that the definition of an imaginary unit is $i = \sqrt{-1}$ and they learn that the solutions for $x^2 = -1$ are $\pm i$. A foundational focus for this standard is for student to understand why taking the square root of a negative number generates a complex number. Students should also develop an understanding that all numbers can be written in the form $a + bi$ when a and b are both real numbers. Students should be able to recognize pure real numbers in the form of $a + 0i$ and pure imaginary numbers in the form of $0 + bi$

to solidify the understanding that both pure real and pure imaginary numbers can be written in complex form. Additionally, students should be able to describe a situation from which complex numbers can emerge, such as a quadratic equation in the form of $x^2 = a$, where a is a negative real number.

Level 4:

As students develop a deep understanding of complex numbers, they will be able to construct a viable argument to describe why taking the square root of a negative number generates an imaginary number. Students should also solve equations from contexts eliciting complex numbers and be able to explain why the context generated a complex solution and whether the solution is viable or not viable. As they solidify their understanding, students will justify their reasoning as to why some solutions are viable while others are not in a real-world situation. To support the study of the magnitude of vectors in future courses, students can be challenged to plot complex numbers in the complex plane. Student thinking could then be pushed for students to apply what they know about the relationship between absolute value and distance to discover how to calculate the absolute value of complex numbers utilizing the complex plane. Students come to realize that when drawing a line from the complex number to the origin and creating a right triangle, they can use their knowledge of right triangles to find the magnitude (length) of the line which represents the absolute value of the complex number.

Standard A2.N.CN.A.2 (Supporting Content)
Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the real and imaginary parts of a given complex number. Identify that all imaginary numbers are like terms and all real numbers are like terms.	Add, subtract, and multiply pure imaginary numbers.	Add, subtract, and multiply complex numbers. Explain when the commutative, associative, and distributive properties are helpful when writing equivalent expressions involving complex numbers. Recognize there is a pattern that will emerge when *i* is raised to positive integer powers.	Write an equivalent expression to a given multi-operational expression that involves adding, subtracting, and/or multiplying complex numbers, identify the properties used within the simplification, and provide justification for why the original and the new expression are equivalent. Explain how the commutative, associative, and distributive properties are helpful when rewriting equivalent expressions involving complex numbers. Identify the pattern that exists when *i* is raised to positive integer powers and explain why it occurs.

Instructional Focus Statements

Level 3:

The complex number system is first introduced to students in Algebra II. It is important that instruction for this standard be closely tied to the conceptual understanding of complex numbers developed in standard A2.N.CN.A.1. The focus of instruction for this standard should initially provide students opportunities to compare and contrast computations with complex numbers to computations with rational and irrational numbers. For example, students may consider expressions such as $(3 - 2\sqrt{5}) + (7 + 8\sqrt{5})$ and (3 – 2*i*) + (7 + 8*i*) noting the similarities and differences that exist between the two. It is important that students not be provided a list of rules and steps to follow when working with addition, subtraction, and multiplication of complex numbers. They need to be given time to develop an understanding of how and why complex numbers interact the way they do with the operations of addition, subtraction, and multiplication.

As students become more fluent in computing with complex numbers, they should begin thinking about how mathematical properties apply to simplifying expressions such as $5(6 - i)$ and $3i\,(4 - 2i)$, when each property is useful, and why. Ultimately, students should self-identify processes for simplifying expressions containing complex numbers.

Students at this level may begin to recognize that there is a pattern existing when *i* is raised to positive integer powers as they work with simplifying increasingly challenging expression.

Students will use their knowledge of complex numbers in future courses when working with vectors.

Level 4:

At this level, students should be challenged to write an equivalent expression to a given multi-operational expression that involves adding, subtracting, and/or multiplying complex numbers. Students with a deep understanding of complex numbers and how each is effected by addition, subtraction, and multiplication should be able to seamlessly move between the operations when presented multi-operational expressions. Additionally, students should be able to identify the properties used within the simplification and provide justification for why the original and the new expression are equivalent.

Students at this level not only recognize that there is a pattern existing when *i* is raised to positive integer powers, but also explicitly identify the pattern and explain why it occurs. This identification should be the by-product of students noticing the pattern as they work to simplify increasingly challenging expressions.

Students could also be challenged to apply their knowledge of operations with complex numbers to solve real-world problems such as finding the impedance of a parallel circuit with two pathways giving students the opportunity to see applications of complex numbers in real world situations.

Standard A2.N.CN.B.3 (Supporting Content)
Solve quadratic equations with real coefficients that have complex solutions.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the real and imaginary part, given the solution to a quadratic equation. Determine if the roots are real or non-real, given the graph of a quadratic equation. Recognize if an equation in the form of $x^2 = r$ would have real or non-real solutions. Calculate the discriminant of a quadratic equation.	Use the discriminant to determine if solutions to a quadratic equation are real or non-real. Solve a simple quadratic equation using inverse operations (including taking the square root of both sides) and identify the solutions as real or complex.	Solve using the quadratic formula and identify solutions as real or complex, given a quadratic equation in the form $ax^2 + bx + c = 0$. Solve by completing the square and identify solutions as real or complex, given a quadratic equation in the form $ax^2 + bx + c = 0$. Explain why quadratic functions may produce real or complex solutions. Determine the reasonableness of solutions by graphing a quadratic function and examining the roots.	Solve the equation and determine if all solutions are viable in the context of the problem, given a real word situation that reflects a quadratic function. Explain why a quadratic equation has a complex solution based on the operations used to solve and the definition of $i^2 = -1$.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

In Algebra I, students begin solving quadratic equations in a variety of ways, but only recognized real number solutions. In Algebra II, instruction should help students realize all solutions to a quadratic equation are not real numbers and that complex solutions can arise. Modeling with graphing tools will help students understand that solutions to a quadratic function may include one x-intercept, two x-intercepts, or no x-intercepts. They should connect a graph with no x-intercepts to an equation with complex roots. Discussion should also take place to help students distinguish between irrational roots and imaginary roots. Many times, students have difficulty understanding that an irrational root falls in the real number system. They assume because an irrational number does not have an exact value that it is a complex number. Students should be provided with quadratic equations which result in irrational roots and discussion should focus on comparing the location of these roots on a graph with complex roots. The connection to the graphical representation of equations can help students visually see the difference and support algebraic understanding.

Instruction should include providing students with quadratic equations in standard form, vertex form, or intercept form, and have them choose a method for solving (factoring, solving by taking the square root, quadratic formula, completing the square, or graphing) then justify why their solution(s) are real or complex. Students should be exposed to various types of equations and asked to justify how they could determine the type of roots algebraically, as well as, graphically. Reinforcing the definition of a complex number as described in standard A2.N.CN.A.1 can help focus students on what to look for as they solve problems algebraically. Additional support can come from providing students with equations in multiple forms and have them to construct tables representing those equations. They could then inspect those tables and find the zero(s) or justify approximately where the zero(s) would be located. Discussion about the patterns seen in the table of values should help students realize that even though they may not find the exact point on the table which represents the zero, they should be able to determine an approximate location on the table due to the shape of a parabola. It is important to include graphing equations from tables (by hand or using technology) to determine the number and placement of zeros. Equations that are provided should include options of two real roots, one real root, or two complex roots. This process should help students compare and contrast functions written in different forms, and as zeros are revealed from the quadratic equations, students should explain why roots are real or complex.

Level 4:

As students develop a deep understanding of solving quadratic equations they should be provided the opportunity to extend their learning to master solving an equation in a way that is most efficient. Discussion should challenge students to analyze and justify solution methods of equations in different forms based on the ability to quickly recognize the type of roots.

Additionally, instruction should include providing students with real-world quadratic problems in order to apply their problem solving techniques. These real-world problems should include those which produce both real and complex solutions. Students determine if all real solutions are viable in the context of their problem or justify why the function in context would not have a real solution. Explaining their reasoning will help students deepen their understanding of quadratic functions embedded in real-world problems.

SEEING STRUCTURE in EXPRESSIONS (A.SSE)

Standard A2.A.SSE.A.1 (Major Work of the Grade)
Use the structure of an expression to identify ways to rewrite it.

Scope and Clarifications:
For example, see $2x^4 + 3x^2 - 5$ as its factors $(x^2 - 1)$ and $(2x^2 + 5)$; see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$; see $(x^2 + 4)/(x^2 + 3)$ as $((x^2 + 3) + 1)/(x^2 + 3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.
Tasks are limited to polynomial, rational, or exponential expressions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a polynomial, rational, or exponential expression that is equivalent to a given expression.	Rewrite polynomial, rational, and exponential expressions into a given form.	Rewrite polynomial, rational, and exponential expressions into a different form and explain why rewriting the expression in that form is beneficial.	Generate multiple forms of a single polynomial, rational, or exponential expression and explain in both verbal and written form the mathematics that was employed to transform the expression. Additionally, explain which form is most useful and provide mathematical justification.

Instructional Focus Statements

Level 3:

Seeing structure in expressions involves critically examining an algebraic expression in which potential rearrangements and manipulations are present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which might not.

Students should be able to provide a mathematical justification for when different forms of expressions are more beneficial. As there are function families overlapping Algebra I and Algebra II, with polynomials, focus needs to be placed on non-quadratic polynomial expressions as they were a focus in Algebra I. Additionally, focus needs to be placed on more complex exponential expressions. Much of the ability to see and use structure in transforming expressions comes from learning to fluently recognize certain fundamental algebraic situations.

Level 4:

Students need to be challenged to write polynomial, rational, and exponential expressions in multiple forms where the initial expressions increase in difficulty over time. The hallmark of this standard is students being able to communicate the importance and benefit gained from writing expressions in various forms. Students should be able to express what the individual terms within the expression mean and how they relate to terms in the other various representations of the same expression.

Standard A2.A.SSE.B.2 (Major Work of the Grade)
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A2.A.SSE.B.2a Use the properties of exponents to rewrite exponential expressions.

Scope and Clarifications: (Modeling Standard)
For example, the expression 1.15^t can be rewritten as $((1.15)^{1/12})^{12t} \approx 1.012^{12t}$ to reveal that the approximate equivalent monthly interest rate is 1.2% if the annual rate is 15%.

i. Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.
ii. Tasks are limited to exponential expressions with rational or real exponents.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize an exponential expression. Recognize properties of exponents. Without a context, choose an equivalent form of an exponential expression.	Choose an equivalent form of an exponential expression and choose the properties used to transform the expression from a real-world context.	Generate an equivalent form of an exponential expression and identify the properties of exponents used to generate the expression from a real-world context.	Generate equivalent forms of an exponential expression, justify each transformation with a property, and explain the benefits of the equivalent expression from a real-world context

Instructional Focus Statements

Level 3:

In Algebra II, the focus of exponential expressions shifts to those with rational and real exponents further widening the real-world situations students can encounter.

It is important to note that this is a modeling standard and that the exponential expressions should be embedded in real-world situations. This provides a context for seeing structure in the expression and allows students to see when and why it is beneficial to view them in different forms. Additionally, it's important to note that the focus is not on writing expressions in simplest form as there really is no simplest form. The form that expressions are written in should be driven by what is being done with the expression in the first place.

Level 4:

Students should continue to demonstrate an understanding of seeing structure in expressions by not only being able to rewrite exponential expressions in various forms, but also in mathematically justifying the steps to reach the desired rewritten form and describing when and why the rewritten form would be beneficial. Students should encounter exponential expressions with real and rational exponents of increasing difficulty in increasingly more complex real-world situational problems.

Standard A2.A.SSE.B.3 (Major Work of the Grade)
Recognize a finite geometric series (when the common ratio is not 1), and use the sum formula to solve problems in context.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the first term, common ratio, and number of terms in a finite series. Discern the difference between an arithmetic and geometric series.	Write out the numerical representation of a geometric series based on a given verbal description. Identify the components in the sum formula. Recognize when real-world problems can be represented by a finite geometric series.	Identify when a geometric series appropriately describes a given real-world scenario. Identify and calculate when necessary the values of a, r, and n using the sum formula. Explain the components in the geometric sum formula. (i.e. where a is the initial value, r is the ratio, and n is the number of terms) Identify the components of the geometric sum formula and recognize the implications of the values of a, r, and n by telling how each affects the series. Write and use the geometric sum formula to solve contextual problems.	Explain phenomena in terms of a finite geometric series. Construct a real-world scenario that can be represented by a geometric series. Explore the possibility of the sum of infinite geometric series.

Instructional Focus Statements

Level 3:

In previous courses, students developed an understanding of sequences of numerical patterns, including generating geometric sequences using a common ratio. This standard builds on the understanding developed in the standard F.LE.A.1 where students connect their learning about arithmetic and geometric sequences verbally, graphically, numerically, and algebraically. Students see how a variety of real-world situations are represented by these sequences. With this foundational knowledge, instruction should focus on contextual problems and guide students to make connections between geometric sequences and series. Students should be given the opportunity to investigate the structure of a set of short geometric series allowing them to discover a, n, and r and their part in the finite geometric sum formula and draw conclusions about the effects of a, n, and r in longer geometric series. Furthermore, students should be able to generalize how to compute the sum of short geometric series and infer how to compute for longer geometric series.

Instruction should provide an opportunity to explore attributes of a series with a negative r value and help students draw the conclusion that the signs of the series of numbers will alternate. It is important to note that having students memorize the formula for the sum of a finite geometric series will not help students develop the necessary conceptual understanding of the structure of the formula which is what ultimately allows them to be successful in solving contextual problems. Students must attend to precision and explain their answer in context. In Pre-Calculus, students will be asked to demonstrate an understanding of sequences by writing them recursively and explicitly. Students will also use sigma notation to represent a series and extend their knowledge of summation by identifying whether a series converges or diverges.

Level 4:

Once students have a strong understanding of geometric series, they should be able to explain fluently the components of the sum formula, and use the sum formula to solve contextual problems, then they should be given the opportunity to explore these concepts with an infinite geometric series. At this level of understanding students should be able to explain the steps in the derivation of the sum formula, so instruction should give students the opportunity to consider and study the proof of the sum formula.

ARITHMETICS with POLYNOMIALS and RATIONAL EXPRESSIONS (A.APR)

Standard A2.A.APR.A.1 (Major Work of the Grade)
Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define factor.	Choose the remainder when a polynomial $p(x)$ is divided by $x - a$. Determine if a given number a is a possible factor for a polynomial $p(x)$.	For a polynomial $p(x)$ and a number a, determine if x-a is a factor of $p(x)$. Identify the remainder when a polynomial $p(x)$ is divided by $x - a$. Identify all possible factors of a polynomial $p(x)$.	Find all factors for a polynomial $p(x)$. Explain the Remainder Theorem using appropriate mathematical vocabulary in both verbal and written form.

Instructional Focus Statements

Level 3:

A particularly important application of polynomial division is the case where a polynomial $p(x)$ is divided by a linear factor of the form x-a, for a real number a. In this case the remainder is a value $p(a)$ of the polynomial at x=a. It is important that this topic not be reduced to simply “synthetic division,” which reduces the method to a matter of carrying numbers between registers, something easily done by a computer, and prevents students from developing conceptual understanding of the Remainder Theorem. It is important for students to see the Remainder Theorem as a theorem, not a technique.

Level 4:

Students with a deep conceptual understanding of the Remainder Theorem can explain the equivalence between linear factors and zeros. This is the basis of much work with polynomials in high school: the fact that $p(a)=0$ if and only if $x-a$ is a factor of $p(x)$. They can deduce that if $x-a$ is a factor then $p(a)=0$. But the Remainder Theorem tells us that $p(x)=(x-a)\, q(x) + p(a)$ for some polynomial $q(x)$. In particular, if $p(a)=0$ then $p(x)=(x-a)\, q(x)$, so $x-a$ is a factor of $p(x)$.

Standard A2.A.APR.A.2 (Major Work of the Grade)
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Scope and Clarifications:
Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Factor a quadratic polynomial with a lead coefficient of 1. Choose the zeros for a given quadratic polynomial with a lead coefficient of 1. Choose a graph to represent a given quadratic polynomial in factored form.	Factor a quadratic polynomial with a lead coefficient of 1, identify the zeroes, and construct a rough graph of the function defined by the polynomial. Explain the mathematical term zero using appropriate mathematical vocabulary in both verbal and written form. Choose a graph to represent a given polynomial presented in factored form. Generate a rough graph to represent a given quadratic polynomial presented in factored form.	Factor a quadratic, cubic, or quartic polynomial, identify the zeroes, and construct a rough graph of the function defined by the polynomial. Generate a rough graph to represent a given non-quadratic polynomial function presented in factored form.	Explain the process for generating a rough sketch of any factorable polynomial function using accurate mathematical language in both written and verbal form.

TN Department of Education

Instructional Focus Statements

Level 3:

Polynomial functions are, on the one hand, very elementary, in that they are built up out of the basic operations of arithmetic. On the other hand, they turn out to be amazingly flexible, and can be used to approximate more advanced functions such as trigonometric and exponential functions in later courses. Experience with constructing polynomial functions satisfying given conditions is useful preparation not only for calculus, but for understanding the mathematics behind curve-fitting methods used in applications to statistics and computer graphics.

The first step in developing this understanding is to construct a rough graph for polynomial functions by using their zeros. Eventually, this progression will lead to constructing polynomial functions whose graphs pass through any specified set of points in the plane.

It is important that students in this early stage continue to develop an understanding of the connection that exists between the graphical and algebraic representation of zeroes and that they are not simply following a rote procedure but provide evidence of an understanding of this connection.

In Algebra II, students are focusing on quadratic, cubic, and quartic polynomials when factors are not provided. Quadratic polynomials were also a focus for Algebra I. Thus in Algebra II, when quadratics are the focus, they should be of appropriate difficulty.

Level 4:

At this level of understanding, students should be demonstrating strong command of the relationship that exists between an algebraic representation that elicits zeroes of a polynomial function and the graphical representation of zeros moving fluidly between the two. Additionally, they should be able to provide a mathematical explanation of the relationship between algebraic and graphical representations of zeroes.

Standard A2.A.APR.B.3 (Supporting Content)
Know and use polynomial identities to describe numerical relationships.

Scope and Clarifications:
For example, compare $(31)(29) = (30 + 1)(30 - 1) = 30^2 - 1^2$ with $(x + y)(x - y) = x^2 - y^2$.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Match polynomial identities with numerical relationships that are examples of the polynomial identity.	Use a polynomial identity, to describe a given numerical relationship.	Identify an appropriate polynomial identity and use it to describe a given numerical relationship.	Identify an appropriate polynomial identity and use it to describe a given numerical relationship and explain the benefit of using that particular polynomial identity to describe the numerical relationship.

Instructional Focus Statements

Level 3:

Polynomials form a rich ground for mathematical explorations that reveal relationships in the system of integers. Instruction should be focused on looking at a wide variety of numerical relationships that are intentionally connected to a polynomial identity. Instruction should not focus simply on the rewriting of numerical relationships, but instead on why it is beneficial to do so.

Level 4:

As students master this standard, they show the most conceptual understanding when they are able to explain the benefit of rewriting numerical relationships in multiple ways. Students should experience numerical relationships that can be rewritten using polynomial identities with increasing variance and difficulty over time.

Standard A2.A.APR.C.4 (Supporting Content)
Rewrite rational expressions in different forms.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Rewrite a polynomial division expression as a rational expression and vice versa.	Choose equivalent forms to represent a rational expression.	Rewrite rational expressions involving addition, subtraction, multiplication and/or division in different forms.	Explain the mathematical relationship that exist between the Remainder Theorem and rewriting rational expressions with a polynomial numerator and a first degree binomial denominator. Rewrite complicated rational expressions involving addition, subtraction, multiplication and/or division in different forms.

Instructional Focus Statements

Level 3:

This standard serves a dual purpose. First, it provides the opportunity for students to interact with long division which is similar to integer long division. When connected to standard A2.A.APR.A.1, it helps support students developing an understanding of the Remainder Theorem. Second, it offers students the opportunity to connect operations on rational numbers to operations with rational expressions. Particular attention should be paid to this connection as opposed to a rote series of steps without any conceptual understanding.

Level 4:

The focus of instruction should emphasize the discovery of the connections that exist between the Remainder Theorem and rational division so that students can explain the relationship. Additionally, they should encounter and work with simplifying rational expressions involving all operations with increased rigor over time.

CREATING EQUATIONS* (A.CED)

Standard A2.A.CED.A.1 (Major Work of the Grade)
Create equations and inequalities in one variable and use them to solve problems.

Scope and Clarifications: (Modeling Standard)
Include equations arising from linear and quadratic functions, and rational and exponential functions.
Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear or quadratic equation in one variable that represents a simple, real-world situation. Create and solve a one variable linear equation or inequality that represents a simple, real-world situation. Solve a simple one variable quadratic or exponential equation. Identify if a real-world situation can be represented by a linear, quadratic, rational or exponential equation. Determine if the solution to a real-world situation requires a one-variable or two variable equation or inequality.	Solve a simple one variable quadratic inequality. Solve a simple one variable exponential inequality. Solve a simple one variable rational equation. Solve a simple one variable rational inequality. Choose a quadratic or exponential equation to represent a simple, real-world situation. Choose a quadratic or exponential inequality to represent a simple, real-world situation.	Create and solve a one variable linear, quadratic, rational, or exponential equation that represents a real-world situation. Create and solve a one-variable linear, quadratic, rational, or exponential inequality that represents a real-world situation.	Create a real-world problem to represent a given linear, quadratic, or exponential equation or inequality.

Instructional Focus Statements

Level 3:

In Algebra II, the variety of function types that students encounter allows students to create even more complex equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations and inequalities that evolve from real-world situations. Students should be formulating equations and inequalities, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems. Real-world situations should elicit equations and inequalities from situations which are linear, quadratic, exponential, and rational in nature. As linear, quadratic, and simple exponential functions are a focus in Algebra I, it is imperative that students have the opportunity to work with polynomials with degree greater than 2, rational, and complex exponential equations and inequalities in Algebra II.

Level 4:

When given an equation or inequality, students can generate a real-world situation that could be solved by a provided equation or inequality demonstrating a deep understanding of the interplay that exists between the situation and the equation or inequality used to solve the problem.

Additionally, students should continue to encounter real-world problems that are increasingly more complex. Students should be using the modeling cycle to solve real-world problems.

Standard A2.A.CED.A.2 (Major Work of the Grade)
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Scope and Clarifications: (Modeling Standard)

i. Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.
ii. Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose equivalent forms of a given quadratic real-world formula.	Rearrange real-world quadratic formulas to highlight a quantity of interest. Choose equivalent forms of a given square root real-world formula. Choose equivalent forms of a given cube root real-world formula. Choose equivalent forms of a given non-quadratic polynomial real-world formula. Choose equivalent forms of a given rational root real-world formula. Choose equivalent forms of a given logarithmic real-world formula.	Rearrange real-world square root formulas to highlight a quantity of interest. Rearrange real-world cube root formulas to highlight a quantity of interest. Rearrange real-world non-quadratic polynomial formulas to highlight a quantity of interest. Rearrange real-world rational formulas to highlight a quantity of interest. Rearrange real-world logarithmic formulas to highlight a quantity of interest.	Rearrange real-world square root, cube root, polynomial, rational, and logarithmic formulas and explain the benefit of solving the formula for the various variables.

Instructional Focus Statements

Level 3:

In previous grades and courses, students have focused on rearranging linear, quadratic, and exponential formulas with integer exponents to highlight a quantity of interest. In Algebra II, students should be working with square root, cube root, polynomial, rational, and logarithmic formulas. As this is a modeling standard, student should be encountering formulas that come from real-world situations. Additionally, students need to be deepening their conceptual understanding of why they might need to write formulas in different ways and what the benefit would be to these various representations of the same real-world formula.

Level 4:

Students need to be exposed to a wide variety of real-world formulas increasing in complexity over time. Additionally, it is imperative that they are able to explain why formulas might need to be expressed in different ways and the benefit that each form provides.

Reasoning with Equations and Inequalities (A.REI)

Standard A2.A.REI.A.1 (Major Work of the Grade)
Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Scope and Clarifications:
Tasks are limited to square root, cube root, polynomial, rational, and logarithmic functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the inverse operations used in solving the equation, given a square root, cube root, polynomial, rational, and logarithmic equation and a list of steps for the solution method. Choose a possible next step to solve the equation, given a square root, cube root, polynomial, rational, and logarithmic equation and a partial list of steps for the solution method. Arrange steps in the order they should be applied, given a square root, cube root, polynomial, rational, and logarithmic equation and a list of unordered steps for the solution method.	Explain the reasoning for each step, given a polynomial equation and a list of steps for the solution method. Explain the reasoning for each step, given a rational equation and a list of steps for the solution method. Identify when it can be determined if no solution or infinitely many solutions exists, given an equation and a list of steps for the solution method.	Solve square root, cube root, polynomial, rational, and logarithmic equations using multiple solution strategies and explain each step in the solution method. Construct a viable argument to justify a chosen solution method used to solve a square root, cube root, polynomial, rational, and logarithmic equation. Compare the steps in each and determine which solution method is most efficient, given an equation with multiple solution methods. Explain why an equation has no solution or infinitely many	Solve the problem, explain each step in the solution path, and justify the solution path chosen, given a real-world problem and an equation that represents the contextual situation. Compare and contrast two given solution paths to a contextual problem and construct a viable argument on which method is most efficient. Correct the mistakes in the solution path and provide an explanation of the misconception using precise mathematical vocabulary, given a list of steps and an inaccurate solution for a square root, cube root, polynomial, rational, and logarithmic equation.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the reasoning for each step, given a one or simple two-step square root and cube root equation and a list of steps for the solution method.		solutions.	

Instructional Focus Statements

Level 3:

In Algebra II, students should develop a conceptual understanding of solving equations as a reasoning process to determine a solution that satisfies the equation rather than a procedural list of steps. Instruction should focus on students creating and determining solution paths or each unique equation and providing a viable argument to justify the chosen solution path. Students should also be able to explain how, when, and why equations have no solution or infinitely many solutions. To help give meaning to these solution types, discussion should focus on the solution being a value of the variable that makes the equation true. This will help students make the connections that an equation has no solution because there is no value that can maintain equivalency and an equation has infinitely many solutions because all values used for the variable create a true equivalency statement.

Students should understand that a problem can have multiple entry points and instruction should be focused on solving equations using a reasoning process of centered around inverse operations and order of operations. Students develop a conceptual understanding of operations in previous grades and they should deepen their understanding of the interplay that exist between the operations. To illustrate maintaining equivalency, a visual and/or concrete model of a balance scale can be used to aid students in understanding that the same inverse operations are being applied to the whole left side and the whole right side of an equation. Emphasizing equivalency is vital in developing a conceptual understanding of solving equations and preventing common misconceptions. A common misconception is applying an exponent to each term individually instead of applying the power to the entire side of an equation as a quantity. For example, when solving $\sqrt{x-1} = x+2$, students may make the common misconception of raising each individual term to the second power instead of raising the quantity to the second power resulting in a the inaccurate next step of $x-1 = x^2+4$. It is imperative in solving equations for students to understand that inverse operations should be applied to the left side of the equation as one quantity and the right side of the equation as one quantity, not to each term individually. Classroom discussion should address the importance of using grouping symbols when necessary and applying the properties of exponents appropriately as students use inverse operations to solve equations.

Students should understand that the solution path they choose to solve any equation must result in a series of equivalent equations all of which have the same solution set. As students apply inverse operations to solve equations, they should be able to explain why equality holds true when performing the selected operation to both sides of the equation. In this course students should be exposed to square root, cube root, polynomial, rational, and logarithmic functions.

Level 4:

As students develop a deeper understanding of solving equations and explaining their solution methods, instruction can be integrated with the application in contextual situation. Students should be able to construct equations that represent a contextual situation as well as create contextual situations to represent a given equation. As students develop a deep understanding of the relationship that exists between the type of function and the context, they can be given functions embedded in real-world situations. When they are given a contextual situation and an equation, students should be able to determine what each part of the equation represents as it relates to the context. They should also be able to solve the equation and create a viable argument to justify their solution path. Students should understand that there are various ways to solve problems and justifying their steps will help them solidify their understanding of solving equations as well as the most efficient solution path. This standards pairs nicely with A2.A.CED.A.1 as it supports the idea of making connections between an equation and its context.

To challenge students to follow a thought process other than their own, they can be asked to critique or correct the solution paths of others. Students will develop a deeper level of understanding if they are given solution paths with incorrect steps in the process or invalid justifications and asked to correct the process or write justifications and defend them.

Standard A2.A.REI.A.2 (Major Work of the Grade)
Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Simplify expressions with rational exponents when the base number is a variable. Write radical expressions in simplest form. Define extraneous solution.	Rewrite radical expressions using rational exponents. Rewrite expressions with rational exponents as radical expressions in simplest form. Simplify rational expressions by factoring. Perform operations of addition, subtraction, multiplication and division on rational expression containing more than one fraction.	Solve rational equations in one variable. Solve radical equations in one variable. Identify extraneous solutions algebraically Justify solutions graphically using technology.	Solve the equation and justify the solution path, given a partial solution method to a rational or radical equation. Critique the reasoning of others by finding errors and justify changes that could be made to correct mistakes, given the steps to solve a rational or radical equation. Explain why extraneous solutions exist.

Instructional Focus Statements

Level 3:

Standard A2.A.REI.A.2 builds on the concept of retaining equivalency when performing the same operation on both sides of an equation and properties of inverse operations. Students reason from A2.A.REI.A.1 that when they perform the same operation on both sides of an equation, equivalency will be retained and the process will end with possible values of an unknown value. Instruction can now focus on solving equations that may have unknown values under the radical or in the denominator.

To get students ready to solve these equations, instruction needs to include the building blocks that help students develop skills needed to solve rational and radical equations. Standards A2.N.RN.A.1 and A2.N.RN.A.2 focus on simplifying expressions involving radicals and rational exponents and rewriting them in either form. Those concepts will be necessary to helping students learn to solve radical equations. To solve rational equations, students must first be taught to simplify rational expressions using methods like factoring trinomials and factoring out common factors in the numerator and denominator. Concepts of fractions, such as finding common denominators to add and subtract rational expressions and how to multiply and divide fractions, will be vital to the mastery of solving various complexities of rational equations.

As students experience solving a variety of radical and rational equations, they should be led to the realization that not all steps are reversible. Asking students to check their solutions algebraically should lead to discussion on why all resulting solutions may not satisfy the original equation. In addition to identifying extraneous solutions algebraically, students should be expected to explain in their own words why extraneous solutions arise and why they are not correct solutions. This can also be supported by exposing students to the graphs of these rational and radical equations and reconnecting their understanding of solutions as x-intercepts of the function.

Level 4:

To deepen the level of understanding, instruction could include providing opportunities for students to follow a given solution method on a rational and radical equations. By presenting partially solved problems and asking students to finish the problem to reveal the solution, students will have to follow the reasoning of others and justify their reasoning. To enforce this same concept, students can be provided rational and radical equations that have been completed solved. Students could then be asked to critique the reasoning of others as they examine the problems and determine if mistakes are made. They should be expected to make corrections and justify their reasoning.

Standard A2.A.REI.B.3 (Supporting Content)
Solve quadratic equations and inequalities in one variable.
A2.A.REI.B.3a Solve quadratic equations by inspection(e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Scope and Clarifications:
In the case of equations that have roots with nonzero imaginary parts, students write the solutions as a ± bi for real numbers a and b.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the solution to a simple one-variable quadratic equation. Determine if a given value is a solution for a quadratic inequality in one variable. Choose an interval that represents a solution for a non-complex quadratic inequality in one variable.	Solve quadratic equations in one variable that generate non-complex solutions using multiple strategies. Determine if a quadratic equation in one-variable has real solutions or complex solutions.	Solve quadratic equations in one variable that generate complex solutions using multiple strategies and express the solutions in the form $a \pm bi$ for real numbers a and b. Solve a complex quadratic inequality in one variable.	Solve one-variable quadratic equations that generate complex solutions, identify the strategy chosen, and explain why the chosen strategy best suits the initial form of quadratic equation.

Instructional Focus Statements

Level 3:

Note: Quadratic inequalities are addressed here as they are a part of standard A2.A.REI.B.3a and they can be overlooked as a part of the standard since they are not readdressed in either part a or b of this standard. Simple quadratic inequalities are introduced in Algebra I, in this course, students should encounter more challenging quadratic inequalities.

It is important that, just like in Algebra I, students begin with the understanding developed in standard A2.A.REI.A.1 that solving any equation is a process. Students should interact with the multiple strategies for working with increasingly more complex quadratic equations discovering and internalizing when each strategy is most beneficial. The strategies should not live in isolation but as a part of a much larger conversation around the merits of each.

Additionally, it is important to note that students are required to find complex roots, writing them in the form $a \pm bi$ for real numbers a and b. That said, students need to develop a strong understanding of when they will get complex solutions.

Level 4:

Students need to be pushed to really solidify their understanding of the many ways to solve quadratic equations suppling mathematical justification to the decisions they make. At this level, students can not only solve quadratic equations, but they can articulate the mathematical underpinnings of the various strategies. Additionally, they are able to connect the various strategies. For example, are the quadratic formula and completing the square related? If so, describe how. If not, tell why.

Standard A2.A.REI.C.4 (Supporting Content)
Write and solve a system of linear equations in context.

Scope and Clarifications:
When solving algebraically, tasks are limited to systems of at most three equations and three variables. With graphic solutions systems are limited to only two variables.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the solution to a system of equations in two variables from a graph. Use substitution to determine if a given solution is correct for the system of equations.	Graph two linear equations and find the solution. Solve a system of equations in two variables algebraically through substitution. Solve a system of equations in two variables algebraically through elimination. Identify the solution to a system of equations in three variables from a table.	Write a system of equations in two variables, From a real world situation. Interpret the solution of a system of equations in two variables in context. Solve a system of equations in three variables algebraically. Justify why a system of three linear equations may have one solution, no solutions, or infinitely many solutions. Use technology to solve a system of three linear equations. Write a system of equations in three variables, from a real world situation.	Create a real world scenario to represent a system of equations in three variables. Determine if the solution to a system of equations is reasonable for a given context. Interpret the differences between one solution, no solutions, or infinitely many solutions, given the graph of a system of equations in three variables.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Interpret the solution to a system of equations in three variables in context.	

Instructional Focus Statements

Level 3:

In Algebra I, A.REI.C.4, students wrote and solved systems of two linear equations graphically and algebraically. Instruction should continue to use multiple representations including graphs and tables to help students visualize the solution and support their ability to solve algebraically. When solving a system of two equations graphically, students may approximate this intersection with technology.

In Algebra II, students will expand their understanding of systems to work with systems of equations in three variables. Students should have experience with the process of substitution and elimination, so discussion should help lead students to recognize how these algebraic methods could be used to solve a system of equations in three variables. As the discussion focuses on elimination, students should be led to discover that the process should begin by rewriting the linear system in three variables as a pair of two equations. The solving process will have more layers to solve for more variables and will require students to have the ability to recognize the limitation of only solving for one unknown value at a time. Support will likely need to be given to help them recognize how to combine the value of one variable and continued use of the elimination method to solve for the other variables.

The use of technology is also important when solving a system of three equations. Since the calculator cannot be used to graph the system as it is with a two-variable system of equations, there is an opportunity for matrices to be introduced as a tool for solving systems. Instruction would need to focus on helping students arrange the equations so they can be entered into a matrix, emphasizing the structure and importance of the coefficients of like variables arranged together. This would allow students to enter coefficients and constants into a matrix and then use operations of matrices in the graphing calculator to solve the system of equations.

Instruction should include engaging students in real-world problems where they write and solve equations from context. In some problems, students must determine if an approximation of a solution or an exact solution is most appropriate for the problem and be able to interpret the meaning of these solutions in terms of the context. Additionally, students should be expected to differentiate among problems where there is one solution, no solutions, or infinitely many solutions and justify the meaning or reason for these results.

Level 4:

As students build a strong understanding of how to solve a system of equations in three variables, they should be challenged to extend their understanding of these equations in context by being asked to write a real-world scenario to represent a system. They will continue to demonstrate understanding by choosing the most appropriate method for solving a system and justify their reasoning throughout the solution process.

Although instruction may not include graphing a system of equations in three variables, discussion on the x, y, and z planes and how they are related can help students visualize the system. This experience can help students understand the visual representation of one solution, no solutions, or infinitely many solutions.

A2.A.REI.C.5 (Supporting Content)
Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify whether a system consisting of a linear equation and a quadratic equation would have no solution, one solution, or two solutions, given a graph. Identify the approximate solution to a system of equations from a graph. Identify the solution to a system of equations from a table. Use substitution to determine if a given solution is correct for the system of equations.	Graph a quadratic equation given in any form. Solve a quadratic equation algebraically. Graph a quadratic equation and linear equation on the same coordinate plane to approximate the solution.	Solve a system of a linear equation and a quadratic equation graphically using technology. Solve a system of a linear equation and a quadratic equation algebraically using substitution. Justify why a system consisting of a linear equation and a quadratic equation may have no solution, one solution, or two solutions.	Justify the solution to a system consisting of a linear equation and a quadratic equation using multiple representations: algebraically using the equation, graphically, using tables of values. Write and solve a real world scenario to represent the system, given a system of one linear equation and one quadratic equation. Construct an argument to explain if the solution to a system with one linear and one quadratic equation is reasonable within a given context.

Instructional Focus Statements

Level 3:

Solving a system of one linear equation and one quadratic equation is a natural extension of systems of linear equations (A.REI.C.4). Instruction should continue to use multiple representations including graphs and tables to help students visualize the solutions and support their ability to solve algebraically. When solving a system of two equations graphically, students may approximate this intersection with technology. Algebraically, systems of one linear and one quadratic equation can be solved using substitution. Students should have experiences using the substitution method with linear equations in Algebra I, so building on this understand, students should revisit the property of substitution and how to substitute an expression for the value of a variable. As students solve for y in the use of technology to graph the equations and view the table, discussion should lead them to recognize the equivalency when both equations are equal to y and help them recognize how to set them equal to each other and solve for the remaining variable.

Students discovered patterns that caused linear systems to have no solution, one solution, or infinitely many solutions, so to build on this understanding graphs of different systems containing a linear and a quadratic could be introduced and students could be asked to compare the differences in the number of solutions for these systems. These discussions should help students recognize how the different number of solutions exist; no solution because the graphs do not intersect, one solution caused when the line is tangent to the quadratic, or two solutions when the line will intersect the quadratic twice. Although this is one of the first times students have seen systems of equations that can have two solutions, students have experience solving quadratic equations that have the same results for similar reasons. It should be pointed out that solutions are now the intersection points with the line created by the linear equation as opposed to the x-axis. As students are working with these systems, they should be encouraged to find exact solutions without technology. By attending to precision, students will leave irrational solutions in simplified radical form when the situation arises.

Level 4:

Instruction can extend learning by challenging students to justify solutions to a system of one linear equation and one quadratic equation using different representations of functions. When given a linear and quadratic system represented one way, students would be expected to justify their solution in more than one other way. Options would include substitution of the solution, solving algebraically, using a table values, or graphically.

Once students have a solid algebraic understanding of systems of this type, they could be challenged to write a real-world problem representing the system, construct a table which represents both equations, find the solution and graph the system either by hand or using technology. Students would then need to be able to explain the meaning and justify the reasonableness of the solution in context.

Standard A2.A.REI.D.6 (Major Work of the Grade)
Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the approximate solutions using technology.

Scope and Clarifications: (Modeling Standard)
Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Tasks may involve any of the function types mentioned in the standard.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the solution(s) for *f(x) = g(x)* when *f(x)* and *g(x)* are linear, quadratic, or absolute value functions, given two equations *f(x)* and *g(x).* Identify the solution of the equation *f(x) = g(x)*, given two linear equations *f(x)* and *g(x)*.	Choose the solution(s) for *f(x)= g(x)* when *f(x)* and *g(x)* are non-linear, non-quadratic polynomial; rational; exponential; or logarithmic functions, given two equations *f(x) and g(x)*. Identify the solution(s) for *f(x) = g(x)* when *f(x)* and *g(x)* are non-linear, non-quadratic polynomial; rational; exponential; or logarithmic functions, given graphs of two equations *f(x) and g(x)*. Approximate the solution(s) for *f(x) = g(x)* using technology when *f(x)* and *g(x)* are linear, quadratic, or absolute value functions, given two equations *f(x)* and *g(x)* embedded in a real-world situation.	Approximate the solution(s) for *f(x) = g(x)* using technology when *f(x)* and *g(x)* are non-linear, non-quadratic polynomial; rational; exponential; or logarithmic functions, given two equations *f(x)* and *g(x)* embedded in a real-world situation. Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x).*	Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)* and explain the meaning of the solution in terms of a real-world context when *f(x)* and *g(x)* are non-linear, non-quadratic polynomial; rational; exponential; or logarithmic functions.

Instructional Focus Statements

Level 3:

In continuing to develop an understanding of what it means to find the solution to two equations using graphing, it is very important that just as we did not want algebraically solving equations to become a series of steps unsupported by reasoning, we want to make sure that graphically solving them the reasoning piece is not left out either. The simple idea that an equation can be solved (approximately) by graphing can often lead to a rote series of steps involving simply finding the intersection point(s) without employing the reasoning of what is actually occurring. Explaining why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ involves a rather sophisticated series of thinking as students must connect the idea of two equations in two variables and how that relates to a single equation in one variable and then understand how both connect to a point(s) on a coordinate plane which is built around two variables. Thus, it is imperative that students reason through this process without being given a truncated set of meaningless steps to follow.

As this is a modeling standard, students should be formulating equations, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions built out of real-world situations.

In Algebra II, students are focusing on non-linear, non-quadratic polynomial, rational, exponential, and logarithmic functions. Students need the opportunity to interact with all of these function types. Additionally, they need to encounter situations where $f(x)$ and $g(x)$ are different function types. These should increase in difficulty over time.

Level 4:

Students should continue to be exposed to a wide variety of non-linear, non-quadratic polynomial, rational, exponential, and logarithmic functions with increasing difficulty embedded in real-world situations. Additionally, they need to explain the meaning of the solution in terms of the real-world context.

TN Department of Education

INTERPRETING FUNCTIONS (F.IF)

Standard A2.F.IF.A.1 (Major Work of the Grade)
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Scope and Clarifications: (Modeling Standard)
Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

i. Tasks have a real-world context.
ii. Tasks may involve square root, cube root, polynomial, exponential, and logarithmic functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify intercepts, maximums and minimums when provided a graphical representation of the function.	Identify intervals where a given function is increasing, decreasing, positive or negative when provided a graphical representation of the function. Identify key features of the graph or table of values, and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a quadratic function embedded in a real-world context. Identify all evident intercepts, maximums and minimums when provided a table of values	Identify all evident key features when provided a table of values representing a square root, cube root, polynomial, exponential, or logarithmic function. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a square root function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a	Graph the function, identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a square root function embedded in a real-world context. Graph the function, identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a cube root function embedded in a real-world context. Graph the function, identify key features of the graph, and interpret the meaning of the key features in

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	representing an exponential function with domain in the integers. Identify key features of the graph and interpret the meaning of the key features in relationship to the context of the problem, given a graph of an exponential function with domain in the integers embedded in a real-world context, Identify evident intercepts, maximums and minimums when provided a table of values representing a square root, cube root, polynomial, exponential, or logarithmic function. Sketch a graph of the function, given a verbal description of the key features of a quadratic function,	graph or table of values representing a cube root function embedded in a real-world context, Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing an exponential function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a polynomial function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a logarithmic function embedded in a real-world context. Sketch a graph of the function, given a verbal description of the key features of a square root, cube root,	relationship to the context of the problem, given an exponential function embedded in a real-world context, Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a polynomial function embedded in a real-world context, graph the function. Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a logarithmic function embedded in a real-world context, graph the function. Create a real-world context that would generate a function with the provided attributes, given key features of a square root, cube root, polynomial, exponential, or logarithmic function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		polynomial, exponential, or logarithmic function,	

Instructional Focus Statements

Level 3:

Functions are often described and understood in terms of their key features and behaviors. Instruction for this standard should in part focus on building on the knowledge gained in Algebra I around identifying key features and behaviors from both graphs and tables and extend this understanding to new function families. In Algebra I, students focused on linear, quadratic, absolute value, and exponential functions with domain in the integers. The new part of this standard for students will be in the function families as opposed to the types of key features/behaviors. Thus, it is important to note that the overarching concept of key features and behaviors is not new to students.

As in Algebra I, instruction should extend beyond simple identification from isolated graphs and tables. As this is a modeling standard, students need opportunities to develop an understanding of the relationship between key features/behaviors and the real-world situation that the function models. The focus should be on developing a strong understanding of the relationship between key features/behaviors and their meaning within real-world situations. Additionally, instruction should provide students with an opportunity to develop an understanding of not only how to identify key features/behavior in graphs and tables, but also on how to generate a graph when provided the key features/behaviors.

Instruction can be very nicely paired with standard A2.F.IF.B.3 where students generate square root, cubed root, exponential, polynomial, and logarithmic graphs from real-world situations. This pairing allows students the opportunity to generate a graph from a real-world situation, identify key features/behaviors, and then discuss their meaning as related to the real-world situation.

Level 4:

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate functions with a pre-determined list of key features/behaviors. Additionally, students with a deep understanding of this standard can interpret key features/behaviors from non-traditional square root, cube root, exponential, polynomial, and logarithmic functions embedded in real-world situations.

Standard A2.F.IF.A.2 (Major Work of the Grade)
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Scope and Clarifications: (Modeling Standard)
i. Tasks have a real-world context.
ii. Tasks may involve polynomial, exponential, and logarithmic functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the average rate of change for an exponential function when given a symbolic representation, table, or graph. Choose the estimated rate of change when given a graph of an exponential function.	Calculate the average rate of change of an exponential function when given a graph. Interpret the rate of change for an exponential function in terms of a real-world context. Choose the estimated rate of change for a specific interval when given an exponential function.	Calculate average rate of change when given an equation or table of a polynomial, exponential, or logarithmic functions. Interpret the average rate of change of a polynomial, exponential, or logarithmic functions Estimate the average rate of change for a specific interval of a polynomial, exponential, or logarithmic functions when given a graph.	Identify the average rate of change for specific intervals of a function as being greater or less than other intervals of the same function. Compare the average rate of change of multiple intervals of the same function and make connections to the real-world situation. Create a contextual situation and identify and interpret the average rate of change with a specific interval.

Instructional Focus Statements

Level 3:

In grades 6 and 7, students began developing the understanding of ratios and proportional relationships. Their understanding of rate of change involved both ratios and proportions using similar triangles to show the additive and multiplicative conceptual underpinnings of the concept. In grade 8, students extended this understanding to functions by examining rate of change in linear functions. In high school, students should solidify this understanding for linear functions and generalize this concept to applying to additional function types. Students should make the connection that the rate of change is the ratio of the change between the dependent and independent variable. For linear functions, students have discovered that this ratio of change is constant between any two points on the line. Students should now make the connection that, for non-linear functions, the ratio of change is not constant due to the functions curvature. This results in the ability to calculate the average rate of change over a specified interval. For example, for the polynomial function $f(x) = x^3$, the average rate of change from x = 1 to x = 4 is $\frac{f(4)-f(1)}{4-1} = \frac{16-1}{4-1} = \frac{64}{3}$. This is the slope of the line from (1, 1) to (4, 64) on the graph *f*. If *f* is interpreted as the volume of a cube of side x, then this calculation means that over this interval the volume changes, on average, 64/3 square units for each unit increase in the side length of the cube.

It is imperative that students gain a conceptual understanding of the average rate of change for a specified interval for non-linear functions. To grasp this idea, students should draw illustrations of the graph and the secant line connecting the intended endpoints. Students should not only be able to calculate the average rate of change, but they should also be able to generate a visual representation and use the visual representation to estimate the average rate of change over a specified interval. Students will gain a deeper conceptual understanding when they compare their estimations to the actual average rate of change for a non-linear function. As students solidify their understanding, they should be able to explain what the average rate of change means in the context of a problem when given symbolic representations, tables, graphs, or contextual situations. As students use multiple representations to evaluate the average rate of change, they should be able to explain the relationship between the multiple representations using both appropriate mathematical language and appropriate justifications.

Level 4:

Students should extend their understanding of average rate of change by comparing the average rate of change of one interval to another interval of the same function. Students should also further their understanding by creating their own contextual situations and interpreting the average rate of change for a significant interval. Students should be intentional in determining which interval or intervals they select and explain the importance of the interval(s) with respect to the context using both precise mathematical vocabulary and precise justifications.

Standard A2.F.IF.B.3 (Supporting Content)
Graph functions expressed symbolically and show key features of the graph, by hand and using technology.
A2.F.IF.B.3a Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions.
A2.F.IF.B.3b Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
A2.F.IF.B.3c Graph exponential and logarithmic functions, showing intercepts and end behavior.

Scope and Clarifications: (Modeling Standard)
A2.F.IF.B.3a: Tasks are limited to square root and cube root functions. The other functions are assessed in Algebra 1.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use characteristics of the symbolic representation of a function to distinguish function type and behavior of the graph. Recognize the parent function from a graph of a transformed square root, cube root, polynomial, exponential, and logarithmic curve.	Describe the behavior of the graph of the square root function by explaining the inverse relationship between squaring a number and taking the square root of a number. Describe the behavior of the graph of the cube root function by explaining the inverse relationship between cubing a number and taking the cube root of a number. Identify key features, such as shape, intercepts, extrema, and end behavior, from a graph of a square root and cube root function. Infer restrictions on the domain and range from a graph. Identify key features, such as intercepts, extrema, and end	Graph a square root, cube root, polynomial with degree greater than two, exponential, and logarithmic function by hand and using technology. Describe end behavior of a polynomial function with degree greater than two given in standard form and factored form. Attend to precision when illustrating intercepts, maxima, minima, and determine the domain, range, and end behavior of a function.	Explain the relationship that exists between a contextual problem and the key features of a graph for a square root, cube root, polynomial with degree greater than two, exponential, and logarithmic function. Critique graphs drawn by others to ensure key features are shown efficiently and appropriately. Write the corresponding function symbolically, given a graph. Explain restrictions on domain and range in context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	behavior from a graph of a quadratic function. Graph a quadratic function by hand and using technology identifying intercepts maxima and minima. Identify the asymptote, given a graph of an exponential or logarithmic curve.		

Instructional Focus Statements

Level 3:

In Algebra I, students use functions to model relationships between quantities and construct a function to model a linear relationship. Students also determine if a function is linear or nonlinear, and they have experience interpreting and representing functions algebraically, numerically, graphically, and verbally. In Algebra II, students will be introduced to additional function families and more key features such as extrema and end behavior. Intercepts, shape, domain, and range take on greater meaning to students through the exploration of a variety of functions. Therefore, it is important for instruction to provide tasks that allow students to explore the behavior and varying parameters of functions. Providing students with models in context, such as connecting the cube root function to solving volume problems, will help develop the meaning of key features and identifying them from a graph, a table, or a verbal description. To meet the rigor of this standard, students should be given the opportunity to work with functions that vary in their symbolic representation such as standard form and factored form of polynomials. This will help students have access to the problem regardless of the symbolic representation, which is further developed in standard A2.F.IF.B.4 as students identify key features through algebraic manipulation.

Students should be able to graph functions by hand and with the use of technology. It is imperative for the teacher to model how to graph with a graphing calculator or other graphing device. This is the first time students will use technology to graph to graph a function type other than a linear function. Furthermore, ample time must be given for students to explore how a table of value can be helpful in identifying key features, domain, and range from a graph. The use of technology should allow students to explore problems whose key features are irrational values, which can be located with the use of a device. Students may have struggled with domain restrictions in previous classes, so continue supporting that understanding by integrating technology to explore domain restrictions so that students to connect the relationship between the algebraic representation and the resulting domain. This exploration of the effect of transformations will support students' mastery of this standard to graph by hand and with technology.

In Algebra I, students focused on linear, quadratic, and parent functions. Instruction in Algebra II should build on the concept of parent functions. This will help students make the connection of how transformations affect the graph, equation, and table of a function, which is explored in standard A1.F.BF.B.2. Students should be presented with problem types whose symbolic representation varies and asked to identify the parent function and describe the transformation from its original, non-transformed graph. This will help students attend to precision as they graph functions of many types and use their understanding of transformations to support the reasonableness of their graph. Instruction should provide ample opportunity for students to compare and contrast the graphs of functions, and it should help them efficiently recognize a parent function when expressed symbolically and graph it fluently.

Additionally, in standard A1.F.LE.A.2 introduced students to exponential functions. Instruction in Algebra II should build on that prior knowledge of geometric sequences in order to graph the exponential function to highlight key features. To continuing tying prior knowledge with new knowledge of the logarithmic function, instruction should connect the inverse relationship that exist between the exponential function's initial value, y-intercept, end behavior, and horizontal asymptote to a logarithmic function's base value, x-intercept, end behavior, and vertical asymptote. Asking students to produce a table with these values will help them sketch a graph of the logarithmic curve.

Graphing polynomial functions should be taught in tandem with standard A2.A.APR.A.2 where students have to identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Instruction for this standard should allow students to discover the end behavior of polynomials through an exploration of varying leading coefficients as the degree of the polynomial increases. Guide the exploration so that students generalize the behavior of positive and negative even-degree functions as well as positive and negative odd-degree functions.

Level 4:

Instruction at this level should provide opportunities for students to create a real-world problem that is modeled by a square root, cube root, polynomial, exponential, or logarithmic function. Students should be required to provide a graph, table, equation, and verbal representation of the problem. Instruction should include posing purposeful questions asking students to show and describe key features from their created problem in context. Instruction should also provide graphs drawn by others and require students to analyze and critique the graphs. Students should be given the opportunity to look at graphs drawn by others so they can analyze and critique their peers work. Through the analysis of many graphs, students should develop an understanding of when key features are efficiently and effectively represented, and, if not, provide a suggestion for representing them more appropriately.

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Standard A2.F.IF.B.4 (Supporting Content)
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
A2.F.IF.B.4a Know and use the properties of exponents to interpret expressions for exponential functions.

Scope and Clarifications:
For example, identify percent rate of change in functions such as $= 2x$, $y = (1/2)x$, $y = 2 - x$, $y = (1/2) - x$.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use the properties of exponents to rewrite exponential expressions.	Identify an exponential function written in symbolic form as an exponential growth or decay. Identify percent rate of change in an exponential function. Determine if the context represents exponential growth or exponential decay, given a real-world problem,	Rewrite an exponential function to reveal the percent rate of change. Rewrite an exponential function to reveal the y-intercept. Write an exponential function given any point and the percent rate of change.	Identify reference points in non-standard forms of exponential functions.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

In Algebra I, students use the properties of exponents to rewrite exponential expressions, interpret key features of an exponential graph, and interpret the parameters in an exponential function in terms of a context. Students build upon this prior knowledge and combine these skills to rewrite exponential functions to reveal key features of the graph and interpret them in a context. Students should be able to identify the y-intercept and percent rate of change (i.e., percent increase or decrease) by rewriting an exponential function into the form $y = ab^x$. Teachers should provide students with opportunities to use exponent properties to convert exponential functions into the form $y = ab^x$. For example, given the exponential function $y = (1/3)^{-x}$, students should write this as $y = ((1/3)^{-1})^x$ or $y = 3^x$. Therefore, students should identify the percent rate of change as a 200% increase. In other cases, students should identify the percent rate of change in a context. For example, students should be able to determine the percent rate of change in the population of a small town, when the population is represented by $p(t) = 15000(.978)^t$. A common misconception held by students is that it represents an increase of 97.8% rather than a decrease of 2.2%. Teachers can model how to write the function as $y = 15000(1 - .022)^t$ to help students see why it is a decrease of 2.2%.

Students can also use exponent properties to reveal the y-intercept of an exponential function. For instance, given the function $y = 2(3)^{x+1}$ students should be able to recognize that the function can be rewritten as $y = 2(3)^1 * (3)^x$, which is equivalent to $y = 6(3)^x$. Thus, the y-intercept is (0, 6). One useful strategy to support students in separating an exponential expression into the product of two exponential expressions is to first provide them with examples of combining the product of two expressions. By doing so, students can use the properties of exponents in ways they are accustomed to prior to reversing the thought process. Although separating the expressions can be useful, teachers should remind students that another strategy to find the y-intercept is to substitute 0 in for x, which is generalizable to all functions.

These concepts builds upon A2.F.BF.A.2, in which students write geometric sequences with an explicit formula and use them to model situations. Therefore, if students are given the ratio and a term in the sequence, they can use this concept to build the function without finding the first term. For example, given the 7th term is 30 and the common ratio is 2, students should be able to write the function $y = 30(2)^{x-7}$ because an input of 7 will result in an exponent of 0, which will produce an output of 30. This can help students work efficiently by making use of the structure of an exponential function.

Level 4:

As students develop a deep understanding of exponential functions, they should recognize reference points in non-standard forms of exponential functions. For example, given the function $y = 3(2)^{x-5}$, students should recognize that the function contains the point (5,3). There are multiple ways that teachers can help students understand this principle. First, students can see the function $y = 3(2)^{x-5}$ as a horizontal translation of the function $y = 3(2)^x$, which has a y-intercept of (0,3). Therefore, instead of having the point (0,3), the function would contain the point (5,3). Second, students should know that anything (except 0) raised to the 0 power is 1 and use this fact to see how making the exponent 0 simplifies the entire second factor to 1. Thus, by inspection, students should quickly see that an input of 5 produces an output of 3 and explain how this concept can be generalized to other exponential functions.

Standard A2.F.IF.B.5 (Supporting Content)
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Scope and Clarifications:
Tasks may involve polynomial, exponential, and logarithmic functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the y-intercept of a function from multiple representations. Identify the slope of a linear function from multiple representations. Identify asymptotes of exponential functions from multiple representations. Describe connections among multiple representations of a linear function. Compare properties of two linear functions each represented in a different way. Compare properties of two quadratic functions each represented in a different way.	Identify the zeros of a function from multiple representations. Identify asymptotes of logarithmic functions from multiple representations. Identify the relative extrema of a polynomial function from multiple representations. Identify the end behavior of a polynomial function from multiple representations. Identify the percent rate of change of an exponential function from multiple representations. Describe connections among multiple representations of a polynomial function.	Compare properties of two exponential functions each represented in a different way. Compare properties of two logarithmic functions each represented in a different way. Compare properties of two polynomial functions each represented in a different way. Compare properties of two functions from different function families each represented in a different way.	Compare properties of two functions within a context. Use precise mathematical vocabulary to explain the relationships of the various representations of a function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Describe connections among multiple representations of an exponential function. Describe connections among multiple representations of a logarithmic function. Move fluently among multiple representations of a function.		

Instructional Focus Statements

Level 3:

Prior to comparing properties of two functions represented in different ways, students need to first identify properties of functions and make connections between different representations of the same function. This is an important standard with respect to achieving access and equity for all students. Teachers should represent a function in multiple ways, especially for English language learners, learners with special needs, or struggling learners, because math drawings and other visuals allow more students to participate meaningfully in the mathematical discourse in the classroom. As students move fluently between representations they must consider relationships among quantities and how each representation provides a unique perspective of the function. Teachers can foster this way of seeing mathematics by having students discuss the similarities among representations that reveal the key features of a function that persist regardless of the form. Through these discussions students can determine which representations are most appropriate for revealing certain key features of the function.

In grade 8 and Algebra I, students compare properties of two linear, quadratic, exponential, and piecewise-defined functions each represented in a different way. Once students have a strong understanding of the various representations of polynomial, exponential, and logarithmic functions in Algebra II, they can begin to compare properties of two functions represented in different ways. For example, given a graph of one cubic function and a table of another, a student should be able to compare their y-intercepts. One strategy that can sometimes be useful is to convert one or both to a different form so that both functions are represented the same way. As students begin to grasp this concept, it is important that teachers provide students with examples that include each function type, with some situated within a context. Therefore, comparing properties in different representations further supports students' understanding of each function type, which means this standard can be paired nicely with other standards that focus on properties and graphs of polynomial, exponential, and logarithmic functions, such as A2.F.IF.A.1 and A2.F.IF.B.3. As students recognize various function types in multiple representations, discussion should lead to the comparison of functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a polynomial function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties.

Level 4:

Students with a deep understanding of the various function types and representations should also be able to compare functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a cubic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties. Once conclusions are formed, teachers can ask further questions related to the context. For example, given a graph of a quartic function and an algebraic representation of an exponential function each describing the cost of a cell phone plan, decide which plan is better. Students should be given the opportunity to describe how to identify function types and compare the properties of functions in various forms. At this level, teachers should expect students to use precise mathematical vocabulary to describe and justify these relationships and qualities.

TN Department of Education

Building Functions (F.BF)

Standard A2.F.BF.A.1 (Major Work of the Grade)
Write a function that describes a relationship between two quantities.
A2.F.BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
A2.F.BF.A.1b Combine standard function types using arithmetic operations.

Scope and Clarifications: (Modeling Standard)
For example, given cost and revenue functions, create a profit function.
For A2.F.BF.A.1a:
i) Tasks have a real-world context.
ii) Tasks may involve linear functions, quadratic functions, and exponential functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Write a function defined by an expression, a recursive process, or steps for calculation to model a linear relationship, given a real-world context. Identify the independent and dependent variable in a real-world context. Identify the first term and common ratio of an exponential function, given a real-world context. Combine like terms of two linear functions to build a new function.	Recognize when a quadratic function should be used to describe the given situation, given a real-world context. Recognize when an exponential function should be used to describe the given situation, given a real-world context. Write a function defined by a recursive process or steps for calculation to model an exponential relationship, given a real-world context. Combine a linear function and	Write a function defined by an expression to model a quadratic relationship, given a real-world context. Write a function defined by an expression to model an exponential relationship, given a real-world context. Compare key characteristics of real-world contexts that can described by various types of functions. Combine multiple functions using arithmetic operations to write a	Create a real-world context that would generate the given function, given a function defined by an expression, a recursive process, or steps for calculation. Explain the various ways a function can be defined and in what real-world situations they would be appropriate. Justify why specific types of functions should be used in combination to describe a given real-world context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	quadratic function using arithmetic operations to build a new function.	function that describes a real-world situation with multiple steps. For example, combine a linear function and an exponential function.	

Instructional Focus Statements

Level 3:

In Algebra I, students create linear, quadratic, and exponential functions from a real-world context defined by an explicit expression, recursive process, or steps for calculation. In Algebra II, students should also extend their understanding to combining function types using arithmetic operations. Instruction for this standard should focus on creating functions from a real-world context and recognizing when multiple functions need to be combined to describe multiple steps. Combining multiple functions is not limited to linear, quadratic, and exponential functions. For example, students should also be able to combine a linear function and an absolute value function to build a new function.

If given a table of values describing a single function, students should first recognize which type of function the table of values represents. Teachers should focus students' attention on the relationship between consecutive points to see if there is a common first difference or constant additive change (linear function), a common second difference (quadratic function), or a common ratio or constant multiplicative change (exponential function). Once students identify the function type, teachers can then help students begin to write the function given the common first difference, second difference, or ratio and other information from the table.

To build coherence, it is important that teachers make connections between linear functions and arithmetic sequences and between exponential functions and geometric sequences. Thus, instruction can be nicely paired with A2.F.BF.A.2, where students generate arithmetic and geometric explicit formulas to model situations. Both linear functions ($y = ax + b$) and arithmetic sequences ($a_n = a_1 + d(n - 1)$) describe additive changes, and students should make connections between the two. For example, b is equivalent to a_0 and a is equivalent to d. Similarly, exponential functions ($y = ab^x$) and geometric sequences ($a_n = a_1r^{n-1}$) both describe multiplicative changes and a is equivalent to a_0 and b is equivalent to r. Students should understand the similarities, but instruction should also help students realize an important difference: arithmetic and geometric sequences are discrete while linear and exponential functions are continuous. This can be done by comparing the graphs of an arithmetic sequence and a linear function, for example.

Level 4:

As students develop a deep understanding of this standard, they should be able to create a real-world scenario given a function or combination of functions. Moreover, they should be able to describe which characteristics of their scenario correspond to each part of the given function. For example, given $y = 10 + 5(2)^{x-1}$ a student might create a scenario similar to the following: The results of a school election are shared with 15 people. 10 of those people keep the secret while the other 5 tell 2 people per day and everyone they tell also tells 2 people per day and those 2 people tell to 2 more and so on. In this function, y represents the number of people that know the results x days after the original 15 people were told. The student should also be able to explain how the $5(2)^{x-1}$ in the function relates to the continuation of everyone telling 2 people per day beginning with 5 of the original people and how the 10 in the function relates to the 10 people that remain constant at 10 because they keep the secret. Similar scenarios can be created to describe half-life or mortgage calculations.

As students continue to work with combining functions, teachers should have students recognize that when combining functions using some operations, the original parent function is maintained, while other operations create new function types. For example, teachers should have students compare adding two quadratic functions with multiplying two quadratic functions.

Standard A2.F.BF.A.2 (Major Work of the Grade)
Write arithmetic and geometric sequences with an explicit formula and use them to model situations.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if a sequence of numbers is an arithmetic sequence, geometric sequence, or neither. Identify the common difference in an arithmetic sequence. Identify the common ratio in a geometric sequence.	Write an arithmetic explicit formula to represent a relationship given by a sequence of numbers. Write a geometric explicit formula to represent a relationship given by a sequence of numbers.	Write an arithmetic explicit formula to model situations, given a real-world context. Write a geometric explicit formula to model situations, given a real-world context.	Create a real-world context that would generate the given function, given a function defined by an arithmetic or geometric explicit formula. Justify why specific real-world contexts should be represented by arithmetic or geometric sequences.

Instructional Focus Statements

Level 3:

In Algebra I, students construct a function defined by an expression to model a linear or exponential relationship. In Algebra II, students build on their knowledge of linear functions to form arithmetic explicit formulas from a real-world context. Instruction should provide students with choice on whether to use the zeroth term ($y = mx + b$), the first term ($a_n = a_1 + d(n-1)$) or others in developing their function. Students should recognize that these equations are equivalent and connect the concepts of slope and common difference.

Students will make connections between geometric sequences and exponential functions created by explicit formulas in a real world context. Instruction should focus on making connections between the table of values and an explicit formula. Students should notice that each output value in the table can be rewritten as the initial value times a power of the common ratio. Then, relating each input value to the power of the common ratio will help students develop a function. Students should also be given choice as to which term to use as the coefficient in their formula. For example, if students are given the fourth term and the ratio, then $a_n = a_4 r^{n-4}$ might be a more accessible formula.

To build coherence, it is important that students make connections between linear functions and arithmetic sequences and between exponential functions and geometric sequences. Thus, instruction can be nicely paired with A2.F.BF.A.1, where students write a function defined by an expression, a recursive process, or steps for calculation to model a linear, quadratic, or exponential relationship.

Level 4:

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate a given function and justify why their scenario represents that function type. Additionally, students with a deep understanding of this standard can create a function from a geometric pattern and describe how each component of their function relates to characteristics of figures in the pattern. For example, students should be able to build a function to represent the number of line segments used to form shapes in a series of shapes following a particular pattern.

TN Department of Education

Standard A2.F.BF.B.3 (Supporting Content)
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Scope and Clarifications:
i) Tasks may involve polynomial, exponential, and logarithmic functions.
ii) Tasks may involve recognizing even and odd functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe, using precise mathematical vocabulary, transformations that would map a geometric figure to its image. Write the function defined by $f(x) + k$, given the function and a positive value of k. Write the function defined by $kf(x)$, given the function and a positive value of k. Write the function defined by $f(x + k)$, given the function and a positive value of k. Write the function defined by $f(kx)$, given a value k and a function $f(x)$,	Compare $f(x)$ and $f(x) + k$ and illustrate an explanation of the effects on the graph using technology. Compare $f(x)$ and $kf(x)$ and illustrate an explanation of the effects on the graph using technology. Compare $f(x)$ and $f(x + k)$ and illustrate an explanation of the effects on the graph using technology. Compare $f(x)$ and $f(kx)$ and illustrate an explanation of the effects on the graph using technology.	Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(x) + k$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $kf(x)$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(x + k)$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(kx)$. Determine if the function is an odd function, even function, or neither, given a function defined by an expression.	Write the equation of a function given the graph by identifying the transformation(s) to the parent function. Apply transformations to a function that has already been transformed. Explain why changes to the argument of $f(x)$ affect the input values and changes outside the function affect the output values.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if the graph is symmetric across the y-axis, given a graph. Determine if the graph has 180 degree rotational symmetry about the origin, given a graph.		Describe multiple effects on a graph for specific values of a, b, h, and k given two functions, $f(x)$ and $af(b(x+h))+k$. Find the value of k for a specific vertical or horizontal translation, stretch, or compression, given two graphs, the image and pre-image.	

Instructional Focus Statements

Level 3:

In grade 8, students verify experimentally the properties of rotations, reflections, and translations of simple figures. Students expanded on this concept in algebra I by applying similar transformations to linear, quadratic, and absolute value functions and describing the transformations using function notation. Finally, in Algebra II, the function types are expanded to include polynomial, exponential, and logarithmic functions. One specific change that teachers need to address is the multiple arguments (i.e., terms including x) found in polynomial functions. In Algebra I, most functions types only had one argument, so students may not realize that they need to change all the terms including x to write $f(x+h)$, for example.

To understand how a, h, and k impact the graph of $f(x)$ when compared to $af(x+h)+k$, students can use technology (i.e., calculator or online graphing tool) to experiment with $f(x)+k$, $af(x)$, and $f(x+h)$. As students vary one value at a time, they can begin to discern how each component affects the graph of $f(x)$. Careful attention should be made to why h translates the graph $-h$ units horizontally. One explanation is to compare $f(x)$ with $f(x-5)$ and see that $f(3)$, for example, will produce the same output value as $f(8-5)$. In this example, an input of 3 into $f(x)$ is equivalent to an input of 8 into $f(x-5)$, and 8 is 5 units to the right of 3, not left. Students can think about it as having to undo what has been done to x inside the argument, which leads nicely to understanding $f(kx)$ stretches the graph horizontally by a factor of $\frac{1}{b}$. Meanwhile, $f(x)$ represents the output values. So, any operations performed to $f(x)$ outside the argument only affect the y values, which results in vertical transformations. Once students understand the effects of each component individually, they should then attempt to describe changes to a graph involving multiple transformations at once.

Connections between transformations and vertex form of a quadratic should be made. Converting to the vertex form of a quadratic reveals the transformations being made to $y=x^2$, which allows students to easily locate the vertex and determine the concavity of the parabola. Making this

connection will help support a deep conceptual understanding of transformations and vertex form. Students should realize that the same transformations will be applied to other function types such as rational and trigonometric functions in future courses.
Additionally, transformations should be used to describe even and odd functions. Given a graph or function, students should be able to determine if $f(-x) = f(x)$ or $-f(-x) = f(x)$ and thus, classify functions as even functions, odd functions, or neither. Recognizing even and odd functions is especially useful in calculus when finding efficient ways to calculate the area underneath a curve.

Level 4:

Students with a deep understanding of this standard should be able to write the equation of a function given the graph by identifying the transformation(s) to the parent function. Teachers should focus students' attention on the order of each transformation. For example, given $-f(x) + 9$, the graph is first reflected across the x-axis, then shifted up 9, rather than shifted up 9 then reflected across the x-axis due to the order of operations. It is also important that teachers place an emphasis on factoring out b from inside the argument so that the horizontal shift can be found. For example, write $(2x-6)^2$ as $(2(x-3))^2$ instead, revealing a horizontal shift of 3 to the right, not 6.

Teachers should help students understand that transformations can be made to functions that have already been transformed. For example, given $f(x) = (x+2)^2$, write an equation for $f(x-5)$ and describe the overall change in the graph.

In addition, students at this level should be able to explain concepts such as why changes inside the argument of a function have the inverse effect on a graph. Taken collectively, these students should understand a function as a process that generates output values from particular input values. Building on this understanding, students should make connections with which transformations perform operations to the input values prior to the function's operations and which transformations perform operations to the output values after the function has been applied.

TN Department of Education

Standard A2.F.BF.B.4 (Supporting Content)
Find inverse functions.
A2.F.BF.B.4a Find the inverse of a function when the given function is one-to-one.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Reflect a graph across horizontal, vertical, and diagonal lines. Solve linear and cube root equations. Identify the domain of the function, given a graph.	Draw the graph of $f^{-1}(x)$ by reflecting $f(x)$ across the line $y = x$, given the graph of $f(x)$. Find $f^{-1}(b)$, given $f(a) = b$, where a and b are real numbers. Write a point on the graph of $f^{-1}(x)$, given a point on the graph of $f(x)$. Find the inverse of a relation from a list of points or a table. Determine if a given graph is one-to-one.	Find the inverse of a function when the given function is one-to-one. Graph the inverse of a given linear function. Graph the inverse of a given cubic function. Graph the inverse of a quadratic function with a restricted domain. Find the inverse of a quadratic function by first restricting the domain to make it one-to-one.	Explain how an inverse function undoes the operations of the original function. That is, $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$. Explain why functions that are not one-to-one do not have an inverse that is a function. Determine if a function has an inverse that is a function, given a function represented by an expression. Restrict the domain of a function so that it is one-to-one.

Instructional Focus Statements

Level 3:

In Algebra I, students used inverse operations to rearrange formulas to highlight a quantity of interest (A1.A.CED.A.4) and solve linear, quadratic, and absolute value equations (A1.REI.A.1). Students also began to understand a function as a process that produces output values corresponding to certain input values. In Algebra II, students continue to use inverse operations to write the inverse of a function, which produces the input values corresponding to particular output values. So that each output value produces one unique input value, the original function must be one-to-one. Otherwise, the inverse of the function would not be a function itself. For example, students should be able to find the inverse of one-to-one functions such as linear, cube root, cubic, exponential, and logarithmic functions or restrict the domain of a function that is not one-to-one (e.g., quadratics).

Instruction should provide students with opportunities to see the need for a inverse function. If a set of output values are given, it would be tedious to solve for the corresponding input values individually. For example, $F = \frac{9}{5}C + 32$ is the formula to convert from Celsius to Fahrenheit. If given multiple Fahrenheit temperatures, it would be time consuming to solve for each of the corresponding Celsius temperatures individually using this function. Instead, the inverse function can be used to convert the values much quicker, and thus create a formula to convert any Fahrenheit temperature to Celsius (i.e., $C = \frac{5}{9}(F - 32)$). Other relationships that may be helpful examples include: the perimeter of a square and its side length ($P = 4x$), the area of a circle and its radius ($A = \pi r^2$), and the circumference of a circle and its radius ($C = 2\pi r$).

Although switching x and y works algebraically, instruction should be carefully organized to ensure that students solve for the input variable to create a new function whose operations undo the original function's operations. This way, students understand that they are using inverse operations in the opposite order to solve for the independent variable (e.g., x), which addresses a common misconception of simply reciprocating numbers and changing operations. Encoding and decoding using functions is a great way for students to think about the undoing process. Also, allowing students to experience inverse functions through multiple representations can strengthen their' understanding of this concept. For example, asking questions from a table of values requires new ways of thinking about an inverse function because it does not have an equation to manipulate.

Level 4:

As students develop a deep understanding of this standard, they should be able to explain inverse functions as an undoing process using the composition of functions. In addition, if the original function is not one-to-one (e.g., $y = x^2$), students at this level should restrict the domain so that every output value corresponds to a unique input value, which means its inverse is a function. Restricting the domain will be very useful in future courses when learning about inverse trigonometry functions.

Furthermore, students should understand the differences when finding the inverse of a function with a context (e.g., $F = \frac{9}{5}C + 32$). Students at this level should understand why switching the variables does not make sense in a real-world context and that they are simply changing which variable is the input and output. Therefore, graphing a function and its inverse on the same coordinate plane no longer makes sense because the axes need to be switched on a separate plane in order to graph the inverse function.

Linear, Quadratic, and Exponential Models (F.LE)

Standard A2.F.LE.A.1 (Supporting Content)
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize a function as linear from a graph, table, a description, or a set of ordered pairs. Recognize a function as exponential from a graph, table, a description, or a set of ordered pairs.	Recognize a function as a sequence and determine if it is arithmetic or geometric. Justify a function is linear using rate of change. Justify a function is exponential using rates of change.	Write a linear function given a graph. Write a linear function given a table of values. Write a linear function given a description of a simple real-world relationship. Write a linear function given a set of input-output pairs (ordered pairs). Write an exponential function given a graph. Write an exponential function given a table of values.	Analyze functions created by others to determine accuracy and explain and correct any errors. Create a real-world situation that may be modeled by a linear function and write the function. Create a real-world situation that may be modeled by an exponential function and write the function. Collect data for a real-world situation that can be represented by a linear or exponential function and write the function that models it. Define the variables and explain in context why the function models the situation.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Write an exponential function given a description of a simple real-world relationship. Write an exponential function given a set of input-output pairs (ordered pairs). Write a function given an arithmetic or geometric sequence or a description of one.	

Instructional Focus Statements

Level 3:

This standard was introduced to students in Algebra I with simple single-step contextual examples. In Algebra II, the examples should increase in complexity and incorporate multi-step context. This standard aligns with several Algebra II standards, including those in the CED, IF, BF, and SSE clusters, and should be incorporated with those rather than as a stand-alone lesson. Instruction should include examples of both function types in multiple representations including graphs, tables, and descriptions to allow students time to determine whether the function is linear or exponential as well as write the function. This is a good opportunity to relate input and output values with independent and dependent variables.

As this is a modeling standard, real-world examples should be provided and students should be required to explain how the function models the context.

This standard also allows for students to connect their learning about arithmetic and geometric sequences to creating the functions that model them. These should also come from multiple representations including graphs, tables, and descriptions of real-world situations.

Level 4:

To increase the level of understanding, students should critique examples of functions created by others. One way to do this is to have students work in pairs with one students constructing a function and the other student checking it. For example, student A constructs a function from a graph while student B constructs a function from a table of values. Then they swap papers and student B graphs student A's function while student A creates a table of values from student B's function. Then each compares their work with the originals. If they do not match, they must determine where the mistake was made and correct it.

Once students have a good understanding of constructing both linear and exponential functions, they can create their own real-world examples. This can also involve students predicting a situation that would provide data that could be modeled by a linear or exponential function, collecting that data, and writing the function based on that data to test their prediction. Students should show and explain why the real-world example represents a linear or an exponential function, including representing it in multiple ways.

Standard A2.F.LE.A.2 (Supporting Content)
For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize an exponential function as one in which the base is a constant, while the exponent is the variable.	Identify the components of an exponential function using correct math vocabulary: base, exponent, argument, coefficient. Identify the components of a logarithmic function using correct math vocabulary: base, exponent, argument, coefficient.	Identify the components of a logarithmic function and explain how they relate to an exponential function. Convert an exponential function into a logarithm function using correct notation. Convert a logarithm function into an exponential function using correct notation. Evaluate a logarithm using technology. Identify and use common logarithms and natural logarithms. Attend to precision when defining components and writing a logarithmic function.	Create a real-world problem to represent a given logarithm equation. Analyze the work of others to determine accuracy and explain and correct any errors.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Know when to use a logarithmic function to solve a contextual problem. Use a logarithmic function with a base of 2, 10, or e to solve a contextual problem.	

Instructional Focus Statements

Level 3:

Students need to have a good understanding of exponential functions before learning logarithms so they can make the necessary connections between these two functions. Exponential function: $ab^{ct} = d$ Logarithmic function: $log_b d^a = ct$, where a is the coefficient of the exponential, b is the base, c is the coeficient of t, t is the exponent, and d is the argument.

It is important for students to attend to precision when writing the notation for a logarithm. Students often do not understand that the base is written as a subscript and therefore can get confused between the base and the argument. One way to help build understanding of these components is to have students write both the exponential and logarithmic functions side by side identifying all the components.

Students will learn additional properties of logarithms in more advanced courses, including that a logarithm is the inverse of an exponential function.

Level 4:

When given an equation or inequality, students can generate a real-world situation that could be solved by a provided equation demonstrating a deep understanding of the interplay that exists between the situation and the equation used to solve the problem.

Students should be asked to critique the reasoning of others by analyzing the set-up and conversion process they did to find the solution as well as the interpretation of the solution in context. Having students explain any mistakes or misconceptions and correct them can be very beneficial to deepening the level of understanding as well.

Standard A2.F.LE.B.3 (Supporting Content)
Interpret the parameters in a linear or exponential function in terms of a context.

Scope and Clarifications: (Modeling Standard)
For example, the equation $y = 5000\,(1.06)^{x}$ models the rising population of a city with 5000 residents when the annual growth rate is 6 percent. What will be the effect on the equation if the city's growth rate was 7 percent instead of 6 percent?
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define slope as a rate of change. Identify the slope and the y-intercept in a linear function written in slope-intercept form. Identify the coefficient, base, and exponent in an exponential function.	Determine the slope and y-intercept of a linear function in a graph. Calculate the slope of a line that passes through two given points. Calculate the y-intercept of a linear function algebraically. Identify the initial value of an exponential function in a graph. Calculate the initial value of an exponential function algebraically. Calculate the growth rate of an exponential function by finding the ratio of successive terms.	Explain the meaning of the slope and y-intercept in context of the real-world situation, given a linear function. Explain the meaning of the coefficient, the base, and the exponent in context of the real-world situation, given an exponential function with a domain in the integers. Predict and determine how a linear function is affected by a change in the slope or y-intercept. Explain this change in context. Predict and determine how an exponential function is affected by a change in the coefficient, base, or exponent. Explain this change in context.	Critique the reasoning of others. Create a real-world scenario that can be modeled by it. Describe the effect of a potential change in the parameters, given a linear function. Create a real-world scenario that can be modeled by it. Describe the effect of a potential change in the parameters, given an exponential function.

Instructional Focus Statements

Level 3:

This standard was introduced in Algebra I in standard A1.F.LE.B.4. In Algebra II, there are no limitations to the exponential functions being interpreted.

As this standard is a modeling standard, examples should connect to a real-world context. The focus on this standard is on how the different components affect each other. Use questions that ask students to interpret the slope and y-intercept of linear functions in the context of a real-world situation. Likewise, ask students to interpret the coefficient, base, and exponent of exponential functions in context of a real-world situation. Then extend their learning by asking them to determine the effect of changes to the parameters on the function. The scope provides an example of an exponential function question.

Have students make a prediction of the effect of a change in a parameter and then verify if their prediction was correct by applying the change and comparing the results. Repetition of this activity will help students develop a better understanding of the properties of the operations within the function. Connecting the function to the context will help them justify their reasoning for their predictions. This repetition will also help students see the structure of the function and make a connection to its use as a general formula for the given real-world situation.

It is important for students to attend to precision in their interpretations and explanations should include units to ensure they are interpreting completely and correctly.

Level 4:

Teachers should provide students examples of linear and exponential functions and ask them to create a real world scenario that could be modeled by them. Students should also describe the effect of any potential change in the parameters. Students should be able to critique others' interpretations of the parameters and correct any mistakes. This standard can be integrated with A2.F.IF.B.3 and interpreting key features of a graph in connection with the function and the context.

Trigonometric Functions (F.TF)

Standard A2.F.TF.A.1 (Supporting Content)
Understand and use radian measure of an angle.
A2.F.TF.A.1a Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
A2.F.TF.A.1b Use the unit circle to find sin θ, cos θ, and tan θ when θ is a commonly recognized angle between 0 and 2π.

Scope and Clarifications:
Commonly recognized angles include all multiples n π/6 and n π /4, where n is an integer.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify and label the degree measure of commonly recognized angles between 0 and 360 degrees on the unit circle, when given a plotted point on the unit circle. Identify and label the radian measure of $\pi/2$, π, $3\pi/2$, and 2π on the unit circle, when given a degree measure. Identify the sine, cosine, and tangent of both acute angles, given all three sides of a right triangle. Convert between radian and degree measure using the ratio of $\frac{\pi}{180}$ or $\frac{180}{\pi}$.	Choose an appropriate definition for a radian measure of an angle is equal to the ratio of the length of the subtended arc of the angle to the radius. Identify and label the radian measure of commonly recognized angles between 0 and 2π on the unit circle. Identify the sin θ, cos θ, and tan θ, where θ is a commonly recognized angle between 0 and 2π, given the coordinates of a point on the unit circle.	Explain that a radian measure of an angle is equal to the ratio of the length of the subtended arc of the angle to the radius. Draw a unit circle to represent θ and find sin θ, cos θ, and tan θ, given a common radian measure for θ.	Choose the radian measure of the central angle formed by the arc, given a circle of radius other than 1 and an arc length that is a whole number multiple of the radius. Find csc θ, sec θ, and cot θ where θ is a commonly recognized angle between 0 and 2π.

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Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:

Instructional Focus Statements

Level 3:

In previous courses, students learned how to measure angles in degrees. In Algebra II, this concept shifts to describing angles of rotation about a point using radians. Radians should be introduced as the number of radius lengths it would take to get from (1, 0) to a specified point on the unit circle (going along the circumference). It can be shown that it takes a little over 3 radians to get from (1, 0) to (-1, 0) on unit circle, which is 180 degrees. In fact, it takes exactly π radius lengths or π radians. Therefore, 180 degrees is equivalent to π radians. Students should make the connection between fractions of 180 degrees and fractions of π. For example, 30 degrees is a sixth of 180 degrees and thus, is $\pi/6$ radians. Students should identify commonly recognized angles in the first quadrant of the unit circle and use multiples of those to describe angles in the other three quadrants. Radian measure, instead of degree measure, will be used to solve problems with greater efficiency in future courses.

In Geometry, students are introduced to special right triangles, 45°-45°-90° and 30°-60°-90°, and define trigonometric ratios. In Algebra II, instruction should extend a students' knowledge of special right triangles to construct the x and y coordinates on the unit circle. Given this information, students will find the cosine and sine of each commonly recognized angle. Special attention should put on the simplicity of using a circle with radius 1, as $\cos \theta = x/r$ and $\sin \theta = y/r$ become simply $\cos \theta = x$ and $\sin \theta = y$. However, students should recognize that all circles would have these same properties since all similar right triangles having the same side ratios. Fluency with trigonometric ratios of commonly recognized angles will aid students in graphing trigonometric functions in future courses, as well as, solve problems involving trigonometric functions.

Level 4:

In addition to seeing the unit circle as a set of points on the coordinate plane and instruction should focus on helping students visualize the cosine and sine of each angle as the legs of the special right triangles formed by the origin and the point. This will allow students to see the mathematics instead of memorizing values. Students at this level can use their knowledge of the unit circle to find the value of the reciprocal trigonometric functions as well. In addition, students with a deep understanding of this standard can calculate the radian measure when the given circle has a radius other than 1.

Standard A2.F.TF.A.2 (Supporting Content)
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the degree measure of commonly recognized angles between 0 and 360 degrees on the unit circle Identify the radian measure of commonly recognized angles between 0 and 2π on the unit circle. Find the two missing side lengths, given one side length of a 45°-45°-90° or 30°-60°-90° special right triangle.	Identify the degree measure of angles coterminal to commonly recognized angles on the unit circle. Identify the radian measure of angles coterminal to commonly recognized angles on the unit circle.	Explain why coterminal angles have the same sine, cosine, and tangent values. Explain how the unit circle can be used to find the sine, cosine, and tangent of all real numbers. Identify sin θ, cos θ, and tan θ when θ is coterminal to a commonly recognized angle on the unit circle, given a graphical representation of theta.	Find sin θ, cos θ, and tan θ given θ is coterminal to commonly recognized angle on the unit circle. Identify multiple angle measures that would have a given sine, cosine, or tangent value and explain why there could be more than one answer. Identify the degree measure of negative angles coterminal to commonly recognized angles on the unit circle. Identify the radian measure of negative angles coterminal to commonly recognized angles on the unit circle

Instructional Focus Statements

Level 3:

Standard A2.F.TF.1 focuses on developing the unit circle and identifying the degree measure, radian measure, sine, cosine, and tangent for all commonly recognized angles between 0 and 2π. Once students understand the values on the unit circle, instruction should shift for A2.F.TF.2 towards expanding the unit circle to all real numbered values of theta. In particular, students should identify angles that are coterminal to those on the unit circle and understand how a single point represents multiple angles of rotation (an infinite amount actually). This includes angles greater than 2π that are coterminal to commonly recognized angles in the unit circle. It is also important that students know that there are an infinite number of angles in between the commonly recognized angles, which together, extends trigonometric functions to all real numbers. Instruction should help students notice the periodic nature of trigonometric functions that results from extending the domain to all real numbers. In future courses, students will build on their understanding to graph trigonometric functions and understand why the graphs have a domain of all real numbers and why the graph is periodic.

Level 4:

As students develop a deep understanding of this standard, they should have opportunities to discover patterns in the unit circle. For example, students can investigate even and odd multiples of π to generalize the cosine, sine, and tangent of both sets of angles. Students may notice that the sine and tangent of any integer multiple of π is 0, odd multiples have a cosine of -1, or even multiples have a cosine of 1. They can then use their understanding to find certain values, such as sin (43pi) or cos (-16pi) to solve problems. In addition, students at this level should extend their understanding of coterminal angles to include negative angles when measuring clockwise on the unit circle.

Standard A2.F.TF.B.3 (Supporting Content)
Know and use trigonometric identities to find values of trig functions.
A2.F.TF.B.3a Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of sin θ, cos θ, and tan θ to evaluate the trigonometric functions.
A2.F.TF.B.3b Given the quadrant of the angle, use the identity $\sin^2\theta + \cos^2\theta = 1$ to find sin θ given cos θ, or vice versa.

Scope and Clarifications:
Commonly recognized angles include all multiples n π/6 and n π/4, where n is an integer.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use the Pythagorean theorem to find the exact value of the third side, given two sides of a right triangle. Find sin θ given cos θ, or vice versa, given the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and the quadrant of θ.	Evaluate the other two trigonometric functions for the same angle, given the sine, cosine, or tangent of an angle, Know the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$	Recognize and use the right triangle ratio definitions of sin θ, cos θ, and tan θ to evaluate the trigonometric functions, given a point on a circle centered at the origin. Know and use the identity $\sin^2\theta + \cos^2\theta = 1$ to find sin θ given cos θ, or vice versa, given the quadrant of θ.	Recognize and use the right triangle ratio definitions of csc θ, sec θ, and cot θ to evaluate the trigonometric functions, given a point on a circle centered at the origin. Use properties from the unit circle and the Pythagorean theorem to explain the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

Instructional Focus Statements

Level 3:

In Geometry, students find missing side lengths using the Pythagorean Theorem and trigonometry functions. In Algebra II, this skill is transferred to the context of the unit circle. By drawing the reference triangle for the given point on a circle centered at the origin, students can label the legs of the triangle using the coordinates and thus, find the tan θ. Finding the hypotenuse using the Pythagorean theorem enables students to also evaluate the sin θ and cos θ. Because students are describing trigonometric functions instead of just side lengths, both positive and negative values are now appropriate based on the given trigonometric function and quadrant. By relating cos θ to x and sin θ to y, students should be able to assign the correct sign without memorizing which trigonometric functions are positive and negative in each quadrant. In future courses, students will use this concept in various situations, including converting from Cartesian coordinates to polar coordinates.

Students can use similar ideas to find the sin θ given the cos θ, and vise versa. However, there is a unique relationship between sin θ and cos θ that can be used more efficiently. As cos θ is x, or the horizontal leg of the reference triangle, sin θ is y, or the vertical leg of the reference triangle, and the hypotenuse is 1, the Pythagorean theorem can be applied to these three side lengths. This produces $\sin^2 \theta + \cos^2 \theta = 1$. Because$|a| = a^2$, this equation is true in all quadrants and even if θ lies on an axis. This equation can be easily displayed by trying examples of commonly recognized angles that students know from the unit circle. For example, for $\pi/6$, this would be $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$. This is known as one of the Pythagorean identities. This trigonometric identity, along with others, will be frequently used in future courses as students simplify trigonometric expressions.

Level 4:

As students develop a deep understanding of this standard, they should be able to explain the Pythagorean identity and how it can be used. In addition, as students study the unit circle, they should begin to recognize relationships between trigonometric values in the unit circle. For instance, students may recognize other identities such as cos (-θ) = cos (θ), sin (-θ) = -sin (θ), and sin (θ) = cos (90 - θ). As students discover these patterns, they can then use these identities to evaluate trigonometric functions more efficiently.

INTERPRETING CATEGORICAL and QUALITATIVE DATA (S.ID)

Standard A2.S.ID.A.1 (Supporting Content)
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine how many standard deviations above or below the mean a data value lies. Explain how the normal curve describes the population density curve for many naturally occurring situations such as height, weight, or strength of adults. Explain the Empirical Rule and its relationship to the normal curve	Use the empirical rule to estimate the percent of the data within one, two, or three standard deviations from the mean, given the mean and standard deviation. Draw and label a normal distribution curve, given the mean and standard deviation. Explain how the symmetry of the normal curve can be used to find missing percentages.	Use the empirical rule to estimate the percent of data above or below various values on the normal curve. Use the empirical rule to estimate the percent of data between values on the normal curve.	Use the empirical rule to estimate various percentages on the normal curve within a context. Use technology or tables to estimate various percentages on the normal curve that are not a multiple of the standard deviation from the mean.

Instructional Focus Statements

Level 3:

In Algebra I, students describe center and spread using median and interquartile range as well as mean and standard deviation. Teachers should build upon this understanding of mean and standard deviation by discussing their definitions and how to apply them to the normal curve and in what types of situations normal curves are used. Students should then be introduced to applying the empirical rule to approximately normal distributions to tell what percent of data values fall within whole-numbered standard deviations from the mean. For example, given a normal distribution with a mean of 15 and a standard deviation of 5, what percent of the data is between 10 and 20? Teachers should then challenge students to use the empirical rule to investigate more complex problems such as: what percent of the data is below 30? above 5? or between 10 and 25? Therefore, instruction should focus on using geometric concepts to estimate areas under the curve that are not directly provided by the empirical rule. For example, what percent of data is above three standard deviations to the right of the mean? Discussion and discovery should lead to the understanding that since 95% of the data falls within two standard deviations from the mean, 5% fall outside of these values and thus, half of that (2.5%) would be each tail.

Level 4:

As students develop a deep understanding of this standard, they should be provided real-world examples and how the normal distribution can be used to inform decision making. Students may also be curious about how to find percentages on the normal curve that are not a multiple of the standard deviation from the mean. Teachers can provide students with the appropriate tables or technology to estimates these percentages using z-scores (value minus mean divided by the mean).

Standard A2.S.ID.B.2 (Supporting Content)
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
A2.S.ID.B.2a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.

Scope and Clarifications:
Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
i. Tasks have a real-world context.
ii. Tasks are limited to exponential functions with domains not in the integers.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear function to fit a given data set. Choose if a given scatter plot is best represented by a linear, quadratic, or exponential function.	Choose a quadratic function that fits a given data set. Use a given linear function to solve a problems in the context of the data. Fit a linear function to a given set of data.	Fit a quadratic function to a given set of data. Fit an exponential function to a given set of data, where exponential functions are limited to domains not in the integers. Solve problems using a linear, quadratic, and exponential function, where exponential function are limited to domains not in the integers, in the context of the data.	Create a contextual situation with an embedded data set derived from a given function. Explain the relationship between the function, data set, and the contextual situation using precise mathematical language and justifications. Use a given function to explain the relationship between two quantities in a created context.

Instructional Focus Statements

Level 3:

In grade 8, students developed an understanding of how to create a scatterplot, evaluate the scatterplot in order to describe any pattern associations between the two quantities, and informally fit a straight line to data when it visually resembled a straight line. In high school, students should extend this understanding to summarize, represent, and interpret data on two categorical and quantitative variables. This allows students to use mathematical models to capture key elements of the relationship between the two variables and explain what the model tells about the relationship. Students should gain a conceptual understanding of how to draw conclusions in addition to finding the equation for the line of best fit. As students' progress through A Algebra it should become apparent to them that many real-world situations produce data that can be modeled using functions that are not linear. The exposure to quadratic and exponential functions broadens the options students have for modeling data sets, where data sets can be represented in tabular, graphical, or as a discrete set of points.

Students should be exposed to real-world situations where it is apparent that the scatter plot suggests a pattern that is more curved than linear in its visual depiction. Thus leading the student to realize that a linear function does not provide the closest fit to the data causing the student to consider other function types. It is imperative that students discover that sometimes obvious patterns may not tell the whole story. Students should develop an understanding that sometimes curves fit better than lines. Students should not only discover this algebraically but also develop an understanding of the connection that exists between the model and the contextual situation that it represents and understand that this connection is essential in identifying and building appropriate models. As students solidify their understanding, they should be able to describe how the variables are related within the context of the situation. Students should also use various forms of technology to explore and represent scatterplots as this will enhance their ability to see the relationship that exits between the variables.

Level 4:

As students extend their understanding, they should be able to create a contextual situation with an embedded data set derived from a given function. Students should also be able to explain and provide justifications for the relationships that exist between the function, data set, and the contextual situation using precise mathematical language. Particular attention should be put on creating situations that differentiate between linear, quadratic, and exponential functions. Students should be able to explain why one function is more appropriate than another function for the contextual situation.

MAKING INFERENCES and JUSTIFYING CONCLUSIONS (S.IC)

Standard A2.S.IC.A.1 (Major Work of the Grade)
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Scope and Clarifications:
For example, in a given situation, is it more appropriate to use a sample survey, an experiment, or an observational study? Explain how randomization affects the bias in a study. There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a study as a sample survey from a given verbal description of the context. Identify a study as an observational study from a given verbal description of the context. Identify a study as an experiment from a given verbal description of the context.	Determine if a sample survey, experiment, or observational study would be most appropriate, given a contextual situation,	Identify bias in a given study. Explain the differences among sample surveys, experiments, and observational studies. Describe what type of situations would be most appropriately studied with sample surveys, experiments, and observational studies. Explain the limitations of sample surveys, experiments, and observational studies related to randomization.	Create a research question best answered through a sample survey and with a sound methodology that minimizes bias. Create a research question best answered through an experiment and with a sound methodology that minimizes bias. Create a research question best answered through an observational study and with a sound methodology that minimizes bias.

Instructional Focus Statements

Level 3:

In Algebra I students learn that correlation does not imply causation. One way to determine causation is to conduct an experiment in which all other confounding factors are controlled. Teachers should emphasize the importance of randomization in selecting a sample for the study and random assignment to the control and treatment groups so that inferences can be made to the intended population. For example, if studying Tennessee males, randomly selecting 200 Tennessee males then randomly assigning 100 to the control group and 100 to the treatment group would suffice. If randomization is not met, the results will not be as useful, as the data may be biased (e.g., if only males from east Tennessee were selected). Although a simple random experiment is the best-case scenario, students should recognize it is often difficult to conduct due to ethical concerns, uncontrollable factors (e.g., weather), or financial barriers. In these situations, an observational study can be conducted instead. Teachers should lead conversations on various research questions and whether an experiment would be most appropriate.

Observational studies are best conducted when natural conditions allow for studying existing control and/or treatment groups (researchers do not intervene in any way). Due to using existing groups, observational studies are rarely, if ever, considered random. For example, an experiment researching the effect of spider bites on children would have major ethical concerns. Therefore, students should realize that researchers in this study would only be able to conduct an observational study with children that have already been bitten by a spider. The sample was not randomly selected, therefore students should have some concerns about bias with this study.

In a sample survey (a type of observational study), participants are surveyed on the research topic. Again, randomly selecting participants from the population is best. If this is not possible, other methods can be used to select participants, such as stratified random sampling, systematic sampling, cluster sampling, or multi-stage sampling. A discussion about other sampling methods including convenience sampling and volunteer sampling should focus on how these can create bias. Another way a sample survey can introduce bias is through the wording of the questions. For example, a question might lead participants to answer in a certain way based on how the question is asked. Students should also consider the location and time when participants are surveyed to decrease bias. For example, students should recognize that researchers should not survey peoples' interest in fruit with people in the fruit aisle of the grocery store or call homes to survey adults at 1pm, while most are at work.

Level 4:

As students develop a deep understanding of the three types of studies, they should progress to designing their own studies. Beginning with a topic that interests them, students should create a research question, describe their methodology, and justify the chosen type of study. Other students should have opportunities to critique the shared methodology and describe its limitations. In future courses, students may learn about hypothesis tests that are used to analyze the data gathered through experiments, observational studies, and a sample survey.

Standard A2.S.IC.A.2 (Major Work of the Grade)
Use data from a sample survey to estimate a population mean or proportion; use a given margin of error to solve a problem in context.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify if a study as a sample survey from a given verbal description of context.	Explain what a margin of error is in either verbal or written form using appropriate mathematical vocabulary. Choose a confidence interval given a population mean and margin of error from a sample survey. Choose a population mean from given data from a sample survey.	Use data from a sample survey to estimate a population mean. Use data from a sample survey to estimate proportion. Solve problems in a contextual situation when given a margin of error.	Explain the relationship to a contextual situation when given a margin of error. Explain how increasing or decreasing a sample size can affect the margin of error. Explain what this means with respect to a contextual situation. Choose a population and use data from a sample survey to estimate the population mean, calculate the margin of error, and explain the margin of error with respect to the population.

Instructional Focus Statements

Level 3:

In previous grades, students developed an understanding of population means and proportions for contextual situations. In high school, students couple this understanding with making inferences about populations through sample surveys. Students should first understand that a sample survey is exactly as it states a representative "sample" of the population rather than the entire population. Students should also understand the relevance for using a sample survey in contextual situations. In conjunction with this understanding, students should understand that the size of the sample survey can vary. A larger sample will result in a more accurate population mean and proportions. A smaller sample will result in a less accurate population mean and proportion. Depending on the context, increasing the size may not always be realistic. For example, in manufacturing sample surveys are used to determine the quality of a product since testing the entire population could result in little to no products left for production. Understanding the importance of using a sample survey is key prior to developing an understanding of a margin of error.

In this standard, the focus is on understanding the use of a given margin of error and not calculating the margin of error. Students should understand that a margin of error is the largest expected size of the difference between an estimate and the actual population value that is being estimated. For example, when students estimate the average weight of a population based on a proper sample and your margin of error is "4 pounds," that is saying that you would be very confident that the actual population mean weight is within 4 pounds of your sample estimate. Thus concluding that if the sample average weight is 45 pounds and the margin of error is 4 pounds, you would be confident that the actual population mean weight would be somewhere between 41 pounds and 49 pounds.

Additionally, students should develop an understanding of when it is appropriate to use a sample population mean and when it is appropriate to use a sample proportion, the fraction of samples which were successes, based on the contextual situation. This should include making judgements of the size of the sample and the meaning of the margin of error.

Level 4:

As students solidify their understanding, they should be able to choose a population and use data from a sample survey to estimate the population mean, calculate the margin of error, and explain the margin of error with respect to the population. They should also be able to estimate the margin of error and explain what it means with respect to the sample population mean and/or proportion. Additionally, students should be able to explain that the sample survey will not always yield a sample estimate that is equal to the value of the population parameter it is estimating. They should understand that there will always be some sample variability. Students should also understand that using a larger sample leads to less sampling variability and results a smaller margin of error when estimating parameters. They should also understand that using smaller sample leads to more sampling variability and results a larger margin of error when estimating parameters. Additionally, students should be able to explain the appropriateness of the sample survey that was used with respect to the contextual situation.

Conditional Probability and the Rules of Probability (S.CP)

Standard A2.S.CP.A.1 (Supporting Content)
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. Identify the outcomes in a sample space which compose the event, for an event described in everyday language (e.g., "rolling double sixes").	Identify the union of two subsets, given a sample space within a context. Identify the intersection of two subsets, given a sample space within a context. Identify the complement of a subset, given a sample space within a context.	Represent sample spaces for compound events as unions, intersections, or complements of other events. Use the symbols ∪ and ∩ to represent sample spaces for compound events.	Create a Venn diagram to illustrate the union, intersection, or complement of events. Create a two-way frequency table to illustrate the union, intersection, or complement of events. Identify more complex subsets (e.g., the complement of $A \cup B$), given a sample space within a context.

Instructional Focus Statements

Level 3:

In grade 8, students represent sample spaces for compound events using lists, tables, and tree diagrams. In Algebra II, students are expected to expand their understanding of compound events by describing them as unions, intersections, or complements of other events. Teachers should build on students’ prior knowledge of compound events by using lists, tables, and tree diagrams to initially describe the union, intersection, and complement of events.

Before moving on to contextual situations, teachers should ask students to find subsets of simple sets of numbers using set notation. Teachers can then relate set notation to different contextual situations, for example, a bag containing various colored marbles. As students become more comfortable with unions, intersections, and complements, they should be expected to describe compound events using the symbols ∪ and ∩. Using a Venn diagram can aid students in visualizing compound events. For example, let's assume there are 18 students enrolled in French (F), 45 students enrolled in Algebra II (A), 7 in both, and 114 enrolled in neither. How many students are there in $F \cup A$? Creating a Venn diagram and coloring in F, A, and the intersection would describe $F \cup A$.

In this situation, the union of French students and Algebra II students would be all those enrolled in French, Algebra II, or both. Students in both subjects should not be counted twice. Therefore, the intersection should be subtracted, and thus, there are 18 + 45 - 7 = 56 students in $F \cup A$. In A2.S.CP.B.6, students will build off of this idea to understand the addition rule with inclusive events. Teachers should remind students to not assume that the complement of F is A. The complement of F is all the students not in French, which would be the total (168) minus those enrolled in French (18), which is 150.

Although Venn diagrams are useful, situations with two categories are best described using a two-way table. For example, 65 students travel on a soccer trip. 43 are players and 12 are left handed. Only 5 of the left-handed students are soccer players. How many students are soccer players or are left handed?

Students should be expected to apply unions, intersections, and complements of events to calculating probability. Subsets can describe the number of ways an event can occur as well as the total number of possible outcomes. Therefore, a deep understanding of this standard sets the foundation for all other S.CP standards.

Level 4:

As students develop a deep understanding of this standard, they should be able to create their own Venn diagram or two-way table to describe various contextual situations. Therefore, students should be exposed to increasingly more complex real-world situations that are best described with a Venn diagram or two-way table. Additionally, students with a deep understanding of compound events can identify complex subsets. For example, students can find the complement of $A \cup B$ or see that the union of all subsets is the sample space.

Standard A2.S.CP.A.2 (Supporting Content)
Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Calculate the probability of two independent events occurring together, in a real-world context.	Calculate the joint probability of A and B occurring together, in a real-world context by examining the sample space. Identify whether theoretical probability events are independent or dependent (e.g., selecting two kings in a row with replacement versus without replacement).	Determine if the joint probability of A and B occurring together is equal to the product of their probabilities, and use this characterization to determine if they are independent.	Explain why two events are independent if the product of their probabilities is equal to the joint probability.

Instructional Focus Statements

Level 3:

In grade 8, students find probabilities of compound events. In Algebra II, students are exposed to both independent and dependent events when calculating joint probabilities. One misconception students have is assuming P(A) and P(B) are independent and thus, they simply multiply P(A)*P(B) without considering if the likelihood of one event occurring affects the probability of the other event occurring. Students should be able to decide if given events are independent or dependent. For example, find the probability of selecting a blue marble from a bag and then selecting a red marble from the same bag after placing the first marble selected back into the bag. This is independent because the probability the first marble is blue does not impact the probability of the second marble being red. Teachers should expose students to events of both types to strengthen their ability to categorize various situations as independent or dependent (e.g., also give students an example where the first marble is not replaced). If students decide the given events are independent, then P(A and B) = P(A)*P(B). Conversely, if P(A and B) = P(A)*P(B), then students can discern that the events are independent.

Many students take advantage of the first statement, but do not immediately understand the second statement of the bi-conditional. Therefore, students should be given opportunities to examine the sample space to decide if the events are independent. For example, let's assume there two light switches. If the light switches are randomly assigned to on or off, what is the probability that the first one is off and the second is on? Students should be able to write the sample space of all the possibilities, which includes OO, OF, FO, and FF (O is on and F is off). In this scenario, students should see that only one of the cases is FO and thus, the probability is 1/4. Since 1/4 is 1/2 times 1/2, then it is equivalent to P(F)*P(O). Therefore, students can decide that the two events are independent and explain how it makes sense because the probability of the second light being on is not impacted by the probability of the first light being off, because they were randomly assigned.

If events are dependent, then students must first consider how the probability of the first event impacts the probability of the second event. The product changes to P(A and B) = P(A)*(B given A), which uses the conditional probability. Instead of memorizing the formula, students should reason and make sense of the problem and how the first event impacts the second. In fact, the formula for joint probabilities is always P(A and B) = P(A)*P(B given A), but with independent events, P(B given A) is the same as P(B) because P(A) does not impact P(B). This idea leads nicely into conditional probability and independence. Thus, instruction can be paired nicely with M2.S.CP.A.3 and M2.S.CP.A.4, in which students will calculate conditional probabilities and use them to decide independence.

Level 4:

As students develop a deep understanding of independent and dependent events, they should be expected to explain why two events are independent if the product of their probabilities is equal to the joint probability. Students can provide an example like light switch problem in Instructional Focus level 3 and explain how if the product of their probabilities is not equal to the joint probability, then the probability of the second event is altered by the probability of the first event, and thus the events are dependent.

Standard A2.S.CP.A.3 (Supporting Content)
Know and understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Calculate the probability of two independent events occurring together, in a real-world context. Recognize conditional probabilities in a real-world context.	Calculate conditional probabilities in a real-world context. Calculate the probability of two dependent events occurring together, in a real-world context.	Explain why events A and B are independent if the probability of B given A is the same as the probability of B. Explain why the probability of B given A is the same as the probability of B if events A and B are independent.	Explain why the probability of A given B is P(A and B)/P(B).

Instructional Focus Statements

Level 3:

Understanding the bi-conditional statement connecting independence to P(B) = P(B given A) continues in this standard as students work with conditional probability. While students develop an understanding of A2.S.CP.A.2, they should begin to understand how to calculate probabilities when the probability of the second event (B) is affected by the probability of the first event (A). By calculating the new probability of B, given that A has already occurred, students are discovering the idea of conditional probability. A2.S.CP.A.3 builds upon A2.S.CP.A.2 to support students in developing a conceptual understanding of conditional probability and why the conditional probability of A given B is P(A and B)/P(B). One way to accomplish this is to begin with the formula for dependent probability, P(A and B) = P(A)*P(B given A). Students can divide both sides of the equation by P(A) to rearrange the formula to reveal P(B given A) = P(A and B)/P(A). Counting methods such as the counting principle, tree diagrams, tables, and lists can be used to help students conceptually understand this principle.

For example, an academic club at school consists of 4 freshman, 6 sophomores, 9 juniors, and 11 seniors. Two students are selected from the club to join the school parade. What is the probability that the second student selected is a junior given the first student selected was a senior? In this scenario, students can simply count how many juniors and total students remain once the senior is selected. This would leave 9 juniors and 19 total students, and thus, P(Junior given Senior has already been selected) would be 9/19. However, students can make the connection by dividing the probability of selecting a senior and junior by the probability of selecting a senior. The counting principle would give P(Senior and Junior)/P(Senior) = $\frac{(11/20)*(9/19)}{(11/20)} = \frac{9}{19}$. In this situation, students can calculate the probability both ways and notice that P(Junior given Senior has already been selected) = P(Senior and Junior)/P(Senior).

However, students aren't always able to write P(A and B) separately as P(A)*P(B), so the formula can be beneficial in those situations. For example, the probability that a randomly selected car has cruise control is 92%. The probability that a randomly selected car has both cruise control and a rear camera is 45%. What is the probability that a car with cruise control also has a rear camera, P(Camera given cruise control)? Students should use the formula to produce $\frac{.45}{.92} = .489$. Understanding the conditional probability B given A as the fraction of A’s outcomes that also belong to B is the focus of A2.S.CP.B.5, so these standards can be nicely paired together.

Level 4:

Explaining why the probability of A given B is P(A and B)/P(B) can take many forms. Students with a deep understanding of this standard should be able to use Venn diagrams, algebra, and contextual situations to explain why the probability of A given B is P(A and B)/P(B). They should be able to describe contextual situations and how conditional probability can be applied to solve problems.

Standard A2.S.CP.A.4 (Supporting Content)
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Scope and Clarifications:
For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Complete a hypothetical 1000 two-way table, given the probability of A, the probability of B, and the probability of A and B.	Calculate conditional probabilities within a context from a two-way table.	Compare conditional probabilities from a two-way table to decide if the events are independent or not independent. Explain why two conditional probabilities from a two-way table with the same conditions means the two events are independent.	Create a real-world context and corresponding two-way table in which the events are independent.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

Two events are independent when knowing that one event has occurred does not change the likelihood that the second event has occurred. Students can use conditional probabilities to tell if two events are independent or not independent from a two-way table. For example, the probability that a student knows what they want to do after high school is 75%, the probability that a student attends extracurricular activities is 40%, and the probability that a student does not know what they want to do after high school and attends extracurricular activities is 30%. Are the events "a student knows what they want to do after high school" and "attends extracurricular activities" independent or not? One way to determine independence is to compare P(extracurricular activities given they know what they want to do after high school), which is P(A and B)/P(A) = 30%/75% = 40% with P(extracurricular activities), which is 40%. In this case, P(B given A) = P(B), therefore, the two events are independent.

It is also helpful for students to create a hypothetical 1000 two-way table to easily organize the information in rows and columns. In the two-way table, students would use 300 instead of 30%, for example. If the probability of attending extracurricular activities given the student knows what they want to do after high school (300/750 = 40%) is equivalent to the probability of attending extracurricular activities given the student does not know what they want to do after high school (100/250 = 40%), then the two events are independent. This should make sense to students because that would mean that the probability of attending extracurricular activities is the same regardless of whether the student knows what they want to do after high school or not. If the probabilities were not equal or relatively close, then students cannot assume the events are dependent, only not independent. Although there may be an association between the two variables, teachers should remind students that association does not mean there is a causal relationship.

<u>Level 4:</u>

As students develop a deep understanding of determining independence by comparing two conditional probabilities with the same condition, they should be able to create their own real-life context with data and discern whether the two events are independent or not independent. Is can be helpful for some students to create data that results in the events being not independent so that other students can be exposed to those situations when examples are shared with the class.

Standard A2.S.CP.B.5 (Supporting Content)
Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

Scope and Clarifications:
For example, a teacher gave two exams. 75 percent passed the first quiz and 25 percent passed both. What percent who passed the first quiz also passed the second quiz? There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize conditional probabilities in a real-world context. Determine what percent a given percent is of another percent (e.g., 5% is what percent of 20%?). Divide two fractions and put into simplest form.	Calculate conditional probabilities in a real-world context. Recognize the conditional whole in contextual problems is not the total.	Use a model (e.g., Venn diagram or table) to represent and solve conditional probability problems.	Explain the conditional probability of A given B as the area of A and B divided by the area of B using a Venn diagram.

Instructional Focus Statements

Level 3:

Based on other conditional probability standards, students should recognize that the probabilities P(B) and P(B given A) are different if the two events are not independent. In standard A2.S.CP.A.3, students develop a conceptual understanding of why P(B given A) = P(A and B)/P(A). In this standard students use this formula to calculate conditional probabilities (i.e., P(B given A)) in various real-world situations. In many cases, students are given the P(A and B) and either P(A), P(B), or both and asked to calculate the conditional probability P(B given A) or P(A given B). Discussion during instruction would need to help students recognize that if they are given both P(A) and P(B), one is superfluous information. It can be helpful to model the probability using a Venn diagram to visualize a percent of a percent. For example, given the probability that a randomly selected student is a female is 56% (A), the probability the student is in drama is 67% (B), and the probability that the student is female and in drama class is 34% (A and B), what is the probability the student is in drama given they are female (B given A)? Since students are given the selected student is female, the total is narrowed to only female students (56%) instead of 100%. Instruction should lead students to recognize that the question is asking what percent of females are in drama which is, mathematically, finding what percent of 56% is 34%. This would lead students to recognize they need to divide .34/.56, which is P(A and B)/P(A). A Venn diagram can depict

the idea that conditional probability is the intersection, P(A and B), divided by the conditional whole, in this case, P(A).
If more than two categories are used, teachers can provide or encourage students to use a table. For example, a frequency table can be used to compare the interest in sports across grade levels. If students are asked to calculate the probability a randomly selected student likes sports given they are in the 10th grade, students could be shown how to cover up the rest of the table to highlight only 10th graders. From there, students can calculate the probability that a student likes sports. Explaining that the condition given narrowed down the subjects to only 10th graders helps students begin to focus on only the values involved in the problem. As students use a table, they can further strengthen their understanding of calculating a percent of a conditional whole.

Level 4:

As students use the conditional probability formula and begin to see conditional probability as a percent of a percent, they should be expected to explain conditional probability using a Venn diagram or table. Teachers should ask students to discuss how the whole is narrowed to a conditional whole and how that impacts the probability of a given event. Using a Venn diagram, students can shade in the area of A and B and show how it is a fraction of the whole (either A or B). Likewise, students could be asked to justify why conditional probability narrows a table to a row or column, in which the probability can then be found. Teachers can ask students to provide an example and write a response about their findings, explaining how the conditional probability was calculated and justifying their thinking. This activity would support the literacy standards in mathematics, as students would be discussing and articulating mathematical ideas while using correct mathematical vocabulary to build a their argument.

Standard A2.S.CP.B.6 (Supporting Content)
Know and apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Scope and Clarifications:
For example, in a math class of 32 students, 14 are boys and 18 are girls. On a unit test 6 boys and 5 girls made an A. If a student is chosen at random from a class, what is the probability of choosing a girl or an A student?
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. Find P(A or B), given two events that are mutually exclusive. Add and subtract fractions and put into simplest form.	Know that P(A or B) = P(A) + P(B) - P(A and B) for both inclusive and mutually exclusive events. Determine if two events are inclusive or mutually exclusive. Identify the union of two subsets, given a sample space within a context. Identify the intersection of two subsets, given a sample space within a context.	Calculate P(A or B) given two events within a real-world context. Interpret P(A or B) in terms of the real-world context.	Explain why P(A or B) = P(A) + P(B) - P(A and B) using a model (e.g., Venn diagram or table). Explain why P(A or B) = P(A) + P(B and the complement of A) is an alternate solution method. Explain why P(A or B) = P(A and the complement of B) + P(B and the complement of A) + P(A and B) is an alternate solution method.

Instructional Focus Statements

Level 3:

In grade 8, student calculate the probability of compound events by examining the sample space using lists, tables, and tree diagrams. In Algebra II, students use their prior knowledge from grade 8 and their understanding of subsets formed in A2.S.CP.A.1 to calculate P(A or B), when A and B are inclusive or mutually exclusive. One misconception students tend to have is that "or" always means add, and therefore, simply calculate P(A or B) as P(A) + P(B). Teachers can break this misconception by giving students examples of inclusive events where P(A) + P(B) is greater than 1. For example, when rolling a 6-sided number cube, what is the probability of rolling an odd number or a number greater than 2? In this case, P(A) + P(B) = 3/6 + 4/6 = 7/6. Discussion should help students recognize that this cannot be correct and realize that by simply adding the probabilities together, they counted 3 and 5 twice. Having students examine the sample space can quickly reveal what the probability should be. At this point, teachers can give students time to discover how to calculate P(A or B) by starting with P(A) + P(B) = 7/6 and asking students to identify the overlap and see that they need to subtract it to calculate the probability correctly. This should help students understand the need to subtract the outcomes that fit in both categories and were counted twice. Thus, students calculate P(A or B) as 7/6 - 2/6 = 5/6.

Instead of giving students the formula, teachers should let students discover why the P(A and B) needs to be subtracted out so that outcomes (i.e., P(A and B)) are not counted twice. It can be helpful to use a Venn diagram so that students can visualize this concept and make the connection to the formula, P(A or B) = P(A) + P(B) - P(A and B). If A and B are inclusive, shading P(A) and P(B) on a Venn diagram reveals to students how the intersection is being shaded twice.

Level 4:

A Venn diagram or two-way table can support students in developing a deep understanding of this standard. Instruction should encourage students to explain why P(A or B) = P(A) + P(B) - P(A and B) using multiple representations, such as both a Venn diagram and a two-way table. By having students create a two-way table, the P(A and B) would be separated out from P(A) and P(B) and students can see these four probabilities shown in the table: P(A and the complement of B), P(B and the complement of A), P(A and B), and P(the complement of B and the complement of A). By displaying data in this form, students should see other ways to calculate P(A or B), such as P(A) + P(B and the complement of A), or P(A and the complement of B) + P(B and the complement of A) + P(A and B). Asking students to separate a Venn diagram into its four parts can also aid in understanding these alternate ways to calculate P(A or B) and often better aligns with students' thinking as they examine the sample space to calculate P(A or B).