TN Department of Education

Module 2:

Developing Early Number Concepts and Number Sense

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Objective

- To introduce a scope and sequence of early number concepts and number sense. This module will look at the development of number ideas for numbers up to 20.

Tennessee Standards

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
2.OA.4 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

TEAM Alignment

- Standards and Objectives
- Presenting Instructional Content
- Activities and Materials
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Participant Activities:

- Participants will read, reflect, and discuss ways to build number sense.
- Participants will learn classroom activities for subitizing (spatial relationships) and building understanding of number relationships within sums of 20.
- Participants will learn activities to help students develop conceptual and procedural understanding of part-part-whole relationships and patterns within 100.

Early Counting

"No matter what prior experiences children have had before coming to school, we need to strive to help all children develop the following four interrelated aspects of early numerical knowledge."

Clements & Sarama, 2009

4 Aspects of Early Numerical Knowledge

- Number Sequence: the names and the ordered list of number words.
- One to one correspondence: counting objects by saying number words in a one to one correspondence with the objects.
- Cardinality: understanding that the last number word said when counting tells how many objects have been counted.
- Subitizing: quickly recognizing and naming how many objects are in a small group without counting. Young children can recognize and name quantities of objects that are less than four without counting.

Clements & Sarama, 2007

Assessment Note

Children have opportunities to develop the four aspects of early numerical knowledge within a smaller set of numbers in pre-K and kindergarten. For children who struggle with grade 1 and grade 2 math content, it is important to assess their knowledge of these four aspects as they may have unfinished learning in those foundational aspects from pre-K and kindergarten. This is when the RTI^2 model will help students fill in the gaps in their foundations through intervention at the Tier II or Tier III level.

Developing Number Sense by Building Number Relationships

Video: Number Sense

https://www.youtube.com/watch?v=Jeel4Qjow4s

As you view the Number Sense video, record your key takeaways below:

Number Sense is not a set of skills that children can develop in a short period of time. It is something that grows and develops over time. These foundational ideas can all be extended to larger numbers, operations, basic facts, and computation.

> "Number sense is a good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms."

Howden, 1989

Turn and Talk

- What are your thoughts on the video and two quotes above?

Relationships between Numbers 1 through 10

Once children have acquired the first three aspect of Early Numerical Knowledge (Number Sequence, One to One correspondence, and Cardinality), too often teachers move directly from the beginning ideas of counting to addition and subtraction, leaving children with a very limited collection of ideas about numbers to bring to these new topics. The result is that often children continue to count by ones to solve simple story problems and have difficulty mastering basic facts. Emphasizing number relationships is key to helping children fully develop number sense.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Small Group Discussion

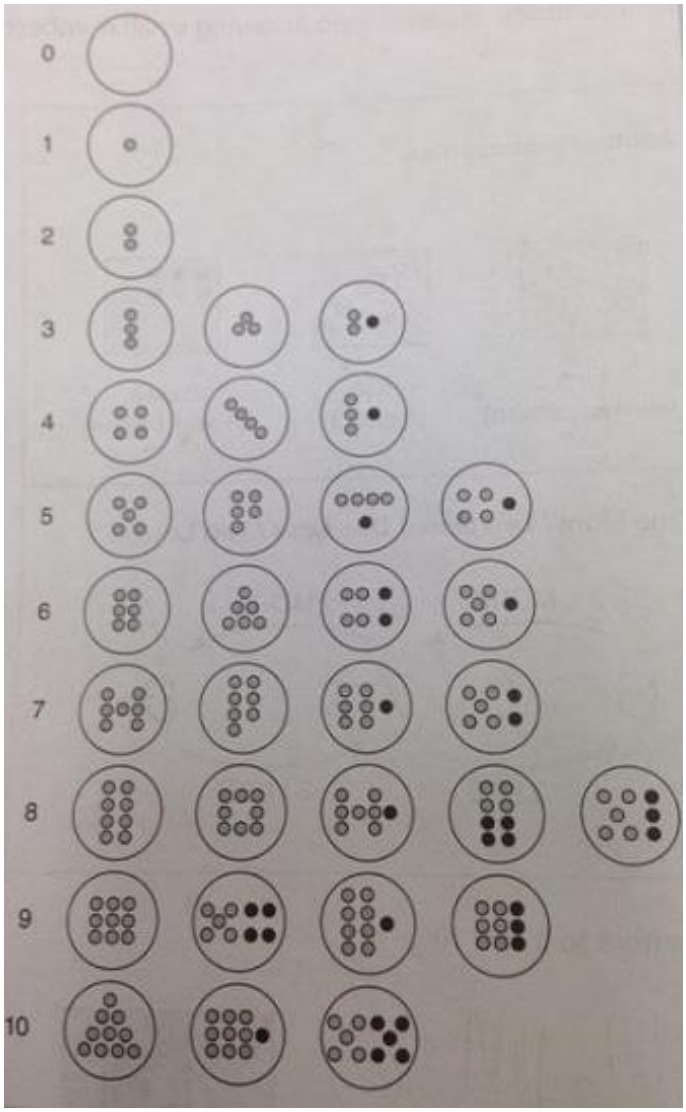
- What does the above paragraph say is the result of children moving too quickly from the beginning ideas of counting straight to addition and subtraction?

- Reflecting on your own practices in the classroom, what is your response to the above paragraph?

Spatial relationships

Before moving from the concepts of counting into addition and subtraction concepts, students need opportunities with the fourth aspect of early numerical knowledge, subitizing. Children can learn to recognize sets of objects in patterned arrangements and tell how many without counting. The figure below shows a set of "dot plates" that can be used to promote subitizing.

DOT PLATES



- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Participant Activity: Make a set of dot plates

Using the materials provided, construct your own set of Dot Plates by choosing five images from the dot plate graphic. These will be used in the activities throughout this professional development and in your classroom. Pay close attention to the coloring of various patterns.

Classroom Activities to Promote Subitizing (Spatial Relationships)

Student Activity: LEARNING PATTERNS

Materials: 10 counters and piece of cardstock per child

To introduce the patterns, provide each child with about 10 counters and a piece of cardstock or paper as a mat.

1. Hold up a dot plate for about 5 seconds and say, "Make the pattern you saw on the plate using the counters on the mat. How many dots did you see? What did the pattern look like?
2. Spend some time discussing the configuration of the pattern and the number of dots.
3. Show the plate so they can self-check.
4. Continue with a few patterns each day.

Van de Walle, Lovin, Karp, & Bay-Williams. *Teaching Student Centered Mathematics*, 2014

Student Activity: DOT PLATE FLASH

1. Hold up a dot plate for only 1-3 seconds and say, “How many dots do you see? What did the pattern look like?”
2. Include easy patterns first and then add more dots as children’s confidence builds.
3. Initially, you may need to show a plate a second time so that children can get a second look.
4. Children like to see how quickly they can recognize and say how many dots.
5. Children can also flash dot plates to each other as a workstation activity.

Van de Walle, Lovin, Karp, & Bay-Williams. *Teaching Student Centered Mathematics*, 2014

Video: Subitizing Activity

https://www.youtube.com/watch?v=3UYn0w-uhZg

Video Notes:

Reflection

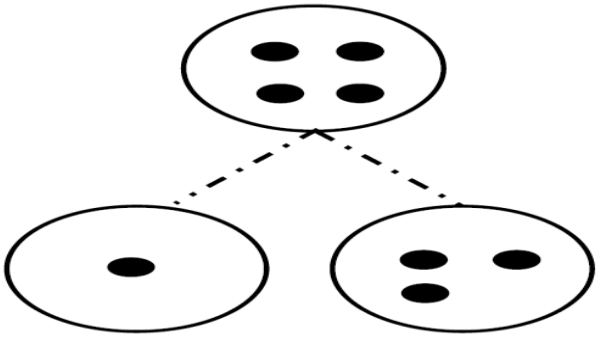
What do you notice about this video and how the images are displayed?

How do you these activities help promote mathematical understanding?

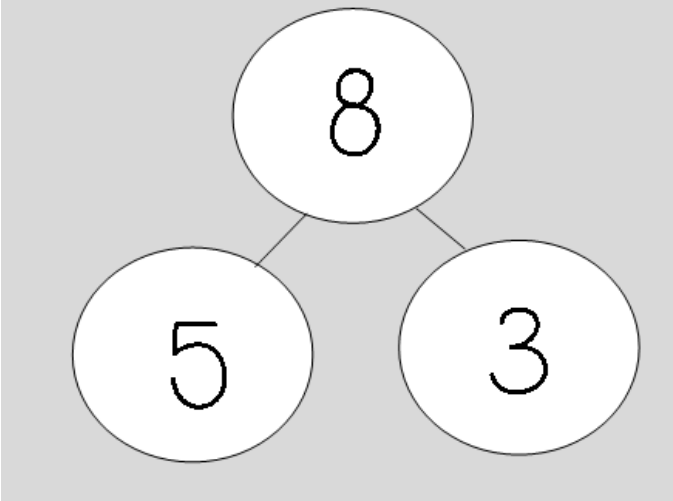
How subitizing activities develop number sense

Journey of the Number Bond

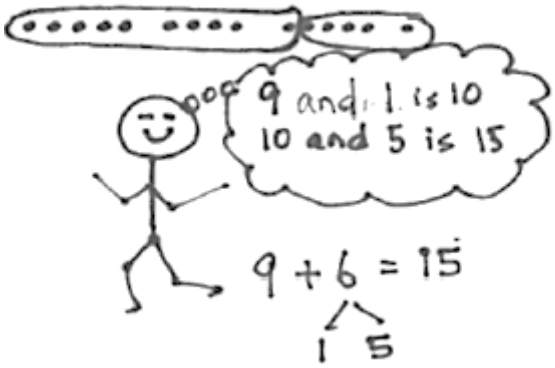
As students become more familiar with dot patterns, teachers may begin to make connections to Number Bonds. A students' ability to understand and manipulate number bonds supports their long-term development of number sense.



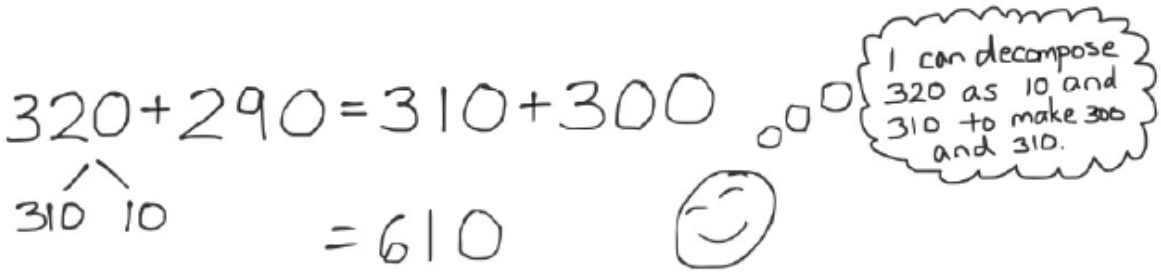
Number Bond with Dot Pattern



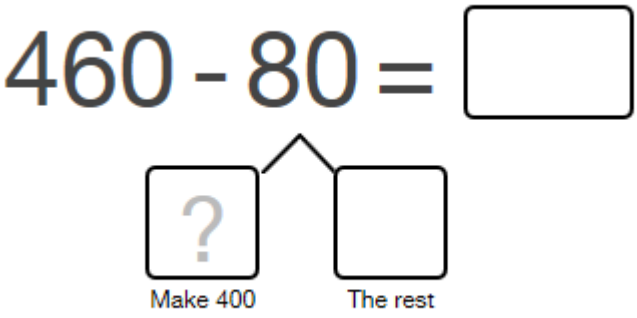
Traditional Number Bond



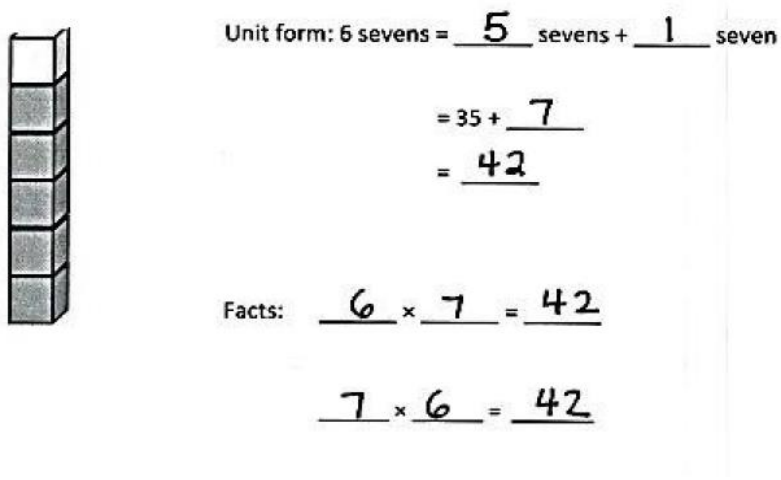
Adding to Make 10



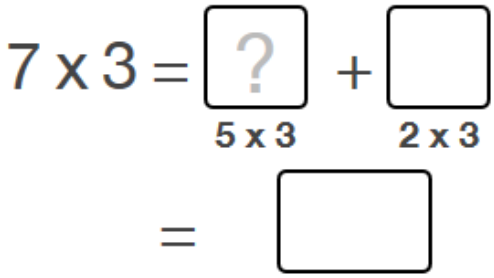
Adding to Make Hundreds



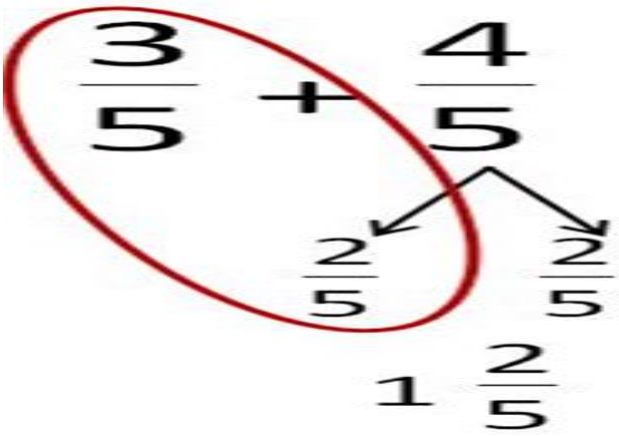
Subtraction



Multiplication (Pictoral)



Multiplication



Adding Fractions

Reflection

How can a students' lack of ability to see a number as a sum of two parts hinder their mathematical understanding?

Reflect on the practices in your district, school, and classroom. Is composition and decomposition of numbers a focus in the early grades? Why or Why not?

Making Number Relationships More Explicit

One and Two More, One and Two Less

When children count, they have no reason to reflect on the way one number is related to another. To learn that 6 and 8 are related by the twin relationships of "two more than" and "two less than" requires reflection on these ideas within tasks that promote counting. Counting on (or back) one or two counts is a useful tool in constructing these ideas.

Student Activity: ONE-LESS-THAN DOMINOES (1.OA.5)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot set dominoes

1. Students play the game in the usual way, but instead of matching ends, a new domino can be added if it is one less than the domino at the end of the board.
2. A similar game can be played for two less, one more, or two more.

Student Activity: MAKE TWO-MORE-THAN SET (1.OA.5)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot Cards

1. Provide children with about six dot cards.
2. For each card, their task is to display a set of counters that is two more than the set shown on the card.
3. Spread out eight to ten cards and allow children to form their own sets.
4. Children should read their set as "Two more than _____ is _____"

Note: You can make a set of dominoes out of cardstock and put a dot pattern on each end. The same patterns can appear on lots of dominoes with different pairs making up one set.

Templates to create paper sets of dominoes can be found here:
https://sites.google.com/site/get2mathk5/home/templates-graphic-organizers

Anchoring Numbers to 5 and 10

The knowledge of 8 as “5 and 3 more” and as “2 away from 10” can play a role in how a child thinks about these examples. Later, similar relationships can be used in the development of mental computation skills on larger numbers, such as 68 + 7.

The most common models to help children anchor numbers to 5 and 10 are five frames and ten frames. Most first and second grade classrooms will use predominantly the ten frame. For a while, many children will count every counter on their ten frame. When making a new number, some children will remove all the counters from the ten-frame and begin from a blank frame. Others will soon learn to adjust numbers by adding on or taking off only what is required, often capitalizing on a row of five without counting. DO NOT pressure children to use one approach or another. With continued practice, all children will grow. How they are using the ten-frame provides insight into children’s current number concept development.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Student Activity: TEN FRAME FLASH CARDS (1.OA.6)

Conklin, Melissa. *It Makes Sense! : Using Ten Frames to Build Number Sense*, 2010

1. Flash ten frame cards to the class or group and see how quickly the children can tell you how many dots are shown.
2. Variations:
    i. Saying the number of empty spaces on the card instead of the number of dots.
    ii. Saying one more than the number of dots (or two more and also less than)
    iii. Saying the “ten fact”, for example “six and four make ten”

Note: Ten frame tasks are surprisingly challenging for children. Children must reflect on the two rows of five, the spaces remaining, and how a particular number is more or less than 5 and how far away from 10.

Literature Connection: **My Little Sister Ate One Hare** by Bill Grossman
A ravenous little sister eats everything from one hare to ten peas.

1st Grade: Students can work collaboratively to find all of the different combinations the sister could eat to total 10 items.
2nd Grade: Students are given the task to find out how many critters in all did the sister eat?
Extension Activity: Ask students how many more disgusting creatures the little sister would have to eat to reach 100 or more. Have them continue with the pattern of 11 creatures, then 12, and so on.

Number Talks with Ten Frames

Another way to utilize ten frames is through number talks. By asking students to come up with a variety of ways to prove "how many" are on each ten frame, students hear various solution paths and learn additional strategies beyond just counting all.

Video: Ten Frame Number Talk

https://www.youtube.com/watch?v=PH5RG4zmmHE

https://www.youtube.com/watch?v=zdE_lxSC3V8

Note: We learned about Number Talks in a previous TDOE training. Some of the resources from that training are included in the Module 2 Appendix.

Small Group Discussion

As you watch the video clip, think about the following questions. Participants will discuss their thoughts with their small group.

- How does the teacher build student fluency with small numbers?

- What questions does the teacher pose to build understanding?

- What strategies are the students using to build meaning of the numbers?

- What opportunities are created for the students to begin building an understanding of ten?

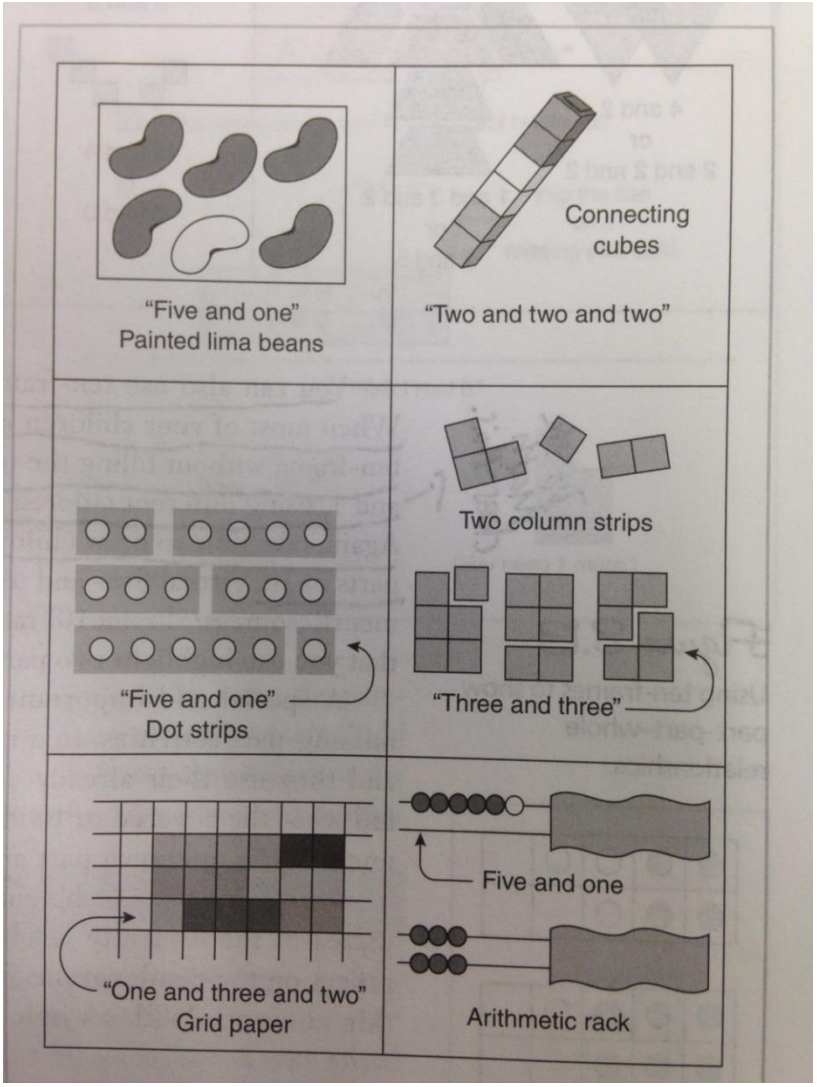
- How does the teacher support student communication during the number talk?

Part-Part-Whole Relationship

Focusing on a quantity as a sum of its parts has important implications for developing number sense. The ability to think about a number in terms of its parts is a major milestone in the development of number sense.

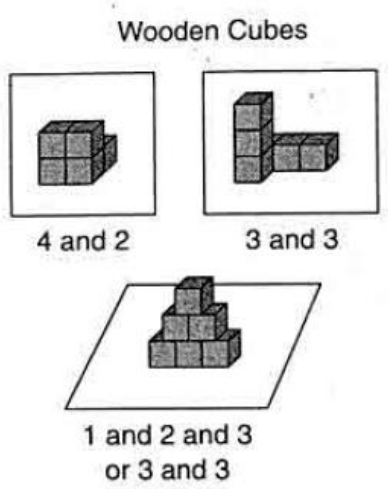
Of the four number relationships we have explored, part-part-whole ideas are easily the most important. It is not unusual to find children in the second grade who have not developed firm part-part-whole constructs for the numbers 7 through 12, even though by that time they should be adding up to 10.

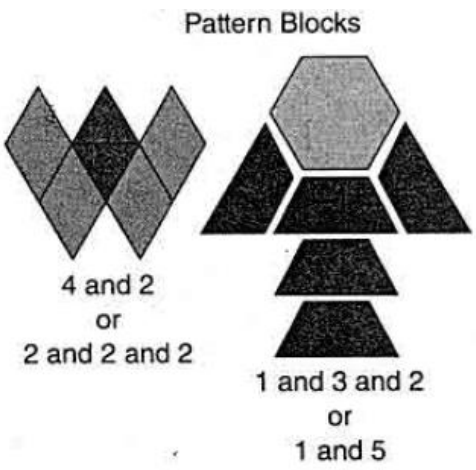
ASSORTED MATERIALS TO BUILD A NUMBER IN TWO PARTS



- Van de Walle, Lovin, Karp, & Bay-Williams. *Teaching Student Centered Mathematics*, 2014

You may add interest and vary the activity by adding a design component. Rather than creating a two-part illustration, children can create an interesting design with an assigned number of elements.





- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Literature Connection:

MATH-terpieces: The Art of Problem Solving by Greg Tang

Greg Tang underscores the importance of four basic rules in problem-solving: Keeping an open mind, looking for unusual number combinations, using multiple skills (like subtracting to add) and looking for patterns. The book involves putting subsets of objects together to make a given sum. The following task can be used.

Ballet Shoes: Find three ways to make a 7.
Lilies: Find four ways to make an 8.
Umbrellas: Find five ways to make a 9.
Peaches: Find five ways to make a 10.
Swirling Stars: Find four ways to make a 7.
Purple Spots: Find six ways to make an 8.
Fish: Find six ways to make a 9.
Eyes: Find six ways to make a 10.
Squares: Find eight ways to make a 7.
Clocks: Find seven ways to make an 8.
Spatters: Find seven ways to make a 9.
Soup Cans: Find ten ways to make a 10.

Retrieved from http://letsreadmath.com/wp-content/uploads/2013/03/March_Teacher_Directions_2-28.pdf

Missing–Part Activities

A special and important variation of part-part-whole activities is referred to as missing-part activities. In a missing-part activity, children are given the whole amount as they use their already-developed knowledge of the parts of that whole to try to tell what the covered or hidden part is. If they do not know or are unsure, they simply uncover the unknown part and say the full combination as they would normally.

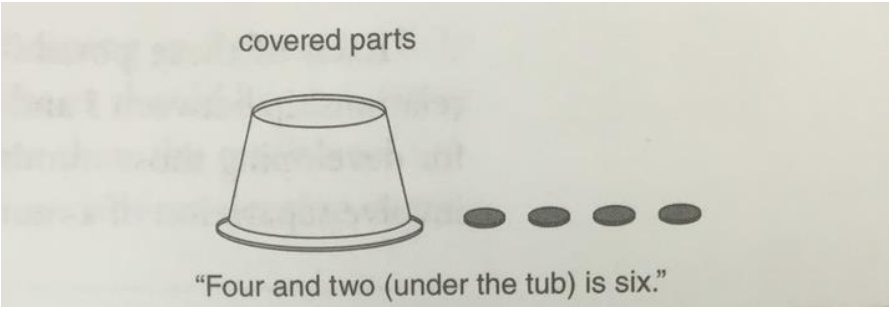
Missing-part activities can be challenging for children not only because the missing part increases the difficulty level, but also because they encourage children to continue to reflect on the combinations for the number. They also serve as a precursor to subtraction concepts. With a whole of 8 but with only 3 showing, the child can later learn to write "8-3=5."

Student Activity: COVERED PARTS (1.OA.8)

Retrieved from www.strategicinterventionsolutions.org

Materials: Counters, Cups

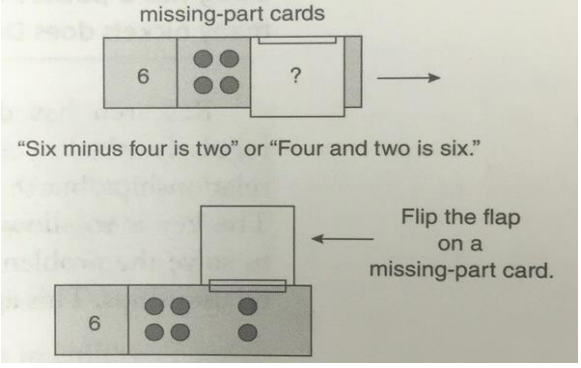
1. A set of counters equal to the target amount is counted out, and the rest is put aside.
2. One child places the counters under a solid cup and then pulls some out into view. (This amount can be none, all, or any amount in between.)
3. The partner says the two parts that makes the whole. For example, "Four and two is six."
4. If there is a hesitation or if the hidden part is unknown, the hidden part is immediately shown.



Student Activity: MISSING-PART CARDS (1.OA.8)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

1. For each number from 4 to 10, make missing part cards using strips of cardstock measuring 3 X 9 inches (this will ensure that a sticky note will fit over it).
2. Each card has a numeral for the whole and two dot sets with one set covered by a sticky note.
3. Children use the cards by saying the two parts that make a whole, such as "four and two is six".



Student Activity: I WISH I HAD (1.OA.8)

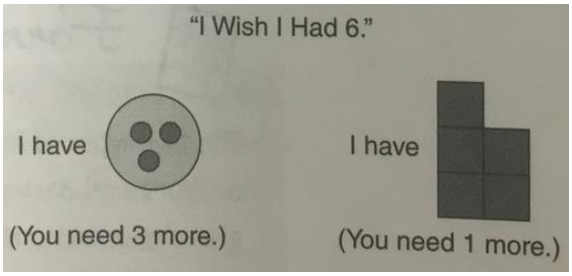
Retrieved from www.strategicinterventionsolutions.org

Materials: Dot Plates, Counters

1. Hold out a dot plate, bar of connecting cubes, or other manipulative and say "I wish I had ____."
2. Students will show the amount of counters needed.

Example: Plate shows 6; "I wish I had nine";

three counters would be shown.



Student Activity: NUMBER SANDWICHES (1.OA.6)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot Cards (one set per 2 students)

Select a number between 5 and 12, and partner children to find combinations of two dot cards totaling that number.

1. Children make a "sandwich" with two cards by placing them back to back with the dot sides out.
2. When they have find at least 10 pairs, the next challenge is for the partner to name the number on the other side. The cards are turned over to confirm.
3. The same sandwiches can then be used again to name the other hidden part.

Student Activity: Sum Search

Sum Search

Find two or more addends (horizontally, vertically or diagonally) to equal the sum in the center square. Circle the addends and write the equation.

6	1	3	8	5	5	8	4	2	1
3	1	4	5	9	4	2	3	1	0
2	3	5	7	2	6	2	3	2	1
6	4	7	5	3	6	8	7	2	5
0	1	1	2	10		0	10	1	2
7	9	1	3			6	2	2	2
8	6	4	0	3	7	3	6	2	1
5	3	4	6	3	6	0	7	2	1
9	3	7	1	3	2	8	7	6	1
5	3	1	1	3	6	8	7	9	1

Sum Search

Find two or more addends (horizontally, vertically or diagonally) to equal the sum in the center square. Circle the addends and write the equation.

5	1	3	2	5	5	8	0	2	1
3	1	4	5	6	4	1	3	1	0
2	5	5	7	2	3	2	1	2	1
6	5	7	5	3	6	8	7	2	5
0	1	1	2			0	10	1	2
7	9	1	3			6	2	2	2
8	6	4	0	3	7	3	6	2	1
5	3	4	1	2	1	0	6	3	5
9	3	7	1	4	2	1	7	6	1
5	2	1	1	3	6	8	7	8	1

9

Retrieved from http://www.k-5mathteachingresources.com/support-files/sum-search.pdf

Student Activity: BUMP (2 Player Addition Games)

Materials

- Need two dice
- Each player starts with a set of 10 same-color chips or clear counters (Player 1: 10 red Player 2: 10 yellow)

Rules:

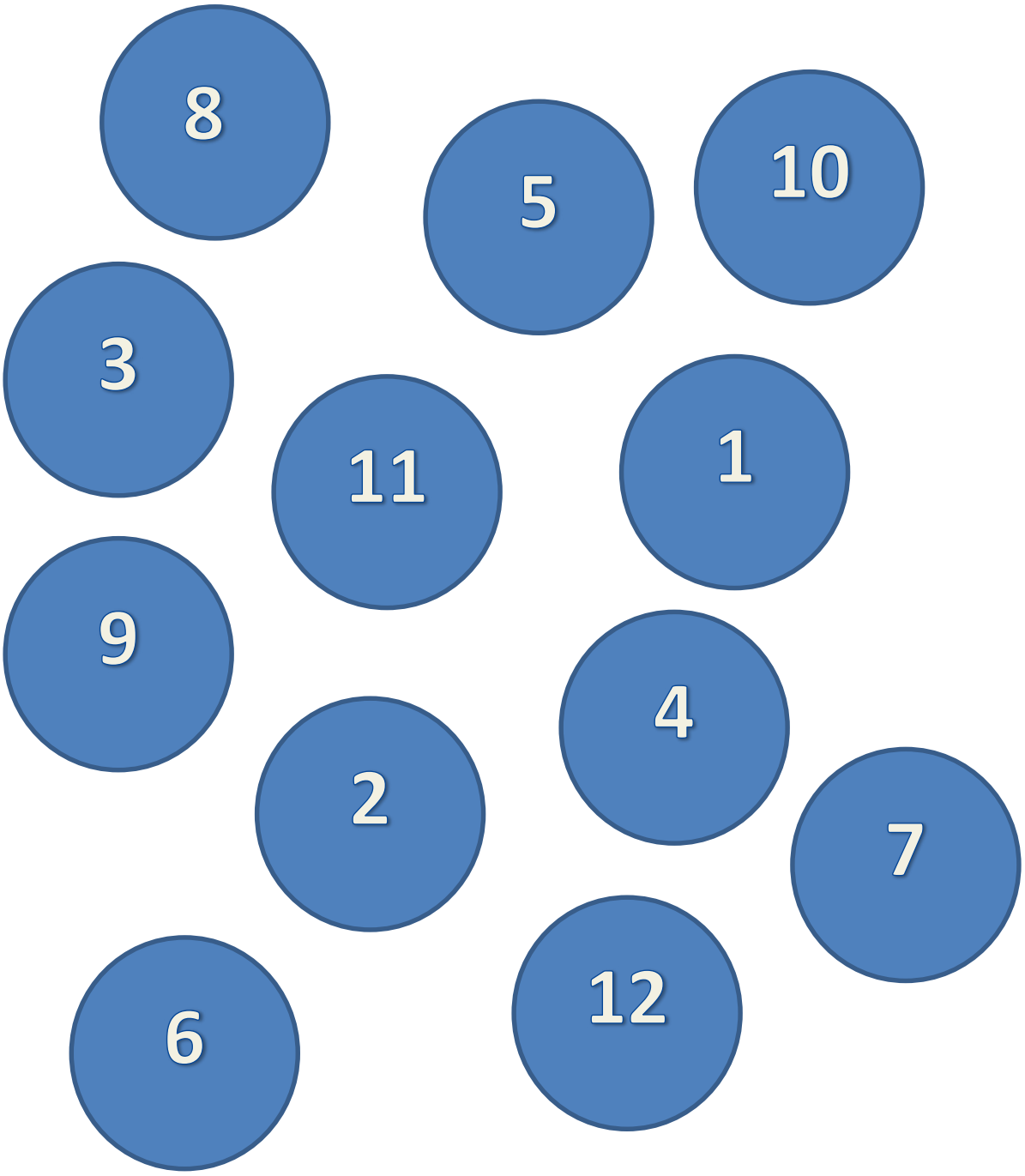
Level 1: Roll dice and put a clear counter on the circle showing the sum of the dice. If another player's counter is on that number, BUMP it off. If your marker is on that number, put an additional counter on top ("crown") and it closes that spot. The winner is the player that uses all of his/her markers first!

- If a player rolls double he gets to roll again.
- If all possible circles (sums and addends) are closed, the player loses a turn.
- The first person to get rid of all 10 chips is the winner.

Variations:

Level 2: Play the same as Level 1, however a player can either use the sum of the dice or the addend(s) of the dice (Example: a 5+4 is rolled, you can place your chip on 9 or 4 and 5).
Level 3: Play the same as Level 2, except players can use any two numbers to equal the sum of the addends (Example: if you roll 4 +5 you can use any two numbers that equal 9, such as 3 +6, 8+1, 7+2, etc.)

BUMP



-Retrieved from www.strategicinterventionsolutions.org

Participant Activity

With a partner, play BUMP using the various rules for each level.

Reflection

- What mathematical thinking does this game require?

- How do these activities help promote number sense and mathematical understanding?

Key Idea #1

Numbers are related to each other through a variety of number relationships. The number 7, for example, is three more than 4, two less than 9, composed of 3 and 4 as well as 2 and 5, is three away from 10, and can be quickly recognized in several patterned arrangements of dots. These ideas further extend to an understanding of 17, 57, and 370.

Relationships for Numbers 10 to 20

Even though pre-kindergartners to second graders experience numbers up to 20 and beyond on a daily basis, it should not be assumed that they will automatically extend the set of relationships that they have developed with smaller numbers to numbers beyond 10. Yet these numbers play a big part in many simple counting activities, in basic facts, and in much of what we do with mental computation. Relationships with these numbers are just as important as relationships involving numbers through 10. In fact, an emerging approach to number instruction is to expose children to numbers beyond 10 and even beyond 20 as soon as possible- even before formal place-value instruction.

(Fosnot & Dolk, 2001)

Although children may count by ones to count sets beyond 10, the experiences of counting and grouping help children build important initial place-value knowledge. The emphasis should be on helping children learn the number words and numerals beyond 10 rather than the traditional notions of place value. That will come as children develop strategies to add and subtract two digit numbers.

(Van De Walle, et al, 2014)

Pre- Place –Value Relationship with 10

There has been found to be a progression of three levels in children's' understanding of ten:

- *An initial concept of ten*- The child understands ten as ten ones and does not see ten as a unit. When children at this level work on a task involving tens, they will count by ones.

- *An intermediate concept of ten*- The child understands ten as a unit composed of ten ones but relies on materials or representations to help complete tasks involving tens.

- *A facile concept of ten*- The child can solve tasks involving tens and ones without using materials or representations. At this level children can mentally think about two- digit numbers as groups of tens and ones.

(Wright et al., 2006)

Stop and think: Say to yourself, "One ten." Now think about that from the perspective of a child just learning to count to 20! What could one ten possibly mean when ten tells me who many fingers I have and is the number that comes after nine? How can it be one? How would you explain this to a child?

Formal Assessment Note:

Ask a child to count out 35 tiles. Watch closely to note whether they count out the tiles one at a time and put them into a pile without any type of grouping or if they group them into tens. Have the child write the number that tells how many tiles they just counted. Some may write "53" instead of "35", a simple reversal. You will likely find that early on children count the tiles one by one and are not yet thinking of ten as a unit (level 1 or initial concept of ten).

In order to help children begin thinking about counting in ways that can advance their understanding of ten, consider providing lots of purposeful opportunities for them to count and group objects.

Participant Activity: Make a set of Place Value (Hide Zero) Cards

Place Value or Hide Zero Cards is a very effective mathematical tool that can be used in a variety of manners. In the module appendix, find and cut out your own set of Place Value cards. We will use these throughout the modules. It is necessary when using them in your classroom that each student has their own set.

Student Activity: BUILD THE NUMBER with HIDE ZERO CARDS (1.NBT.2, 2.NBT.1, and 2.NBT.3)

Hazekamp, Jana. *Why Before How: Singapore Math Computation Strategies*, 2011

1. From cardstock create a set of cards. The tens card should be twice as long as the ones card, and the hundreds cards three times as long as the ones card.

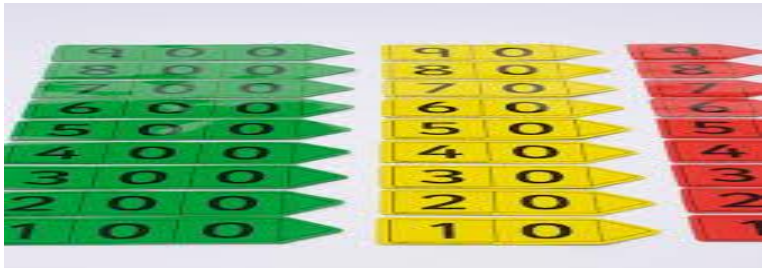
2. Children use the cards to create two-digit numbers.

3. Write a two digit number where children can see it, example 26.

4. Children are to find the two cards that can be used to make 26 (20 and 6).

5. Select children to demonstrate to the class how to make the given number. **Point out that you can still see the 20 hiding under the overlay of the 6.**

6. Repeat the activity with different two-digit numbers. When children are ready you can extend the activity to three-digit numbers. You may also want children to model the numbers with other materials such as base ten materials or ten frames.



Student Activity: TEN AND SOME MORE (1.NBT.2)

Van de Walle, Lovin, Karp, & Bay-Williams., 2006

Materials: Two Part Mat; up to 19 counters per child

1. Using a simple two-part mat, have children count out ten counters onto one side.
2. Have them put five counters on the other side.
3. Together count all of the counters by ones.
4. All together say the combination, "ten and five is fifteen."
5. Turn the mat around, "five and ten is fifteen."
6. Repeat with other numbers in a random order but without changing the side of the mat.

This activity is designed to teach new number names and therefore does require a certain amount of direct teaching. Following this activity, explore numbers to 20 in a more open-ended manner. You may provide children with a two-part mat that has two ten frames drawn one under the other. In random order, have children show numbers to 20 on their mats. There is no preferred way to do this as long as the number of counters is correct.

Formal Assessment Note:

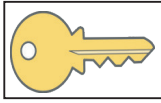
It is interesting to discuss how the counters can be placed on the mat so that it is easy to see how many are there. Have children share their ideas. Not every child will use a full set of ten, but as this idea becomes more popular the notion that 10 and some more is a teen amount will soon develop. Don't forget to include numbers less than ten as well, and if your students are up for the challenge you may include larger numbers, such as 27.

Student Activity: MORE AND LESS EXTENDED (1.NBT.5 and 2.NBT.8)

Conklin, M., 2010

Materials: projector, counters, filled ten frame

1. On a projector, show seven counters and ask what is two more, two less, one less, and so on.
2. Now add a filled ten frame to the display (or ten in any pattern) and repeat the question.
3. Pair up questions by covering and uncovering the ten frame.
4. This activity can be extended to larger numbers.



Key Idea #2: Having number sense means that you can think about and use numbers and their relationships in many different ways.

Numbers to 100: Early Introductions

According to the Tennessee Academic Standards, kindergartners are expected to be able to count to 100 by the end of the school year. Therefore, early exposure to numbers to 100 is important even in kindergarten. Although it is extremely unlikely that children in kindergarten or first grade will have a true understanding of tens and ones or place value, they can learn much about the sequence of numbers to 100, and beyond. Most important at this early level is for children to become familiar with the counting patterns to 100.

The hundreds chart is an essential tool for every K-2 classroom.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Participant/Student Activity: PATTERNS ON A HUNDREDS CHART

Conklin, M & Sheffield S. *It Makes Sense! : Using the Hundreds Chart to Build Number Sense*, 2010

Directions:

1. Work in pairs to find patterns on the hundreds chart.
2. Solicit ideas orally from the class.
3. Have children explain patterns found by others to be sure that all understand the ideas.

Small Group Discussion

- What patterns do you focus on with your students?
- What other patterns do you see that may support Number Sense, Place Value, or Addition and Subtraction Concepts?

Patterns on a Hundred Chart

Module 2 Closing Activity

Think back to the areas we have explored concerning Number Sense Concepts:

- Spatial Relationships
- One More/Less, Two More/Less
- Anchoring to Numbers 5 and 10
- Part-Part-Whole Relationships
- Relationships to Numbers 10 to 20
- Numbers to 100

Note: Although we have looked at a variety of activities, this is not an exhaustive list.

Reflection

- What other strategies/activities do you use in your classroom to promote Number Sense Concepts?

- Discuss these activities/strategies in your small group. Be prepared to share whole group. Record any ideas that you would like to incorporate in your classroom in the following pages.

Collaborative Notes/Activities – Number Sense

Collaborative Notes/Activities – Number Sense