Tennessee Mathematics Standards

Approved July 30, 2010

Calculus
#3113

Standard 1 – Mathematical Processes

Course Level Expectations

CLE 3113.1.1 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely in mathematical reasoning.

CLE 3113.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including testing cases, estimation, and then checking induced errors and the reasonableness of the solution.

CLE 3113.1.3 Develop inductive and deductive reasoning to independently make and evaluate mathematical arguments and construct appropriate proofs; include various types of reasoning, logic, and intuition.

CLE 3113.1.4 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.

CLE 3113.1.5 Recognize and use mathematical ideas and processes that arise in different settings, with an emphasis on formulating a problem in mathematical terms, interpreting the solutions, mathematical ideas, and communication of solution strategies.

CLE 3113.1.6 Employ reading and writing to recognize the major themes of mathematical processes, the historical development of mathematics, and the connections between mathematics and the real world.

CLE 3113.1.7 Use technologies appropriately to develop understanding of abstract mathematical ideas, to facilitate problem solving, and to produce accurate and reliable models.

Check for Understanding (Formative/Summative Assessment)

- ✓ 3113.1.1 Model a written description of a physical situation in each of the following contexts: a function, a differential equation, and an integral.
- ✓ 3113.1.2 Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.
- ✓ 3113.1.3 Recognize the applications of calculus concepts in the world.

- ✓ 3113.1.4 Evaluate definite integrals and numeric derivatives of functions by hand and with appropriate technology; illustrate each with appropriate diagrams and/or graphs.
- ✓ 3113.1.5 Use appropriate notation to represent derivatives and integrals.
- ✓ 3113.1.6 Use appropriate technology (i.e., spreadsheet) to approximate definite integrals (Riemann sums, Trapezoidal rule, or Simpson's rule).
- ✓ 3113.1.7 Discuss the historical synergies accompanying the discovery of calculus and the impact of politics and rivalries on its development. Identify major mathematicians who influenced the development and understanding of calculus and place it in the context of events in world history.
- ✓ 3113.1.8 Correctly expand and contract formulas expressed in summation notation and in open form

Standard 2 – Functions, Graphs, & Limits

Course Level Expectations

CLE 3113.2.1 Use calculus to predict and to explain the observed local and global behavior of a function.

CLE 3113.2.2 Demonstrate an understanding of the concept of the limit of a function.

CLE 3113.2.3 Describe the asymptotic and unbounded behavior of functions.

CLE 3113.2.4 Develop an intuitive understanding of continuity.

Check for Understanding (Formative/Summative Assessment)

- ✓ 3113.2.1 Analyze the graphs of polynomial, rational, radical, piecewise, and transcendental functions using appropriate technology. Discuss which functions behave nicely with respect to algebraic properties and which do not. Justify your discussions.
- ✓ 3113.2.2 Represent complex numbers in both rectangular and polar form.
- ✓ 3113.2.3 Apply the trigonometric form of complex numbers in calculations.
- ✓ 3113.2.4 Apply DeMoivre's Theorem to find roots and powers of complex numbers.
- ✓ 3113.2.5 Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity.
- ✓ 3113.2.6 Define continuity at a point using limits; define continuous functions.
- ✓ 3113.2.7 Apply Intermediate Value Theorem and Extreme Value Theorem to continuous functions.
- ✓ 3113.2.8 Calculate limits (including limits at infinity) using algebra.
- ✓ 3113.2.9 Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition to a variety of piece-wise functions.
- ✓ 3113.2.10 Determine whether a given function is continuous at a specific point.

- ✓ 3113.2.11 Given a complete set of algebraic information and calculus information, construct a sketch of a function that matches that data. Display several functions that each satisfy one set of data; describe their differences and their similarities.
- ✓ 3113.2.12 Given a sketch of a graph of a function, completely describe the function in mathematical terms so that the sketch could be replicated from the description and would be close to the original graph.
- ✓ 3113.2.13 Draw a sketch that illustrates the definition of the limit; develop multiple real world scenarios that illustrate the definition of the limit.
- ✓ 3113.2.14 Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior.

Standard 3 – Derivatives

Course Level Expectations

CLE 3113.3.1 Demonstrate an understanding of the concept of derivative.

CLE 3113.3.2 Recognize the derivative as the slope of the tangent line to a curve at a given point.

CLE 3113.3.3 Use the first and second derivatives of a function to characterize the function and vice versa.

CLE 3113.3.4 Apply derivatives in problem solving situations.

CLE 3113.3.5 Apply basic rules for differentiation.

Check for Understanding (Formative/Summative Assessment)

- ✓ 3113.3.1 Represent and interpret the derivative of a function graphically, numerically, and analytically.
- ✓ 3113.3.2 Interpret the derivative as an instantaneous rate of change.
- ✓ 3113.3.3 Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph.
- ✓ 3113.3.4 Approximate both the instantaneous rate of change and the average rate of change given a graph.
- ✓ 3113.3.5 Relate the increasing and decreasing behavior of f to the sign of f' both analytically and graphically.
- ✓ 3113.3.6 Apply the Mean Value Theorem.
- ✓ 3113.3.7 Understand the relationship between Rolle's Theorem and the Mean Value Theorem.
- ✓ 3113.3.8 Relate the concavity of f to the sign of f'' both analytically and graphically.
- ✓ 3113.3.9 Model rates of change, including related rates problems. In each case, include a discussion of units.
- ✓ 3113.3.10 Illustrate the relationship between differentiability and continuity.
- ✓ 3113.3.11 Write the equation of the tangent line to a curve at a given point.
- ✓ 3113.3.12 Analytically locate the intervals on which a function is increasing or decreasing.
- ✓ 3113.3.13 Translate verbal descriptions into equations involving derivatives and vice versa.

- ✓ 3113.3.14 Identify critical points, maxima and minima, and points of inflection on graphs of functions and using formulas of functions; include information about asymptotes when appropriate.
- ✓ 3113.3.15 Use optimization to find extreme values (relative and absolute).
- ✓ 3113.3.16 Find the derivative of the inverse of a function.
- ✓ 3113.3.17 Use differentiation to solve problems involving velocity, speed, and acceleration.
- ✓ 3113.3.18 Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric).
- ✓ 3113.3.19 Calculate the derivatives of sums, products, and quotients of basic functions.
- ✓ 3113.3.20 Apply the chain rule to find the derivative of a composite function.
- ✓ 3113.3.21 Implicitly differentiate an equation in two or more variables.
- ✓ 3113.3.22 Set up and solve related rates problems and max/min problems. Where applicable, solve both symbolically and graphically.
- ✓ 3113.3.23 Use tangent lines to approximate function values and changes in function values when inputs change (linearization).
- ✓ 3113.3.24 Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using limits and the definition of the derivative and using the derivative rules.

Standard 4 – Integrals

Course Level Expectations

CLE 3113.4.1 Explore the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change.

CLE 3113.4.2 Apply techniques of antidifferentiation.

Check for Understanding (Formative/Summative Assessment)

- ✓ 3113.4.1 Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay, volume).
- ✓ 3113.4.2 Recognize differentiation and antidifferentiation as inverse operations.
- ✓ 3113.4.3 Evaluate definite integrals using the Fundamental Theorem of Calculus.
- ✓ 3113.4.4 Apply basic properties of definite integrals.
- ✓ 3113.4.5 Use Riemann sums and the Trapezoidal Rule to approximate definite integrals.
- ✓ 3113.4.6 Use a definite integral to find the area of a region.
- ✓ 3113.4.7 Use substitution of variables to calculate an antiderivative.
- ✓ 3113.4.8 Develop facility with finding antiderivatives.
- ✓ 3113.4.9 Correctly write a Riemann sum that represents the definition of a definite integral.