Module 3: Place Value and Base Ten

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Objective

- To understand how children progress through the levels of understanding regarding "ten"

Tennessee Standards

1.NBT 2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number line diagram.

TEAM Alignment

- Standards and Objectives
- Presenting Instructional Content
- Activities and Materials
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Participant Activities:

- Participants will discuss the foundational ideas of place value and consider the oral language involved.
- Participants will reflect on the different models used for developing place value understanding.
- Participants will engage in and review multiple student tasks for developing place value understanding.

Place Value Development

"A complete understanding of place value, including extensions to decimal numeration, develops over the elementary and middle grades. However, the most critical period in this development occurs in grades PreK–2. In kindergarten and 1st grade, children count and are exposed to patterns in the numbers to 100. Most importantly they begin to think about groups of ten objects as a unit. The (Tennessee State) Standards recommends that kindergarteners work with numbers between 11 and 19 and by composing and decomposing them into tens and ones using materials and drawings. By second grade, these initial ideas are extended to three-digit numbers. As a significant part of this development, children should engage in composing and decomposing numbers in a wide variety of ways as they solve addition and subtraction problems with two- and three-digit numbers. In other words, there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation."

John Van De Walle, et al (2014)

Place Value Warm-Up Task

Use the Place Value Cards we made yesterday to complete this activity.

1. What number is 10 less than 56?
2. What number is 10 more than 89?
3. What number is 30 more than 42?
4. What number is 40 less than 46?
5. What number is 10 more than 963?
6. What number is 100 less than 809?

Note: This task acts as a bridge between understanding place value and using strategies based on place value for addition and subtraction.

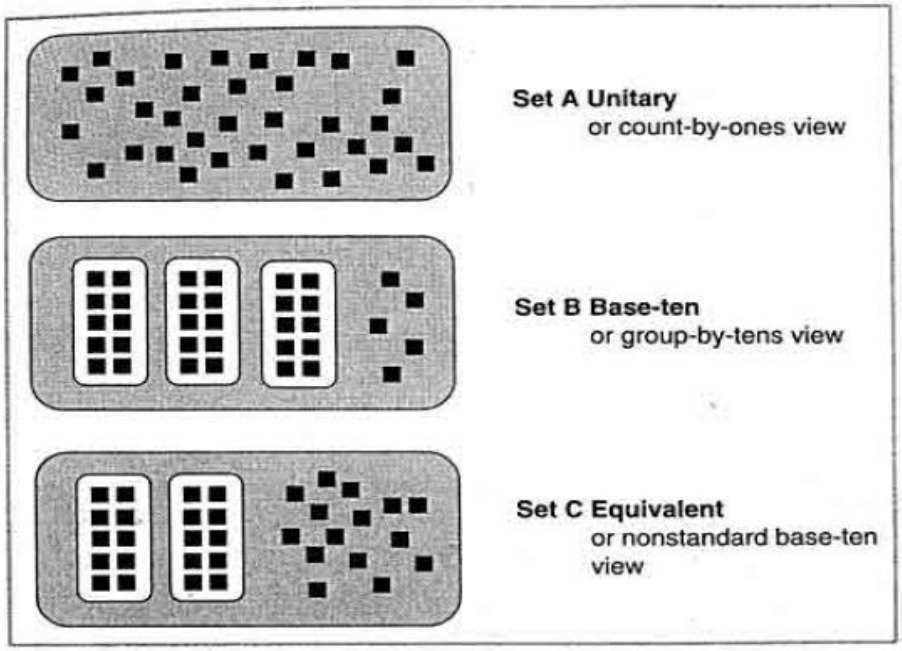
Adapted from
https://www.illustrativemathematics.org/content-standards/2/NBT/A/1/tasks/94

Foundational Ideas in Place Value

Place value understanding requires an integration of new and sometimes difficult-to-construct concepts of grouping by tens (the base ten concept) with procedural knowledge of how groups are recorded in our place value system and how numbers are written and spoken.

Integration of Base Ten Groupings with Counting by Ones

Once you recognize that children can count out a set of 35 by ones, you want to help them see that making groupings of tens and leftovers is a way of counting the same quantity.



- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

You want children to construct the idea that all of these are the same and that the sameness is evident by virtue of the groupings of tens. There is a subtle yet profound difference between children at this stage. The children in the pre-base ten stage may not be sure how many they will get if they count the tiles in set B by ones, or if the groups were “ungrouped “ how many there would be.

Set C – Groupings with fewer than the maximum number of tens are referred to as equivalent groupings or equivalent representations.

Understanding the equivalence of sets B and C indicates that grouping by ten is not just a rule that is followed, but that any grouping by tens, including all or some of the singles, can help tell how many. Many computational techniques (i.e., regrouping in addition and subtraction) are based on equivalent representations of numbers.

The Role of Counting in Constructing Base Ten Ideas

Counting plays a key role in constructing base ten ideas about quantity and in connecting these concepts to symbols and oral names for numbers. Each approach helps children think about mathematics in a different way.

1. *Counting by ones*- Before base ten ideas develop, counting by ones is the only approach by which children can be convinced that all three sets are the same amount.
2. *Counting by groups and singles*- (Set B) This may sound like " One, two, three groups of 10, and one, two, three, four, five singles." This method may be very novel for a student that has never thought about counting a group of objects as a single item. Notice this counting does not tell directly how many items there are. This counting must be coordinated with counting by ones before it can be a means of telling "how many".
3. *Counting by tens and ones*- (Set B and C) This is the way adults would typically count. Although this count ends by saying the number of objects it is not as explicit as the second method in counting the groups.

Reflection

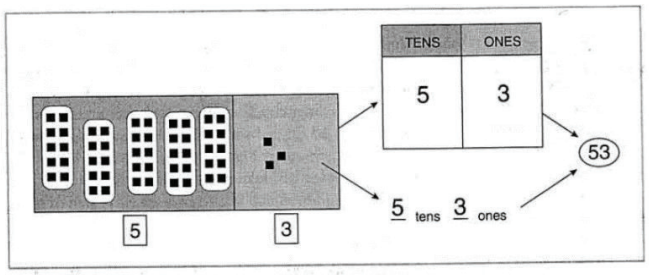
- What are some defining characteristics of children with pre-place value understanding (Module 2) and children who understand place value?

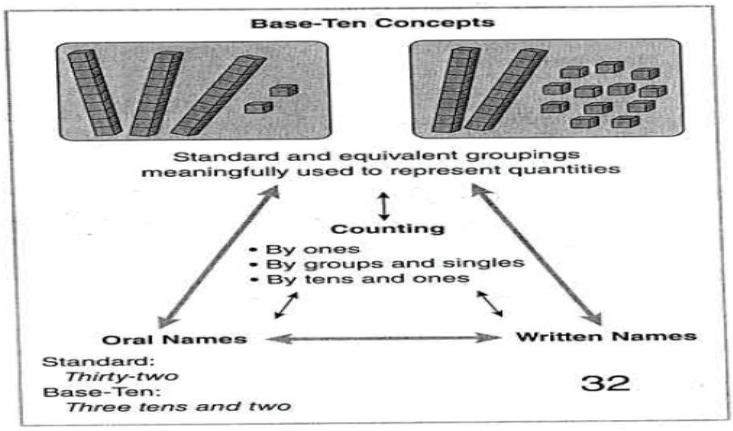
Integration of Grouping with Words

The way we say a number such as “thirty-five” must also be connected with the groupings by ten concept. There are several variations of base ten language for 35: “3 tens and a 5”, “3 tens and 5 ones”, “3 groups of 10 and 5 ones”, “ 3 tens and 5 singles” and so on. Each may be used interchangeably with the standard name. If you have ESL or ELL students, it is best to select one variation and consistently connect it to the standard language.

Integration of Groupings with Place Value Notation

The symbolic scheme that we use for writing numbers (ones on the right, tens to the left of the ones, and so on) must be coordinated with the grouping scheme. Activities can be designed so that children physically associate groupings of tens and ones with the correct recording of the individual digits.





Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Reflection

- What do you notice your students have difficulty with when interpreting and using base ten language?

Base-ten Models for Place Value

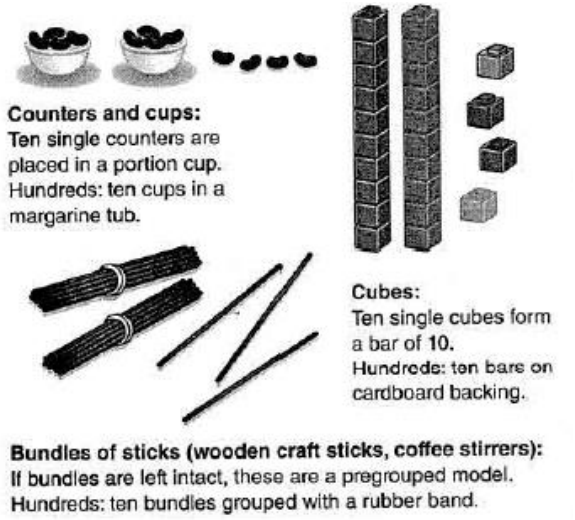
Physical models for base ten concepts can play a key role in helping children develop the idea of “a ten” as both single entity and as a set of 10 units.

Remember, though, that physical models do not “show” the concept to the children. The children must mentally construct the “ten makes one” relationship and impose it on the model. When first teaching place value, the base-ten model for ones, tens and hundreds should be proportional. That is a model for the ten is physically ten times larger than the model for a one, etc.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Groupable Models

Models that most clearly reflect the relationship of ones, tens, and hundreds are those for which the ten can actually be made and grouped from single pieces. These could also be called “put-together-take-apart “models.

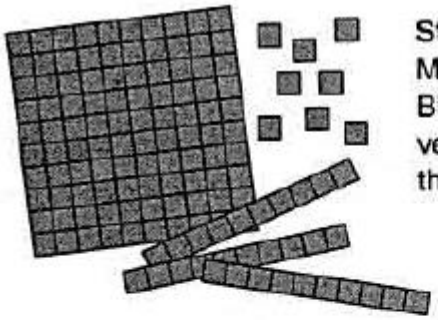


- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

As children become more and more familiar with these models, collections of tens can be made in advance by the children and kept as ready-made tens. This is a good transition into the pre-grouped models.

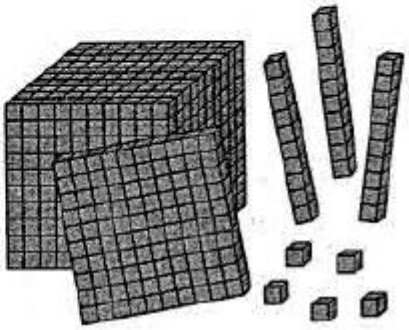
Pre-grouped or Trading Models

Models that are pre-grouped are commonly shown in textbooks and are often used in instructional activities. Pre-grouped models cannot be taken apart. When 10 single pieces are accumulated they must be exchanged for a ten.



Strips and squares: Make from cardstock. See Blackline Master 13. Plastic versions are available through catalogs.

Base-ten blocks: Wooden or plastic units, longs, flats, and blocks. Expensive, durable, easily handled, the only model with 1000.



- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

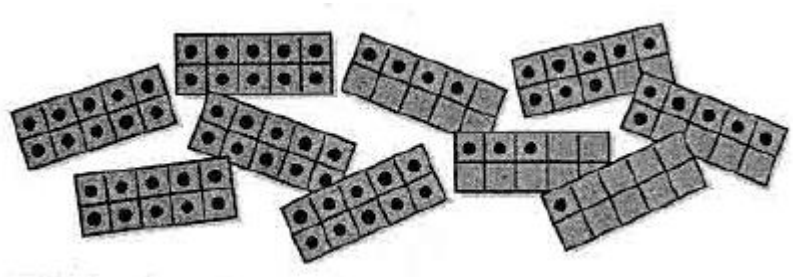
With pre-grouped models you need to make extra efforts to make sure that children understand that a ten piece really is the same as 10 ones. Here children combine multiplicative understanding (each piece is ten times the value of the place to the right) with a positional system (each place has a value) - something hard to do prior to multiplication!

Reflection

- What may be some disadvantages to using the pre-grouped physical models?

LITTLE TEN FRAMES

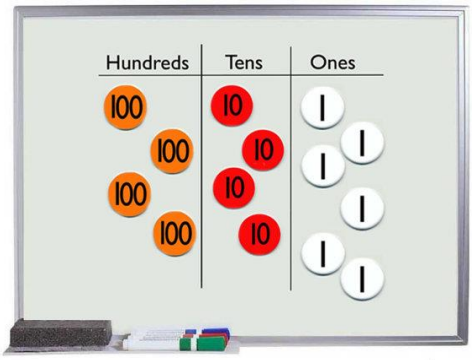
Little ten frames are less common but are very effective. If children have been using ten frames to think about numbers to 20, the value of the filled ten frame may be more meaningful than the ten rods and squares of base ten materials.



Although the ones are fixed on the cards, this mode has a distinct advantage of always showing the distance to the next decade. For example, when 47 is shown with four ten cards and a 7 card, a child can see that three more ones will make five full cards, or 50. This aids in using an open number line for addition.

Non-proportional Models

Non-proportional models can be used by children who understand that 10 units make a “ten”. In non-proportional models, the ten is not physically ten times larger than the one, such as in money, an abacus, or different chips assigned to different place values. Non-proportional representations should not be used to introduce place value concepts: they can be used once children have a conceptual understanding of the numeration system and need additional reinforcement.



Video: Brain Crossing

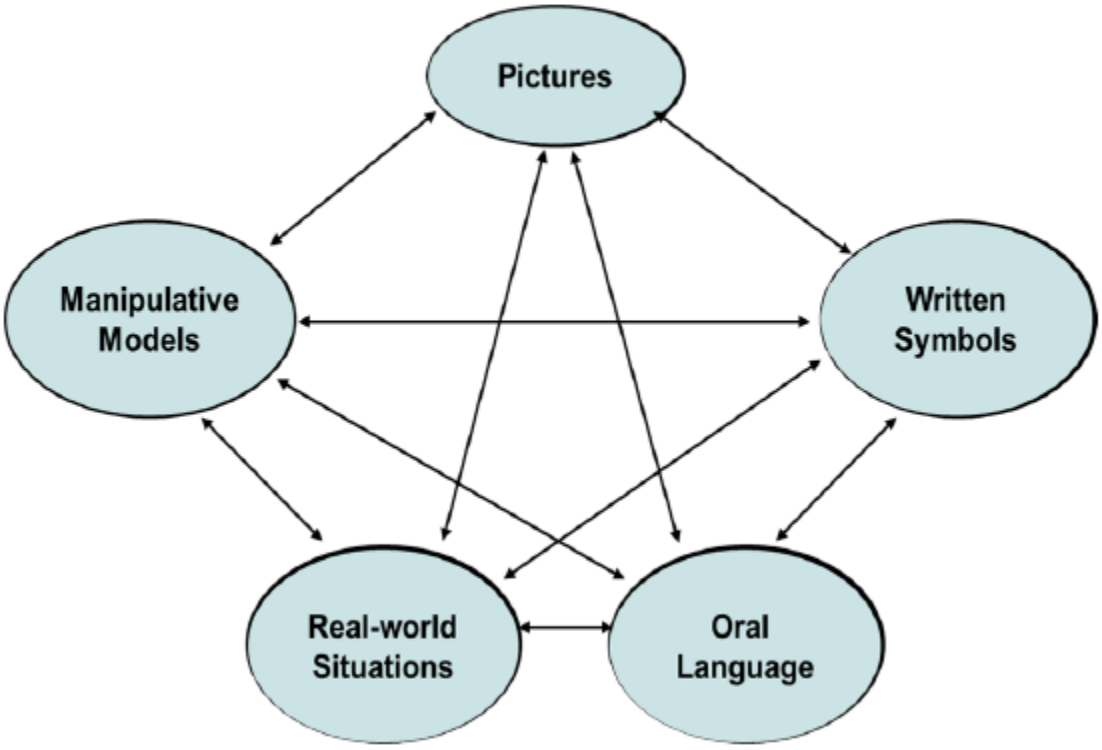
https://www.youtube.com/watch?v=qZBjub36Bvs

How does using both sides of our brain help us learn math better?

Discussion

The graphic below was introduced in previous math trainings. How does it relate to the brain crossing video?

Linking to Research/Literature
Connections between Representations



Adapted from Lesh, Post, & Behr, 1987

Developing Base-Ten Concepts

Connecting the important ideas of base ten concepts or grouping by tens with the oral and written names for numbers should be developed simultaneously with conceptual ideas.

Reflection

Think for a moment on how strange it must sound to say “seven ones.” Children have never said they were “seven ones” years old. The use of the word ten as a singular group name is even more mysterious. Consider the phrase “ten ones makes one ten.” The first ten carries the usual meaning of 10 things, the amount that is 1 more than 9 things. But the other ten is a singular noun, a thing. How can something the child has known for years as the name for a lot of things suddenly become one thing?

Because children come to their development of base-ten concepts with a count-by-ones idea of number, you must begin there. You cannot just impose grouping by ten on children. Children need to experiment with showing amounts in groups of like size and eventually come to an agreement that ten is a very useful size to use.

- Battista, M. *Cognition- Based Assessment & Teaching of Place Value: Building on Students’ Reasoning*, 2012

Student Activity: Counting in Groups (1.NBT.2)

The activity is designed as an example of a first effort at developing grouping concepts.

Materials: countable items (tub of cubes, # of shoes in the class, # of crayons in crayon box)

1. Find a collection of items that children might be interested in counting. The quantity should be countable, somewhere between 25 and 100.
2. Pose the question, "How could we count our ____ in some way that would be easier than counting by ones?"
3. Whatever suggestions you get, try to implement them.
4. After trying several methods, have a discussion about what worked well and what did not.

If no one suggests counting by tens, you might casually suggest that as an idea to try.

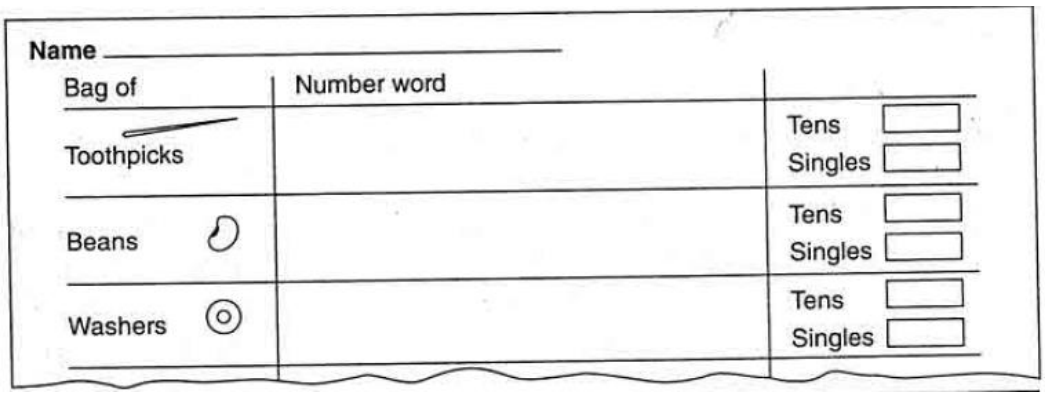
Student Activity: Groups of Ten (1.NBT.2)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Materials: bags of materials, recording sheet

In this activity children make groupings of ten and record or say the amounts. Number words are used so that children will not mechanically match tens and ones with individual digits. It is important that children confront the actual quantity in a manner that is meaningful to them.

1. Prepare bags of different types of objects, such as toothpicks, buttons, beans, plastic chips, etc. The bags can be placed at stations around the room or given to pairs of children.
2. Children should have a recording sheet similar to the image below.
3. Children empty the bags and count the contents.
4. The amount is recorded as a number word.
5. Then the objects are grouped in as many tens as possible.
6. The groupings are recorded on the form.
7. After returning the objects to the bags, bags are traded or children move to another station.



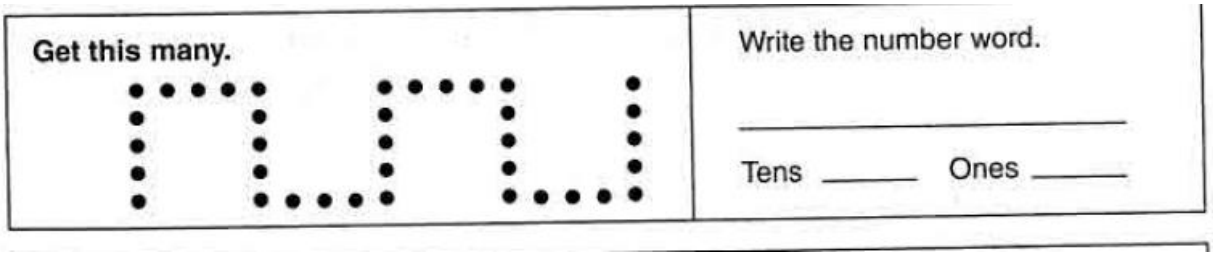
Name _____

Bag of	Number word	
Toothpicks		Tens ☐ Singles ☐
Beans		Tens ☐ Singles ☐
Washers		Tens ☐ Singles ☐

Variations for Groups of Ten

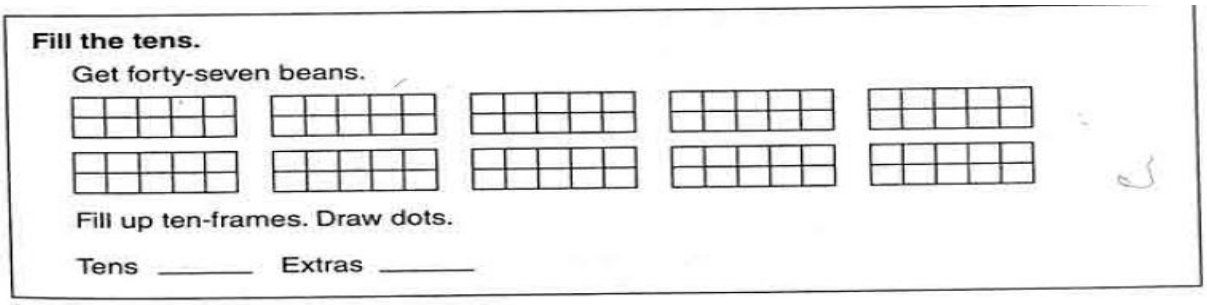
GET THIS MANY

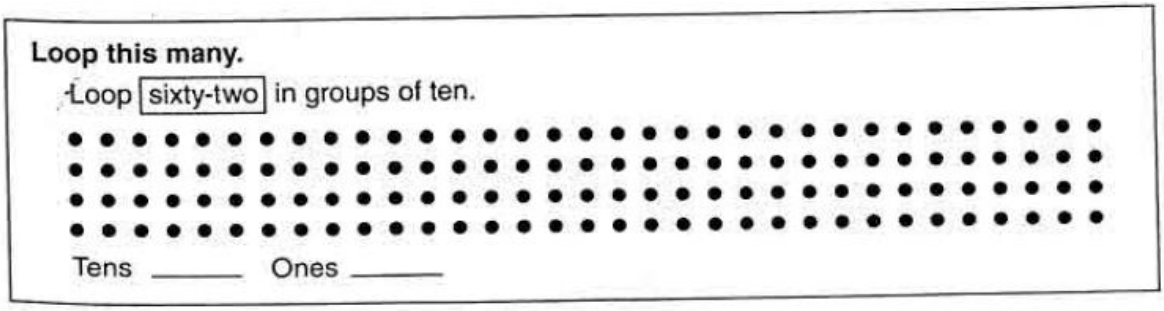
Children count the dots and then count out the corresponding number of objects. Provide small cups to put the groups of ten in. Notice that the activity requires children to first count the set in a way they understand (counting by ones), record the amount in words, and then make groupings.



FILL THE TENS OR LOOP THIS MANY

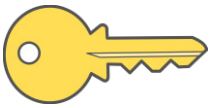
These sheets begin with a verbal name and children must count the indicated amount and then make groups.





Assessment Note

As you watch children doing these activities, you can learn a lot about their base-ten concept development. How do the children count the objects? Do they make groupings of ten? Do they count to 10 and then start again at 1? Children who do that are already using the base ten structure. But what you will more likely see early on is children counting a full set without stopping at tens and without any effort to group the materials in piles.



KEY IDEA #3

Children progress through three levels of understanding the concept of "ten" starting with understanding ten not as a unit but only as ten ones. They then move to seeing ten as a unit by relying on physical or mental reconstructions of models to help them work with units of ten. Finally, they are able to easily work with units of ten without the need of physical or mental reconstructions of base-ten models.

Small Group Discussion

You ask a child to count a small jar of beans. Then ask the child "If you were to place each group of 10 beans in a cup, how many cups would you need?" The child has no idea and makes a random guess.

- What does this tell you about the child's knowledge of place value?

Grouping Tens to Make 100

In second grade, numbers up to 1,000 become important. Here the issue is not one of connecting a count-by-ones approach to a group of 100, but rather seeing how a group of 100 can be understood as a group of 10 tens as well as 100 single ones, and seeing how a group of 1000 can be seen as a group of 10 hundreds as well as 1000 single ones.

Student Activity: Too Many Tens (2.NBT.1)

- Battista, M. *Cognition- Based Assessment & Teaching of Place Value: Building on Students' Reasoning*, 2012

Materials: 150 to 1000 of the same item (beans, cubes, etc), small cups, at least 10 large baggies.

In this activity it is important to use a groupable model so that children can see how the 10 groups of ten are the same as 100 individual items. This connection is often too implicit in the display of a hundreds flat or paper hundreds square in the pre-grouped base ten models.

1. Show students any quantity with 150 to 1000 items. (Ex. beans in a plastic bag)
2. Have the students record estimates of how many of the item are in the container. Discuss with children how they determined their estimates.
3. Give portions of the item to pairs or triads of children to put in cups of 10. Collect leftover beans and put these in groups of ten as well.
4. Ask," How can we use these groups of ten to tell how many _____ we have? Can we make new groups from the groups of ten? What is 10 groups of ten called?" Be prepared to have large baggies or containers in which the 10 cups can be placed. 5.

When all groups are made, count the hundreds, the tens, and the ones separately. Record the totals on the board as "4 hundreds + 7 tens+ 8 ones".

Equivalent Representations

An important variation of the grouping activities is aimed at the equivalent representations of numbers. After children have just completed the "Groups of Ten" activity with a bag of objects pose the following task:

> "What is another way you can show 42 besides 4 groups of ten and 2 singles? Let's see how many you can find."
> "What is another way you can show 551 besides 5 groups of hundred, 5 tens, and 1 one? Let's see how many you can find."

Small Group Discussion

- What do you think would be most students' answer? Take a moment and work with a partner to see if you can come up with a list. Think about what knowledge or understanding is taking place as you are finding different representations.

Student Activity: Can you make the Link? (precursor activity for 2.NBT.7)

- Battista, M., 2012

Materials: items that can be partially grouped

1. Show a collection of materials that is only partially grouped in sets of ten. For example, you may have 5 bundled popsicle sticks of 10 and 17 individual sticks. Be sure the children understand that each bundle is a group of 10 sticks. (You can vary this activity by using larger numbers, i.e., 12 bundled popsicle sticks of ten and 17 individual sticks.)

2. Have children count the number of sticks and also count the singles, in any way they wish to count.

3. Ask, “How many in all?” Record all responses and discuss how they got their answers.

4. Change the groupings (make a ten from the singles or break apart one of the tens). Repeat the questions and discussion. Do not change the total number from one time to the next.

5. Once children begin to understand that the total does not change, ask in what other ways the items could be grouped if using tens and singles.

Student Activity: Three Other Ways (2.NBT.7)

Battista, M., 2012

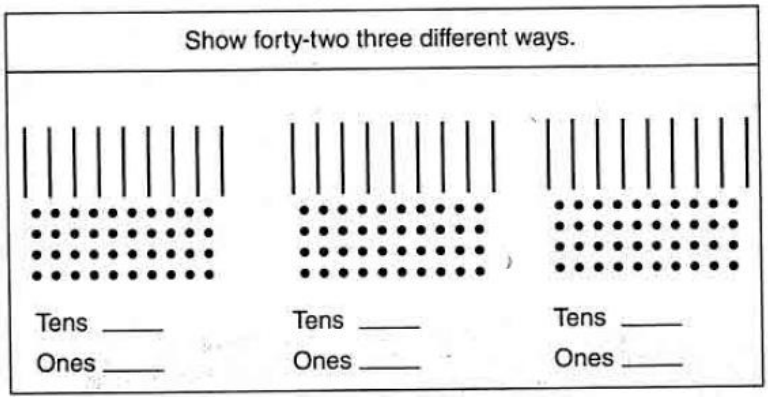
Materials: place value disks or base ten materials, student recording sheet.

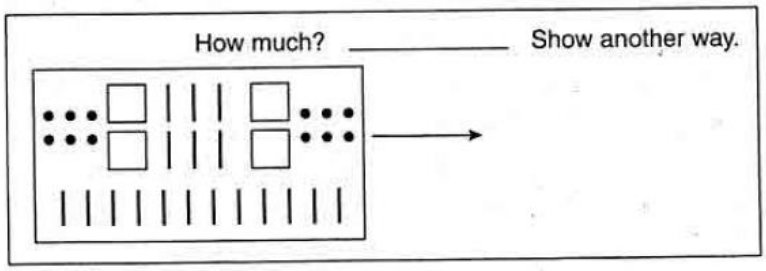
If you teach second grade, equivalent representations for hundreds as groups of tens can help children with the concept of a hundred as 10 tens.

1. Children work in groups or pairs. First they show a three-digit number on their desks with base ten materials in the standard representation.
2. They find and record at least three other ways of representing this quantity.

Variation - Challenge children to find a way to show an amount with a specific number of pieces. For example, ask children to show 463 with 31 pieces.

After children have had sufficient experiences with pre-grouped materials, a semi-abstract "dot-stick-square" notation can be used for recording ones, tens, and hundreds. Use the drawings as a means of telling the children which type of pieces to use to solve problems and also as a way for children to record results.





Participant/Student Activity: Base-Ten Riddles (2.NBT.7)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

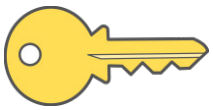
Base ten riddles can be presented orally or in written form. In either case, children should use base ten materials to help solve the riddles. After children solve the riddles, they can write new ones.

Take a few minutes and solve each of the following:

1. I have 23 ones and 4 tens. Who am I? _____
2. I have 4 hundreds, 12 tens and 6 ones. Who am I? _____
3. I have 30 ones and 3 hundreds. Who am I? _____
4. I am 45. I have 25 ones. How many tens do I have? _____
5. I have 13 tens, 2 hundreds, and 21 ones. Who am I? _____
6. If you put 3 more tens with me, I would be 115. Who am I? _____
7. I have 17 ones. I am between 40 and 50. Who am I? _____ How many tens do I have? ____

Reflection

Why would this activity be beneficial to students?



KEY IDEA #4

The groupings of ones, ten, and hundreds can be taken apart in different but equivalent ways.

Oral and Written Names for Numbers

It is vital that we give opportunities to help children connect oral and written names for numbers with their emerging base ten concepts of using groups of ten or one hundred as efficient methods of counting.

NOTE: The ways we say and write numbers are convention, not concepts. Children must learn these by being told rather than through problem-based activities.

Two-Digit Numbers

In kindergarten and first grade, children need to connect the base ten concepts with the oral number names they have used many times. They know the words but may have not thought of them in terms of tens and ones. In fact, early on they may want to write twenty-one as 201.

Almost always use base-ten models while teaching oral names. Initially rather than use standard number words, use the more explicit base-ten language (i.e. “4 tens and 7 ones” instead of “forty-seven”) Base-ten language is rarely misunderstood. When it seems appropriate, begin to pair base-ten language with standard language.

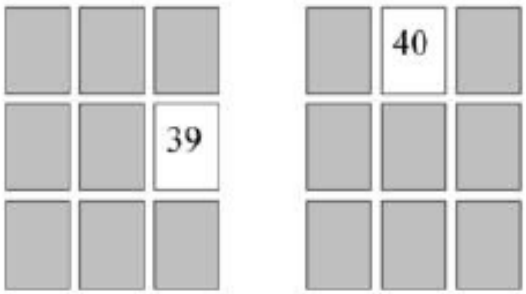
Emphasize the teens as exceptions. Acknowledge that they are formed “backward” and do not fit the patterns.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Student Task: Crossing the Decade Concentration

You will need to create a set of number cards for each of the pair of numbers that cross the decade, i.e., 19 and 20, 29 and 30, 39 and 40, 49 and 50, etc.

Students place all the number cards that end with "_9" face down in an 3x3 array on the left and all the number cards that end with "_0" face down in a 3x3 array on the right. Working in pairs or trios, students take turns. The first student selects a card from the left array, stating the number name and the counting number that follows ("I have 39, I need 40").



He or she then picks one card from the array on the right (the "_0" numbers), hoping to find the target number. If the student does not find a pair, both cards are replaced face down in their original spots. It is now the second student's turn to choose a card from the "_9" array and to try to find the appropriate "_0" card. Students should try to remember where each number is located. (The game is called "Concentration" not "Guessing.")

When a student finds a matching pair he or she keeps that pair of cards. Play continues until all cards have been matched. The student with the most matched pairs wins.

IM Commentary

- One of the most common areas that young children struggle with when learning to count forward is crossing from one family to the next, for example getting to 29 in the counting sequence and not knowing what comes next or stating a random decade number. This game supports student development in this area. Students should have beginning knowledge of the counting sequence beyond the "teen" numbers before playing this game.
- This game can be introduced whole group on the board by making slightly larger cards and using a sentence pocket chart (or magnetic tape on a magnetic board) to arrange the cards in two arrays face down against the board. The teacher then plays against the rest of the class, modeling the process of picking from the left to begin, stating the number name and the number after and then picking from the right.
- It is very important to train the students to draw a card from the left and state what they need before they draw from the right. This will encourage them to think about and problem solve the next decade number. When the students get in the habit of picking up two cards simultaneously the game become much more about luck (although they do have to confirm that it is a pair, so do get some practice) and students are less likely to internalize the information and use it when counting.
- Students who become proficient with playing the game to support counting forward can gain experience in backward counting by picking from the right array first (the "_0" numbers) and then looking for the correct "_9" number. Changing the cards to 20-21, 30-31, 40-41, 50-51 etc with the "_0" numbers on the left and the "_1" numbers on the right is very supportive for another common error in backward counting which is to leave out the decade number when counting backward ie. "33, 32, 31, 29, 28".

Solution

Cards will be matched 19-20, 29-30, 39-40, 49-50, 59-60, 69-70, 79-80, 89-90, 99-100. This game can be modified when playing for the first time or for a struggling student by only making cards up to 50 and making 2 of each pair.

https://www.illustrativemathematics.org/content-standards/tasks/405

Student Activity: Counting Rows of 10 (1.NBT.2)

- Van de Walle, Lovin, Karp, & Bay-Williams., 2006

1. Using a 10 X 10 array of dots on a projector, cover up all but two rows.

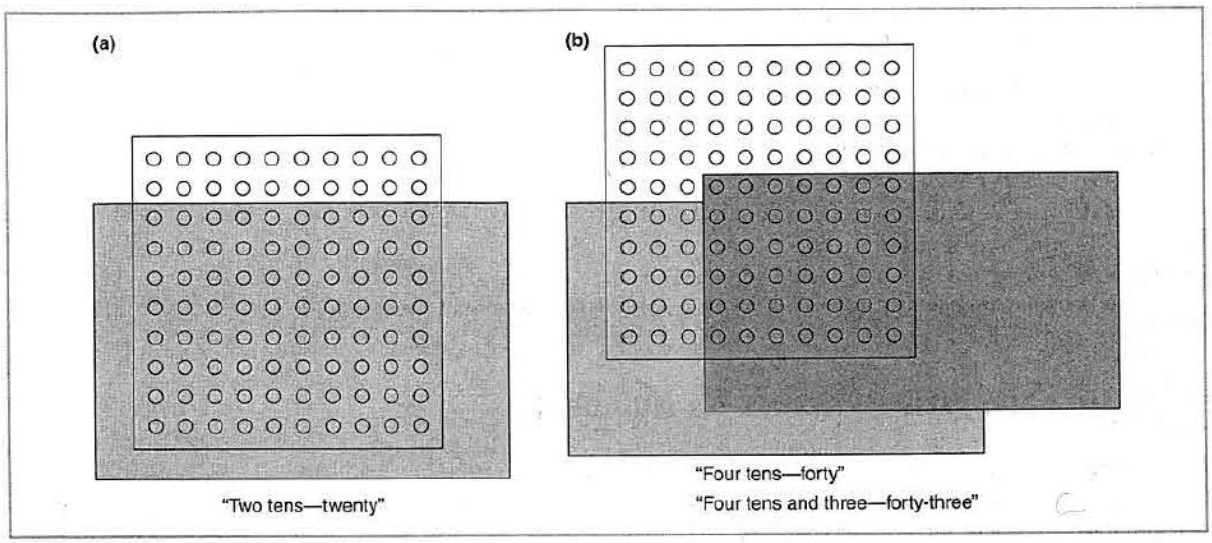
2. Ask, " How many tens? (2) Two tens is called twenty." Have the class repeat.

3. Show another row, and continue in this manner of questioning. Three tens is thirty. Four tens is forty. Five tens could have been fivety but is just fifty. The names sixty, seventy, eighty and ninety all fit the pattern.

4. Slide the cover up and down the array, asking how many tens and the name for that many. Some students may not hear a difference in numbers such as 50 and 15 or 60 and 16, so explicitly compare these words and clearly enunciate and even overemphasize the word endings.

5. Use the same 10 X 10 array to work on names for tens and ones. For example, show four full lines then expose one dot in the fifth row. Say "four tens and one. Forty-one."

6. Add more dots one at a time. "Four tens and two. Forty-two." When the pattern is established, repeat with other decades from 20 through 90.



Student Activity: Counting with Base Ten Models (1.NBT.5)

- Battista, M., 2012

Materials: Base ten materials (tens and ones) OR groupable tens and ones materials

1. Show some ten pieces on a project or carpet in a mixed arrangement.
2. Ask “How many tens?”
3. Add a ten or remove a ten and repeat the question.
4. Add some ones.
5. Always have the children give the base ten name and the standard name.
6. Continue to make changes in the materials displayed by adding or removing one or two tens and by adding or removing ones.

Avoid the standard left to right order for tens and ones so the emphasis is on the names of the materials not the order they are in.

Reverse the activity by having children use base ten blocks at their desks. You say a number - “Make 63”. The children make the number with their materials and then give the base ten name and standard name.

Two-Digit Numbers

Student Activity: Tens, Ones, and Fingers (1.NBT.2)

- Burns, Marilyn. *About Teaching Mathematics: A K-8 Resource*, 1998

Materials: Students

1. Ask the class, “Can you show 6 fingers (or any amount less than 10)?"

2. Ask, “How can you show 37 fingers?” Some children will figure out that at least 4 children are required.

3. Line up four children, have three of the children hold up 10 fingers while the fourth child holds up 7. Have the class count the fingers by tens and ones.

4. Ask other children to show different numbers.

5. Emphasize the number of sets of 10 fingers and the single fingers (base-ten language) and pair this with standard language.

NOTE: It is important occasionally to count an entire representation by ones. Remember that counting by ones is the young child’s principle linkage with the concept of quantity. For example, suppose you just had children use linking cubes to make 35. Try asking, “Do you think there are really 36 blocks there?” Many children may not be convinced, so the counting by ones is very significant.

Three Digit Number Names (2.NBT.1 & 2.NBT.3)

The approach to three-digit number names is essentially the same as for two digit names.

- Show mixed arrangements of base-ten materials
- Have children write the base-ten name and the standard name.
- Vary the arrangements for one example to the next by changing only one type of piece.
- Have children at their desks model numbers that you give to them orally using the standard names.

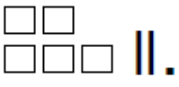
Student Activity: Make Six Numbers (2.NBT.3)

Make Six Numbers

Materials: numeral cards (1-9)

1. Turn over three cards.
2. Use all three cards to make **six** different numbers.

   Example: 5 1 2 512, 521, 152, 125, 215, 251
3. Record your numbers.
4. Order the numbers from least to greatest.
5. Show each number using base ten blocks, in word form, and in expanded form.

   Example: 521  five hundred twenty one 500 + 20 + 1

Retrieved from http://www.k-5mathteachingresources.com/support-files/make-6-numbers.pdf

Student Activity: Subtract 10 and 100 (2.NBT.8)

Subtract 10 and 100

5 3 6	536 - 10 = 526 536 - 100 = 426
6 3 5	635 - 10 = 625 635 - 100 = 535

Materials: set of numeral cards (0-9)

1. Work with a partner. Shuffle the cards and place them facedown. Turn over three cards to create a 3-digit number.
2. Subtract 10 from the number and record the equation. Subtract 100 from the original number and record the equation.
3. Change the order of the three cards to create a different 3-digit number. Subtract 10 and record. Subtract 100 and record.
4. Change the order of the three cards again to create a different 3-digit number. Subtract 10 and record. Subtract 100 and record.
5. Repeat steps 1-4 with a different set of three cards.

The difference between ___ and 10 is ___.
The difference between ___and 100 is ___.

___ minus 10 equals ___. ___ minus 100 equals ___.

Retrieved from http://www.k-5mathteachingresources.com/support-files/subtract-10-and-100.pd

Student Task: Looking at Numbers Every Which Way Task

a. 127 is a number.

- Write it as a sum of 100's, 10's, and 1's.
- Write its name in words.
- Draw a picture to represent the number.
- Locate it on the number line.

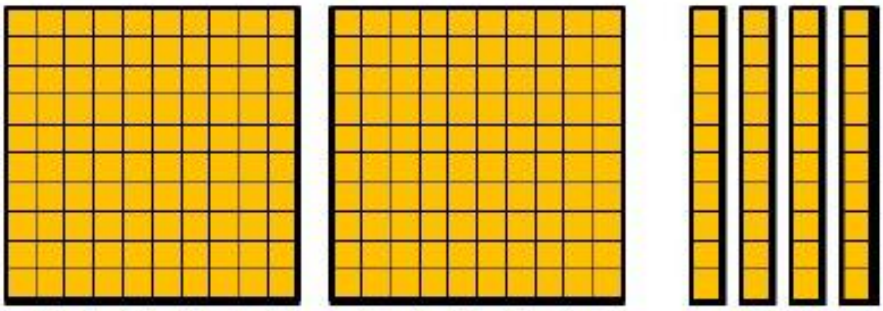
b. 500+60+8 is a number.

- Write it as a three-digit number.
- Write its name in words.
- Draw a picture to represent the number.
- Locate it on the number line.

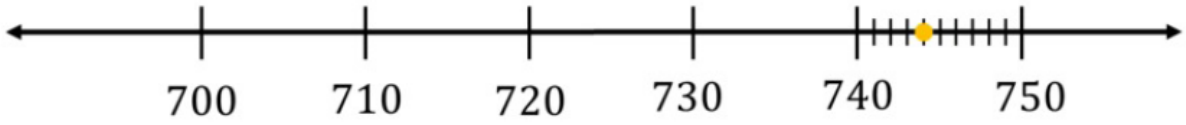
c. Six hundred and nine is a number.

- Write it as a three-digit number.
- Write it as a sum of 100's, 10's, and 1's.
- Draw a picture to represent the number.
- Locate it on the number line.

d. The picture represents a number. The big square represents 100, the rectangle represents 10, and the small square represents 1.



e. A number is shown on the number line.



- Write it as a three-digit number.
- Write it as a sum of 100's, 10's, and 1's.
- Write its name in words.
- Draw a picture to represent the number.

IM Commentary

This task gives students the opportunity to work with multiple representations of base-ten numbers. The standard 2.NBT.3 asks students to read and write numbers to 1000 using base-ten numerals, number names, and expanded form. This task addresses all of these and extends it by asking students to represent the numbers with pictures and on the number line, which supports the understanding described in 2.NBT.1. Students who are still grappling with the meaning of base-ten numerals might benefit from having base-ten blocks on hand. Attached is a black line master for a place-value mat which can help scaffold students who are having trouble. Eventually, students should be able to do this task without concrete representations, however.

http://s3.amazonaws.com/illustrativemathematics/attachments/000/007/755/original/public_task_1236.pdf?1442499271

Discuss with your group:

What makes this a high-level task?

What can a teacher learn about his/her students' understanding of place value when looking at student responses?

Written Symbols

Place value mats are simple mats divided into two or three sections to hold ones and tens or one, tens, and hundreds pieces. You can suggest to children that the mats are a good way to organize their materials when working with base-ten blocks.

Assessment Note

Be aware of how easy it is for a child to show a number on a mat using base-ten blocks and learn to write the number without any understanding of what the number represents. First and second grade textbooks often show a picture of base ten materials and have children record numbers in this manner:

7 tens and 3 ones is 73 in all.

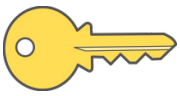
Reflection

- What strategies do you use to avoid this problem and to help show true understanding?

Here is one idea to the address the issue:

As children use their place-value mats, they can be shown how the left-to-right order of the pieces is also the way that numbers are written. To show how numbers are built, use the Place Value (or Hide Zero) cards (Module 2) and as children place the materials of a number (i.e., 457) on the mat they also place the matching card (400, 50, and 7) below the materials. Then, starting with the hundreds card, layer the others on top, right aligned.

This approach will show how the number is built while allowing children to see the individual components of the number. This is especially helpful when there are zero tens. The place value mat and the matching cards illustrate the important link between the base ten models and the written form of the numbers.



KEY IDEA #5

Children's ability to label the tens place and the ones place or to count by tens does not guarantee they understand that one ten is the same as ten ones.

Patterns and Relationships with Multi-digit Numbers

The Hundreds Chart

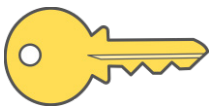
We want our children to look at patterns in our number system and how numbers are related. In particular, we are interested in helping children develop an understanding of the relationships of all numbers to special numbers, called "benchmark numbers". These ideas begin to provide a basis for computation.

In first and second grades, children can use the hundreds chart to develop base ten understanding, noticing that jumps up or down are jumps of ten, while jumps to the right and left are jumps of one.

Student Activity: Finding Neighbors on the Hundreds Chart (precursor to 1.NBT.4)

Conklin, M & Sheffield S. *It Makes Sense! : Using the Hundreds Chart to Build Number Sense*, 2010

1. Begin with a blank or nearly blank hundreds chart. Circle a particular missing number.
2. Children are to fill in the designated number and its “neighbors”. This can be done with a full class or with blank hundreds chart worksheets. It is important to use a “think aloud” to describe what key features of the numbers you think about as you determine the missing number and its neighbors. If there are children in your class who are ready, allow them to share their strategies instead of you.
3. After children become comfortable naming the neighbors of a number, ask what they notice about the neighboring numbers. What about those numbers on the diagonal?
4. By discussing these relationships on the chart, children begin to see how the sequence of numbers is related to the numerical relationships.



KEY IDEA #6

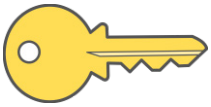
There are patterns to the way numbers are formed.

Student Activity: Models with the Hundreds Chart (1.NBT.4)

Conklin, M & Sheffield S., 2010

Many children at the first grade level will not understand the corresponding numeric relationships such as those discussed in the previous activity. In this activity number relationships on the chart are made more explicit by including the use of base ten materials.

1. Use any base-ten model for two-digit numbers with which the children are familiar. The little ten frames are recommended.
2. Give children one or more numbers to first make with the model and then find on the chart. Use groups of two or three numbers either in the same row or the same column.
3. Ask children, "How are the numbers in the row (or column) alike? How are they different?" To emphasize place value ask them "What place value is changing?"
4. Indicate a number on the chart. Ask children, "What would you have to change to make it into each of its neighbors?"



KEY IDEA #7

The positions of digits in numbers determine which value they represent.

Student Activity: The Thousands Chart (used to begin work with 2.NBT.2 and 2.NBT.3)

Van de Walle, Lovin, Karp, & Bay-Williams., 2006

It is also helpful for children to have a chart that extends to 200, even in the first grade. Perhaps a more powerful idea is to extend the hundreds chart to 1000.

1. Provide children with several sheets of the blank hundreds chart.
2. Assign groups of three or four children the task of creating a 1 to 1000 chart.
3. The chart is made by taping 10 blank hundreds charts together in a long strip.
4. Children should decide how they are going to divide up the task of filling in the chart with different children working on different parts of the chart.

The thousands chart should be discussed as a class to examine how numbers change as you count from one hundred to the next, what the patterns are, and so on. In fact, the earlier hundreds chart activities can be extended to a thousands chart.

Video: Lesson Idea—Trash Can Game

https://www.teachingchannel.org/videos/second-grade-math-lesson

Video Notes

Small Group Discussion:

- How does the repetition in the game allow for practice without redundancy?

- In what ways does the partner and whole group work help to scaffold learning?

- How are students developing oral language skills while playing either game?

Participant Activity: Play the Trash Can Game

Trash Can Game

_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)	_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)
_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)	_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)
_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the least value.)	_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the least Value.)
_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the least value.)	_____ _____ _____ _____ Trash Can _____ _____ (Try to make the number with the least Value.)

101 and Out

Player 1_____

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Total

Player 2_____

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Total

Relationships with Benchmark Numbers

One of the most valuable features of both the hundreds chart and the little ten frame cards is how clearly they illustrate the distance to the next multiple of 10. Multiples of 10, 100, and occasionally other special numbers such as multiples of 25 are referred to as benchmark numbers. Children might learn to use this term as they work with informal methods of computation. No matter the terminology used, understanding how numbers relate to these special numbers is an important step in children's development of number sense and place value understanding.

In addition to the hundreds chart, the number line is an excellent way to explore these relationships.

Participant/Student Activity: Who am I? (2.MD.6)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

1. Sketch a line (or use a piece of cash register tape) and label 0 and 100 at opposite ends.
2. Mark a point with a "?" that corresponds to your secret number.
3. Children use estimation to try to identify your secret number.
4. For each estimate, place and label a mark on the line.
5. Continue marking each estimate until your secret number is discovered.
6. Have children explain how they are making their estimates. Highlight the use of any benchmark numbers in their estimations.
7. As a variation, the endpoints can be numbers other than 0 and 100. For example, try 0 and 1000, 200 and 300, or 500 and 800.

Participant/Student Activity: Who could they be? (2.MD.6)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

1. Label two points on a number line (not necessarily the ends) with benchmark numbers.
2. Show children different points labeled with letters. Ask what numbers these might be and why the children think that.
3. Question students about where other numbers might be (other than the letters), or how far apart do you think A and D are? Do you think A is more than 100?

Student Activity: Ordering Numbers Task

Malik is given a list of numbers:

$$1 \qquad 5 \qquad 10 \qquad 50 \qquad 100$$

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right):

$$49, 7, 22, 98, \text{ and } 3$$

Where in the list should he put each of these numbers?

IM Commentary

The purpose of this task is to give students an opportunity to compare numbers less than 100 to benchmark numbers. Even though a number line is not explicitly given in the task, it is useful for students to list the numbers in the order they would appear on the number line; this allows them to focus on the relative ordering without worrying about the exact placement on the number line. Students should also have a chance to do similar tasks with a number line.

1.NBT.3 asks students to record comparisons with the symbols $>, =$ and $<$. While this task does not ask for this, it helps build up an understanding of the relative magnitude of numbers which is a precursor to proficiency with the more abstract symbolic comparisons.

https://www.illustrativemathematics.org/content-standards/tasks/6

Discussion

How can you adapt this task for second grade?

Closing Activity

Connections to Real–world Ideas

As children study place value concepts, encourage them to see numbers in the world around them. Children in the first grade should be thinking about numbers to 100 and second graders should be thinking about numbers up to 1000.

Reflection

- Where are numbers like this?

Module Reflection

Think back to the areas we have explored concerning Place Value and Base Ten Concepts:

- Counting by Ones & Tens with Models and Words
- Base-Ten Grouping (Ones making Tens, Tens making Hundreds)
- Equivalent Representations
- Oral & Written Names for Numbers
- Patterns & Relationships with Multi-digit Numbers
- Relationships with Benchmark Numbers
- Connections to the Real World

Although we have looked at a variety of activities, this is not an exhaustive list.

Reflection

- What other activities do you use in your classroom to promote Place Value and Base Ten Concepts?

Discuss these strategies in your small group. Be prepared to share whole group. Record any ideas that you would like to incorporate in your classroom in the following pages.

Collaborative Notes/Activities – Place Value & Base Ten