PBS Lesson Series

Math: Grade 7, Lesson 16, Circumference

Lesson Focus: This lesson will explore circumference of a circle when the problem provides different pieces of contexts within a problem. Students will then use the circumference formula in order to solve real world problems.
Practice Focus: Students will focus on practicing applying the circumference formula and substituting giving information in order to solve real world contextual problems involving circumference of a circle.
Objective: Students will use the circumference formula to solve for a variety of variables with a focus on real world problems.
Key Vocabulary: Perimeter, Circumference, Radius, Diameter, and Pi
TN Standards: 7.G.B.3
Teacher Materials:

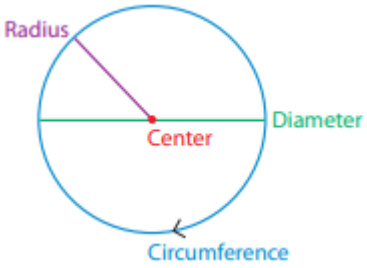
- Paper or white board
- Pen/pencil/marker
- Prepared copies of the examples (to save time)
- Student Practice Packet

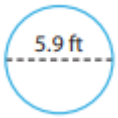
Student Materials:

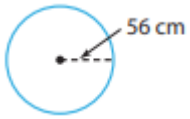
- Paper and a pencil, and a surface to write on

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 7st graders out there, though all children are welcome to tune in. This lesson is the sixteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about the circumference of a circle in mathematics! Before we get started, to participate fully in our lesson today, you will need:** • **Paper, a pencil and a surface to write on and the optional student packet and optional calculator** **Ok, let's begin!**	Students get materials ready for the lesson.
Intro (5 min)	

Can you think of a few items near you right now that are circles? [Pause] **What about items you have seen somewhere outside of your house?** [Pause] **Those are great examples! Each one of those circles has what is called a circumference. Before we dive into what a circumference is let's learn a few key terms that we will need in order to achieve todays learning target.**	Student thinks about examples of circles in their home and outside of their home.
[Teacher draws a circle on the board with the center labeled as "Center".] **Go ahead and draw a circle on your paper. It doesn't have to be a perfect circle but something that looks similar to mine will be great! Also, go ahead and label your center of your circle just as I have done.** [Pause]	Student constructs diagram of special terms in a circle for reference later.
The first term that we are going to go over is radius or radii for plural. [Teacher draws a radius on their circle and labels it "radius".] **A radius is a line segment with one endpoint at the center of the circle and the other endpoint on the circle. The length of a radius is called the radius of the circle. A line segment is a straight line which links two points without extending beyond them. Can you draw a radius on your circle?** [Pause]	
Great! Similar to the radius there is a line segment called a diameter. [Teacher draws a diameter on their circle and labels it "diameter".]	
A diameter of a circle is a line segment that passes through the center of the circle and whose endpoints lie on the circle. The length of the diameter is twice the length of the radius. The length of a diameter is called the diameter of the circle. Go ahead and draw the diameter on your circle! [Pause]	
Perfect! Next we have the Circumference of the circle. [Teacher traces the outside of the circle and labels the circumference as "circumference" outside of the circle.] **The circumference of a circle is the distance around the circle. It is very similar to the perimeter. Do you remember what the perimeter is of an object?** [Pause]	
Right! It is the distance around an object. Similar to fence around a yard. Circumference is a special word for the perimeter of a circle. [Teachers diagram will look similar to the one below when completed.]	
Does your circle look like mine? [Pause] **Great! This visual will be great to have as you work though problems later.**	

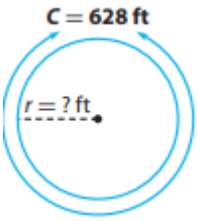
 **This next part is extremely interesting. In this next section we will learn where the circumference formula comes from. Are you ready?** [Pause] **Let's go!** **The ratio of the circumference to the diameter $\frac{C}{d}$ is the same for all circles.** [Teacher writes $\frac{C}{d}$ on the board.] **This ratio is called π or Pi.** [Teacher writes π and Pi on the board.] **You can approximate it as 3.14.** [Teacher writes 3.14 on the board.] **You are able to use π to find a formula that will calculate the circumference of a circle! For any circle $\frac{C}{d} = \pi$.** [Teacher writes $\frac{C}{d} = \pi$ on the board] **Can you solve the equation for "C"?** [Pause] **What operation do you need to do in order to have "C" all by itself on one side of the equal sign?** [Pause] **Yes! You need to multiply both sides of the equation by "d"! Go ahead and do this on your paper as I do it on the board.** [Teacher shows solving process on the board that is shown below.] $\frac{C}{d} = \pi$ — The ratio of the circumference to the diameter is π. $\frac{C}{d} \times d = \pi \times d$ — Multiply both sides by d. $C = \pi d$ — Simplify. **From our solving we now know that circumference is equal to pi multiplied by the diameter. Recall that the diameter is how much greater than a radius?** [Pause] **Exactly! A diameter is twice the length of a radius. This means you can use the equation $C = \pi d$** [Teacher points to formula on the board.] **to find the formula for circumference C in term of radius r. What can we substitute in for "d"** [Pause] **Correct! We are going to substitute in "2r" for d because the diameter is twice as long as the radius. Do this on your paper please!** [Teacher does solving process on the board as student does it on their paper. Solving process should look as it is shown below.]	Student learns definition of circumference and formulas.

$C = \pi d = \pi(2r) = 2\pi r$ **The two equivalent formulas for circumference are $C = \pi d$ and $C = 2\pi r$.** [Teacher writes both equations on the board.] **Please write these equations on the top of your paper because they will be extremely important as we move forward!** [Pause] **Let's use our new found formula!**	
Teacher Model (10 minutes) Objective 1: Student can calculate circumference when given a diameter. **Let's apply one of our formulas to a problem!** [Teacher draws diagram below on the board.] **Draw the circle on the board on your paper.** [Pause] [Teacher will write and read aloud the directions.] **Find the circumference of the circle. Use 3.14 for pi. Round to the nearest hundredth if necessary.**  **Analyze the picture that you have just drawn.** [Pause] **What does the 5.9 ft represent in the circle?** [Pause] **Great! It represents the diameter of the circle. Since the problem provides us with the diameter which formula should we use?** [Pause] **That's the one I would use too! We will use the $C = \pi d$ formula because the problem provides us with diameter and not the radius.** [Teacher writes the formula on the board for reference.] **What is the problem asking us to solve for?** [Pause] **Yes. The problem is asking us to solve for circumference which is represented by which variable?** [Pause] **"C" Right again! With that in mind what variable are we going to substitute 5.9 ft in for?** [Pause] **Did you say "d"?** [Pause] **Great! That is exactly what we are going to do. Do that on your paper as I do it on the board.** [Pause, teacher does work below on the board.] $$C = \pi\, d$$ $$C = \pi\, (5.9\, ft)$$ **What does π stand for again?** [Pause] **It stands for Pi which from the directions we can assume is 3.14. Go ahead and**	Objective 1: Student will be learning how to use circumference formula to solve for circumference when given a diameter. Student answers prompts.

substitute that in your equation. [Teacher substitutes 3.14 in for π.] $$C = 3.14\ (5.9\ ft)$$ **What is our last step in solving for circumference?** [Pause] **We multiply the 3.14 and the 5.9! If you have a calculator you can use it here! Go ahead and then we will compare answers.** [Pause] **Did you get 18.53 ft?** [Pause] **Did you round your answer properly? Remember the problem tells you to round to the nearest hundredth!** [Pause] **Great job! You just solved for the circumference of that circle! Let's move on to the next one!**	
Objective 2: Student is able to solve for circumference when given a radius. **Let's apply one of our formulas to a problem again!** [Teacher draws diagram below on the board.] **Draw this circle on the board on your paper.** [Pause] [Teacher will write and read aloud the directions.] **Find the circumference of the circle. Use 3.14 for pi. Round to the nearest hundredth if necessary.**  	Objective #2: Students solve for circumference using the circumference formula when given the radius.
Analyze the picture that you have just drawn. [Pause] **What does the 56 cm represent in the circle?** [Pause] **Yes! It represents the radius of the circle. Since the problem provides us with the radius which formula should we use?** [Pause] **That's the one I would use as well! We will use the $C = 2\pi r$ formula because the problem provides us with radius this time and not the diameter.** [Teacher writes the formula on the board for reference.] **What is the problem asking us to solve for?** [Pause] **Yes. The problem again is asking us to solve for circumference which is represented by which variable?** [Pause] **"C" Right! Thank you so much for your help! With that in mind what variable are we going to substitute 56 cm in for?** [Pause] **Did you say "r"?** [Pause] **Great! That is exactly what we are going to do. Do that on your paper as I do it on the board.** [Pause, teacher does work below on the board.] $$C = 2\pi r$$ $$C = 2\ (\pi)\ (56\ cm)$$	Student Analyzes picture. Student answers prompts.

What does π stand for again? [Pause] **It stands for Pi which from the directions we can assume is about 3.14. Go ahead and substitute that in your equation.** [Teacher substitutes 3.14 in for π.] $$C = 2\ (3.14)\ (56\ cm)$$ **What is our last step in solving for circumference?** [Pause] **We multiply the 2, 3.14 and the 56! Go ahead and then we will compare answers. I will give you some extra time here since this one is more difficult than our last one.** [Pause] **Did you get 351.68 cm?** [Pause] **Did you round your answer properly? Remember the problem tells you to round to the nearest hundredth!** [Pause] **Great job! What does the 351.68 cm represent?** [Pause] **It does represent the circumference, but what is the circumference?** [Pause] **Exactly. It is the distance around the circle. So if you started at one point and walked all the way around the circle you would walk 35.68 cm! Now let's apply our formula to a real world problem.**	
Objective 3: Student is able to solve for circumference in a contextual problem. [Teacher reads problem aloud and writes it on the board.] **A circular fountain has a radius of 9.4 feet. Find the diameter and the circumference to the nearest tenth of a foot.**	Objective 3: Students will solve for the circumference in a contextual problem.
Can you draw a picture to represent this fountain? [Pause] **Go ahead and do it! Make sure you label the radius appropriately. This will help you visualize what is given and what we are solving for.** [Teacher draws a circle and labels the radius "9.4 feet".]	Student draws picture.
Now that we have a diagram to help us, let's move forward with problem. First the problem asks you to find the diameter. What do you know about the relationship between the diameter and a radius? [Pause] **Got it. The diameter is twice as long as the radius. If we are told that the radius is 9.4 feet long then how long would the diameter be?** [Pause] **Did you say 18.8 feet?** [Pause] **How did you calculate that?** [Pause] **Good. You multiplied the 9.4 feet by 2 and the answer to that is 18.8 feet which is correct. Next the problem asks you to solve for the circumference. Are you ready?** [Pause] **You can do it!**	Student answers prompts.

Analyze the picture that you have drawn. [Pause] **What does the 9.4 feet represent in the circle?** [Pause] **Yes! It represents the radius of the circle. Since the problem provides us with the radius of the fountain which formula should we use?** [Pause] **We will use the $C = 2\pi r$ formula because the problem provides us with radius and not the diameter.** [Teacher writes the formula on the board for reference.] **What is the problem asking us to solve for?** [Pause] **Yes. The problem again is asking us to solve for circumference which is represented by which variable again?** [Pause] **"C" Right! You are getting the hang of this! What variable are we going to substitute 9.4 feet in for?** [Pause] **Did you say "r"?** [Pause] **Great! That is exactly what we are going to do. Do that on your paper as I do it on the board.** [Pause, teacher does work below on the board.] $$C = 2\pi r$$ $$C = 2\ (\pi)\ (9.4\ ft)$$ **You might be thinking, Why did we not use the $C = \pi d$ equation since we calculated the diameter? You certainly could have and you will end up getting the same answer. What does π stand for again?** [Pause] **It stands for Pi which from the directions we can assume is about 3.14. Go ahead and substitute that in your equation.** [Teacher substitutes 3.14 in for π.] $$C = 2\ (3.14)\ (9.4\ ft)$$ **What is our last step in solving for circumference?** [Pause] **We multiply the 2, 3.14 and the 9.4! Go ahead and then we will compare answers. I will give you some extra time again since this one is more difficult. If you get stuck look above to the previous problem for help!** [Pause] **Did you get 59 feet?** [Pause] **Did you round your answer properly? Remember the problem tells you to round to the nearest tenth!** [Pause] **You are doing so well at calculating the Circumference of a circle. We have just a few more problems to work on!**	
<u>Guided Practice</u> (10 minutes) I do: [Teacher reads problem aloud and writes it on the board.] **A circular pod has a circumference of 628 feet. A model boat is moving directly across the pound, along a radius, at a rate of 5 feet per second. How long does it take the boat to get from the edge of the pond to the center?** [Pause]	Student listens and reads problem to determine important pieces of information.

I want you to visualize what this problem is asking. Picture a pond and a little model boat floating across that pond. Do you have a picture in your head? [Pause] **Great!** **First we need to calculate the distance of the radius of this circular pond. Where are the two endpoints of the radius?** [Pause] **The two endpoints of the radius are on the circle and at the center of the circle. Can you help me draw a picture of this problem? Make sure you label that the circumference is 628 feet and we are trying to solve for the radius in your drawing.** [Pause, Teacher draws picture below on the board.]  **This is the drawing that I came up with. Does it look similar to yours?** [Pause] **Great. Let's keep going! Because we are given the circumference and we are solving for the radius which equation do we need to use?** [Pause] **That's right! We need to use the $C = 2\pi r$ equation.** [Teacher writes equation on the board.] **What variable do we need to substitute the 628 feet in for?** [Pause] **Yes! We are going to substitute it in for "C" since the problem tells us this is the circumference. What are we trying to solve for in this problem?** [Pause] **"r" or radius! Go ahead and substitute in the 628 feet for "C" on your paper.** [Teacher does the substitution process and writes it on the board for students to compare.] $$628 = 2\pi r$$ **Does your equation look like the one I have written on the board?** [Pause] **Glad we are in the same spot! Ok since we are solving for "r" I need to get "r" on one side of the equation and everything else on the other. How do I do that?** [Pause] **That's it. I divide by two and by Pi on both sides. Can you do that for me on your paper?** [Pause] **Thank you!** [Teacher will do work on the board while student is doing work on their paper. Work should look similar to what is below.] $C = 2\pi r$ $628 \approx 2(3.14)r$ $\frac{628}{6.28} \approx \frac{6.28r}{6.28}$ $100 \approx r$	Student draws a figure to represent problem. Student answers prompts by teacher. Student compares equation.

From our work it appears that the radius of the pond is roughly 100 feet in length. What is the problem asking us to solve for in the very end? [Pause] **The problem is asking us to find how long it will take the boat to go from the edge of the pond to the center. From the problem we are told that the boat is traveling at 5 feet per second. How are we going to calculate how long it will take the boat to travel 100 feet?** [Pause] **Nice!** [Teacher wil read script below and show work below on the board.] $$Time = \frac{100\ feet}{5\ feet\ per\ second}$$ $$Time = 20\ Seconds$$ **If we divide the 100 feet by 5 feet it will tell us how long it will take. 100 divided by 5 is 20 so it takes the boat 20 seconds to travel the length of the radius! Wow! Great job on that problem! Here is another one that I need your help with!** We do: [Teacher reads problem aloud and writes it on the board.] **A round swimming pool has a circumference of 66 feet. Carlos wants to buy a rope to put across the diameter of the pool. The rope costs $0.45 per foot. And Carlos needs 4 feet more than the diameter of the pool. How much will Carlos pay for the rope?** [Pause]	Student listens and reads problem to determine significant information.
Just like the pond can you visualize Carlos' pool? Picture the rope that he is trying to tie across the diameter of the pool. Where would that be? Can you draw a picture? [Pause] **Good job!** **First we need to calculate the distance of the diamter of this circular pool. Where are the two endpoints of the diamter?** [Pause] **The two endpoints of the diameter are on the circle and and the diameter must go through the center of the circle.**	
Because we are given the circumference and we are solving for the diameter which equation do we need to use? [Pause] **That's right! We need to use the $C = \pi d$ equation.** [Teacher writes equation on the board.] **What variable do we need to substitute the 66 feet in for?** [Pause] **Yes! We are going to substitute it in for "C" since the problem tells us this is the circumference. What are we trying to solve for in this part of**	Student figures out which equation is most helpful to use.

the problem? [Pause] **"d" or the diameter! Go ahead and substitute in the 66 feet for "C" on your paper and solve for the diameter.** [Teacher does the substitution process, solves and writes it on the board for students to compare.] $$66\ feet = \pi d$$ $$\frac{66\ feet}{\pi} = \frac{\pi d}{\pi}$$ $$\frac{66\ feet}{3.14} = d$$ $$21\ feet \approx d$$	Student answers prompts by teacher. Student solves for diameter.
From our work it appears that the diameter of the pool is roughly 21 feet in length. The problem is asking us to find how much it will cost Carlos to purchase rope if the rope costs $0.45 per foot. From the problem we are told that Carlos needs 4 more feet that the diameter. How many actual feet will Carlos need to purchase? [Pause] **Right. He will need to purchase 25 feet of rope total. Now you can calculate the amount of money that Carlos is going to need. If it costs $0.45 per foot of rope and Carlos needs 25 feet total, how much money will Carlos need? Go ahead and do the work on your paper! Keep in mind that we are able to do this because of proportions from our previous lessons!** [Pause. While student is doing work teacher will write work that is below on the board for student to compare answer when finished.] $$Total\ Cost = (\$0.45)(25)$$ $$Total\ Cost = \$11.25$$ **Did you get that it will cost Carlos $11.25? That's perfect! You are doing so well! See if you are able to figure this one out.**	Student makes since of the problem and how to finish solving for the final price. Student solves for final price.
You do: [Teacher reads problem aloud and writes it on the board along with diagram.] **The Ferris wheel shown make 12 revolutions per ride. How far would someone travel during one ride?** [Pause]	

	Student listens and reads problem in order to pick out important pieces of information.
What information does the problem give you about the circle? [Pause] **You are correct. It tells you that the diameter is 63 feet in the length. What is the problem asking for?** [Pause] **The distance someone travel during a ride. What is the first information you may want to solve for first?** [Pause] **Great. I would solve for the circumference as well. Before we get too far into the solving process let's think about what your final answer will look like. If you solve for the circumference for the Ferris wheel, how many revolutions will that represent?** [Pause] **Only one revolution. How many revolutions are there per ride?** [Pause] **12 revolutions is right. Keep these numbers in mind as you are solving but continue solving for the circumference of the Ferris wheel. I am going to be quite for a few moments while you select the proper formula and attempt to solve the problem. Don't worry you can do it and feel free to use a calculator to speed up the solving process!** [Teacher Pauses for approximately two minutes.]	Student answers prompts.
Which formula would you choose to use with the information provided? [Pause] **I would use the $C = \pi d$ formula as well. What variable are you able to substitute for?** [Pause] **Great! What are you are able to substitute in for "d"?** [Pause] **63 feet is correct! Go ahead and calculate the circumference!** [Teacher does the work as student calculates it on their paper.] $$C = \pi d$$ $$C = \pi\,(63\ feet)$$ $$C = 3.14\,(63\ feet)$$ $$C = 197.82\ feet$$	Student solves for circumference.
The circumference of the Ferris wheel is 197.82 feet. What does this actually mean? [Pause] **This means that in one revolution the Ferris wheel travels 197.82 feet. From the problem we are trying to calculate the distance traveled in one ride which is how many revolutions?** [Pause] **12 revolutions. Calculate the total distance traveled during 1**	Student makes sense of what final answer will look like. Student calculates total distance.

ride. [Teacher does the work as student calculates it on their paper.] $$\boldsymbol{Total\ Distance} = (\mathbf{197.82}\ \boldsymbol{feet})(\mathbf{12})$$ $$\boldsymbol{Total\ Distance} \approx \mathbf{2,373.8}\ \boldsymbol{feet}$$ **The total distance traveled per ride is 2,373.8 feet! That's almost a half of a mile! Wow! Awesome job!** Additional Problems (if needed): Sam is placing rope lights around the edge of a circular patio with a diameter or 18 feet. The lights come in lengths of 54 inches. How many strands of lights does he need to surround the patio edge?	
Independent Practice (3 min) **Great work, 7th grade! Today, we worked on how to use circumference of a circle when the problem provides different pieces of contexts within a problem, and then use the circumference formulas to solve real world problems! You sure did a great job! After the video, you will have some problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **I enjoyed calculating the circumference and applying them to real world scenarios with you today! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	