Tennessee Higher Education Commission

Supplementary Task: Presenting Data (What's the "Big Whoop"?)

This task is designed to illustrate the strengths and weaknesses of data presented in a table, as a graph, or as a function. The task can be used at any point in a college algebra course when students are using data to develop a model for an application.

These examples support the objective:

Analyze data and apply concepts to solve problems.

in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example 1: Whooping Cranes Population

Whooping cranes are the tallest birds in North America. The whooping crane nearly vanished in the mid-20th century, with a count of only 16 birds in 1941. Captive breeding programs and reintroduction efforts have increased the number of wild birds to over 200, with roughly the same number living in captivity. The last remaining natural migratory flock of whooping cranes is called the Western flock. (In 2001, experts began to introduce an Eastern flock as part of reintroduction efforts.) The population of the whooping crane Western flock from 1940 until 2010 is given in the table below.

Year	1940	1950	1960	1970	1980	1990	2000	2010
# Cranes	22	34	33	56	76	146	177	281

a) What kinds of information does this table of values give you?

b) Graph the data given in the table. Does the graph give you more or different information about general trends in the data? If so, what kind of information does the graph give you that the table does not?

c) Find a function to model the data given in the table. Graph your function on the graph from part (b). How well does the graph of your function fit the data from the table? What information does a function allow you to infer that would not be apparent from a table or a graph?

d) Using the function, predict the population of the whooping crane Western flock in 2025. Does this value seem reasonable?

Resource: www.learner.org/jnorth/tm/crane/Population.html

Example 2: Pertussis (Whooping Cough) Cases

Pertussis is a highly contagious respiratory disease known for uncontrollable, violent coughing. Pertussis most commonly affects infants and young children. Diagnosed pertussis cases are reported by states to the Centers for Disease Control and Prevention (CDC). The number of reported pertussis cases at five-year intervals from 1925 until 2010 are included in the table below.

Year	1925	1930	1935	1940	1945	1950	1955	1960	1965
# Reported Cases	152003	166914	180518	183866	133792	120718	62786	14809	6799

(table continued)

Year	1970	1975	1980	1985	1990	1995	2000	2005	2010
# Reported Cases	4249	1738	1730	3589	4570	5137	7867	25619	27550

a) What kinds of information does this table of values give you?

b) Graph the data given in the table. Does the graph give you more or different information about general trends in the data? If so, what kind of information does the graph give you that the table does not?

c) Find a function to model the data given in the table. Graph your function on the graph from part (b). How well does the graph of your function fit the data from the table? What information does a function allow you to infer that would not be as apparent from a table or a graph?

d) Use your function to predict the number of reported pertussis cases in 2025. Does this value seem reasonable?

e) The number of reported pertussis cases began to rise between 1980 and 1985, and there was a significant increase between 2000 and 2005. What do you think caused this increase?

Resource: Centers for Disease Control and Prevention, www.cdc.gov/pertussis/surv-reporting/cases-by-year.html (Note: This site includes data for all years 1922-2011.)

Prior Knowledge Needed

Students should have a basic working knowledge of using data from a table to create a scatterplot (most likely using technology). Students should also know how to use technology to create a regression function (linear, quadratic, cubic, quartic, exponential, etc.).

It is also helpful if the technology used provides a value of the coefficient of determination (r^2). If the technology provides a value for r^2, it will be useful for students to know that higher r^2 values imply better fits between the model and the data.

Prerequisite Common Core State Standards for Mathematical Content that support these examples

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Build a function that models a relationship between two quantities

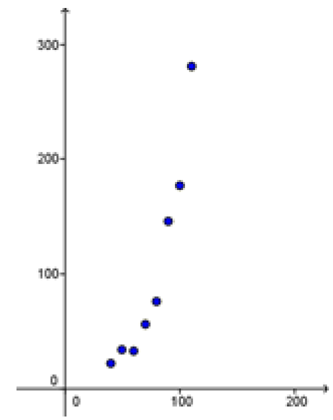
Construct and compare linear, quadratic, and exponential models and solve problems

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

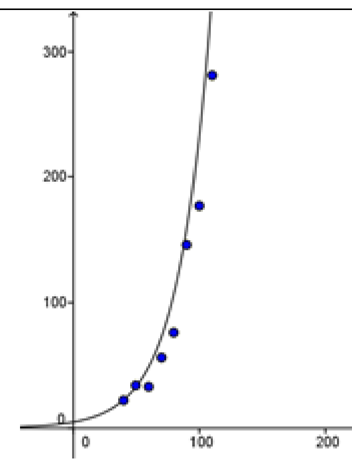
Solutions

Example 1, part (a): A table of values will give specific information at specific points in the problem. Sometimes it is possible to identify overall trends (increasing, decreasing, constant, for example), but it is usually difficult to determine other information such as rates of increase or decrease or predicted values for points not given in the table.

Example 1, part (b): The graph appears to the right. Note that the graph indicates a very steep rise in the population in this case, so there is apparently a high rate of increase in the population. The shape of the graph also suggests an exponential model.



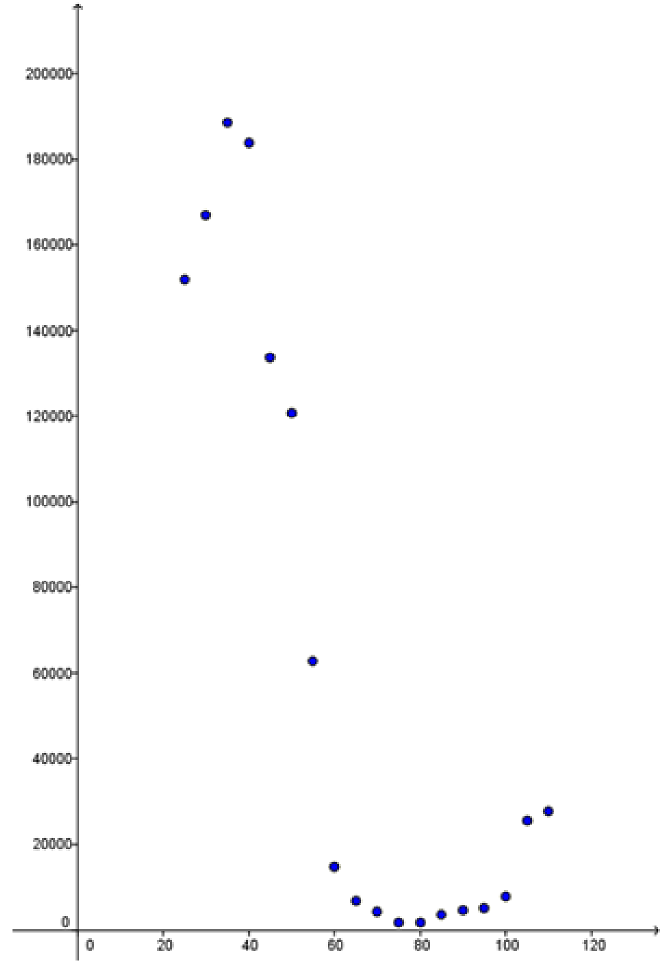
Example 1, part (c): Based on the shape of the graph, an exponential function should be a good fit for the model. Using exponential regression on the calculator, we can construct an exponential model: $P(t) = 4.62(1.04)^t$, where P(t) represents the population t years after 1900. (Note that values from the regression are rounded to two decimal places.) The graph of the model appears to the right. The function provides an easy way to estimate values of the population at points not originally given in the table.



Example 1, part (d): Using P(t) (with the rounded values) from part (c) and using t = 125, we would predict the population in 2025 to be approximately 622 whooping cranes in this population. This is reasonable given that the year of the prediction is 15 years after the last value given in the table.

Example 2, part (a): A table of values will give specific information at specific points in the problem. Sometimes it is possible to identify overall trends (increasing, decreasing, constant, for example), but it is usually difficult to determine other information such as rates of increase or decrease or predicted values for points not given in the table.

Example 2, part (b): The graph is given on the right. From the graph, it is clear that there is a rise in the number of cases, followed by a sharp decline, and then a smaller rise. This suggests that perhaps a higher-degree polynomial model might be a better fit than an exponential, linear, quadratic, logarithmic, or logistic model.

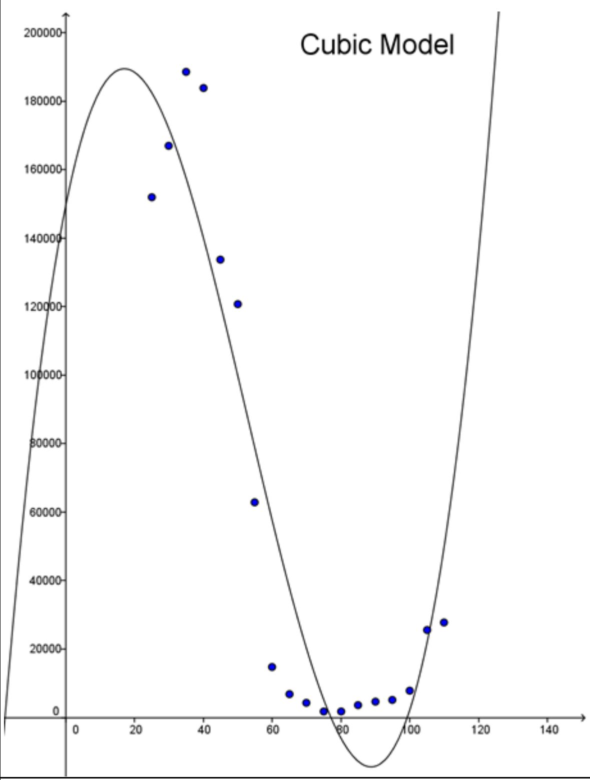


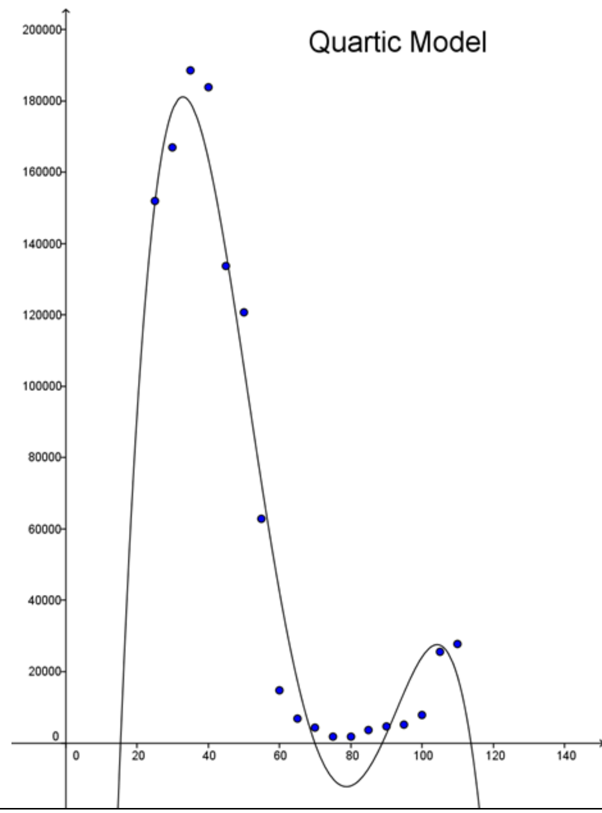
Example 2, part (c): Both a cubic and a quartic model are given. The data were input into a graphing utility using, as the t-value, the number of years since 1900. Values given by the graphing utility were rounded to four decimal places due to the sizes of the coefficients and the spread of points in the graph. Keeping as many decimal places as possible will result in closer fits to the data.

The quartic model is a slightly better fit than the cubic model. The function provides an easy way to estimate values of the population at points not originally given in the table.

Cubic model: $N(t) = 1.1075t^3 - 175.8012t^2 + 5034.9750t + 149256.0743$

Quartic model: $N(t) = -0.0619t^4 + 17.8206t^3 - 1761.9939t^2 + 66873.2765t - 673861.8579$





Example 2, part (d): Values are calculated using the N(t) functions given in part (c). The value of t is 125 (years since 1900). Cubic model: N (125) = 194820 (rounded to the nearest integer) – This does not seem reasonable as this value is higher than any of the numbers of reported pertussis cases given in the table. We would expect that modern medicine would be able to cope with the increase in these numbers and would be able to stop the rapid increase. Quartic model: N(125) = -152302 (rounded to the nearest integer) – This does not seem reasonable since we cannot have a negative number of reported cases of pertussis.
Example 2, part (e): According to the Center for Disease Control, reported cases of whooping cough vary from year to year and tend to peak every 3-5 years. This pattern is not completely understood. Many factors contribute to the number of reported pertussis cases, including improvements in diagnosis and the reporting of cases in adolescents and adults.

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