TN Department of Education | 2015 Summer Training

Math Grades 6-8

Participant Packet

Tennessee Department of Education | 2015 Summer Training

MATH

GRADES 6-8

Table of Contents

Module 1
Examining the Beliefs and Knowledge of Effective Mathematics Instruction

Guiding Principles

Across all grade bands and subjects there are common guiding principles for each TNCore training:

- All students are capable of achieving at a high level.
- Students rise to the level of expectation when challenged and supported appropriately.
- Students learn best when they are authentically engaged in their own learning.
- We must continuously improve our effectiveness as teachers and leaders in order to improve student success.
- We must make every minute with our students count with purposeful work and effective instruction.

Norms for Collaboration

- Keep students at the center
- Be present and engaged
- Monitor air time and share your voice
- Challenge with respect
- Stay solutions oriented
- Risk productive struggle
- Balance urgency and patience

Rationale

"All of us who are stakeholders have a role to play and important actions to take if we are finally to recognize our critical need for a world where the mathematics education of our students draws from research, is informed by common sense and good judgment, and is driven by a non-negotiable belief that we must develop mathematical understanding and self confidence in all students."

- NCTM Principles to Action, 2014.

Goals

Participants will:

- Examine productive and unproductive beliefs about mathematics education
- Learn about the different types of knowledge effective mathematics teachers must have
- Connect work from previous TNCore trainings to NCTM's Teaching Practices

Session Activities

Participants will:

- Complete mathematics tasks across the grade band
- Read and discuss NCTM's productive and unproductive beliefs
- Define Mathematical Knowledge for Teaching using a Frayer Model

Overview

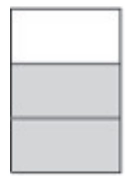
Module 1 focuses on the importance of the productive vs. unproductive beliefs (NCTM, 2014) around teaching mathematics that teachers hold. Participants engage in high level tasks and discuss the different mathematical knowledge types (Ball, 2012) and mathematical teaching practices (NCTM, 2014) that support effective and rigorous mathematics instruction.

Practice Problems

1. Tonya and Chrissy are trying to understand the following story problem for $1 \div \frac{2}{3}$.

One serving of rice is $\frac{2}{3}$ of a cup. I ate 1 cup of rice. How many servings of rice did I eat?

To solve the problem, Tonya and Chrissy draw a diagram divided into three equal pieces, and shade two of those pieces.



Tonya says, "There is one $\frac{2}{3}$-cup serving of rice in 1 cup, and there is $\frac{1}{3}$ cup of rice left over, so the answer should be $1\frac{1}{3}$."

Chrissy says, "I heard someone say that the answer is $\frac{3}{2} = 1\frac{1}{2}$. Which answer is right?"

Is the answer $1\frac{1}{3}$ or $1\frac{1}{2}$? Explain your reasoning using the diagram.

https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/463

2.

On the number line above, the numbers a and b are the same distance from 0. What is $a+b$? Explain how you know.

https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/310

3. Without using the square root button on your calculator, estimate $\sqrt{800}$ as accurately as possible to 2 decimal places. (Hint: It is worth noting that $20^2 = 400$ and $30^2 = 900$.)

https://www.illustrativemathematics.org/content-standards/8/NS/A/tasks/338

4. Ojos del Salado is the highest mountain in Chile, with a peak at about 6900 meters above sea level. The Atacama Trench, just off the cost of Peru and Chile, is about 8100 meters below sea level (at its lowest point).

A. What is the difference in elevations between Mount Ojos del Salado and the Atacama Trench?

B. Is the elevation halfway between the peak of Mount Ojos del Salado and the Atacama Trench above sea level or below sea level? Explain without calculating the exact value.

C. What elevation is halfway between the peak of Mount Ojos del Salado and the Atacama Trench?

https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/1987

5. Use the computation shown below to find the products.

$$\begin{array}{r} 189 \\ 16\overline{)3024} \\ \underline{16} \\ 142 \\ \underline{128} \\ 144 \\ \underline{144} \\ 0 \end{array}$$

1. 189×16

2. 80×16

3. 9×16

www.illustrativemathematics.org/content-standards/6/NS/B/2/tasks/270

6. Select all equations that are correct.

- A $\sqrt[3]{8} = 2$
- B $\sqrt{125} = 5$
- C $\sqrt[3]{99} = 33$
- D $\sqrt{169} = 13$
- E $\sqrt[3]{27} = 3$

MICA

7.

Which is equivalent to $\frac{x + 14}{7}$?

Ⓐ 2

Ⓑ $x + 2$

Ⓒ $\frac{x + 2}{7}$

Ⓓ $\frac{1}{7}x + 2$

MICA

8.

David surveyed 200 middle school students about their favorite season of the year and the season of their birthday. His results are shown in the table.

Birthday and Favorite Season Survey

	Season of Birthday	Favorite Season
Winter	54	37
Spring	58	31
Summer	44	111
Fall	44	21

Select each statement that is a reasonable conclusion based on the survey data.

A About 20% of all middle school students have their birthday in the spring.

B Most middle school students' favorite season is the same season as their birthday.

C More than half of all middle school students would likely choose summer as their favorite season.

D About 60 students from a group of 600 middle school students would likely choose fall as their favorite season.

MICA

NCTM Unproductive and Productive Beliefs

Beliefs About Teaching and Learning Mathematics	
Unproductive Beliefs	**Productive Beliefs**
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Independent/Small Group Time

- What are your biggest takeaways after reading the unproductive and productive beliefs?

- How can the differences in beliefs about teaching and learning mathematics impact students' opportunity to learn?

Mathematical Knowledge for Teaching

What do teachers need to know and be able to do in order to teach effectively? Or, what does effective teaching require in terms of content understanding?

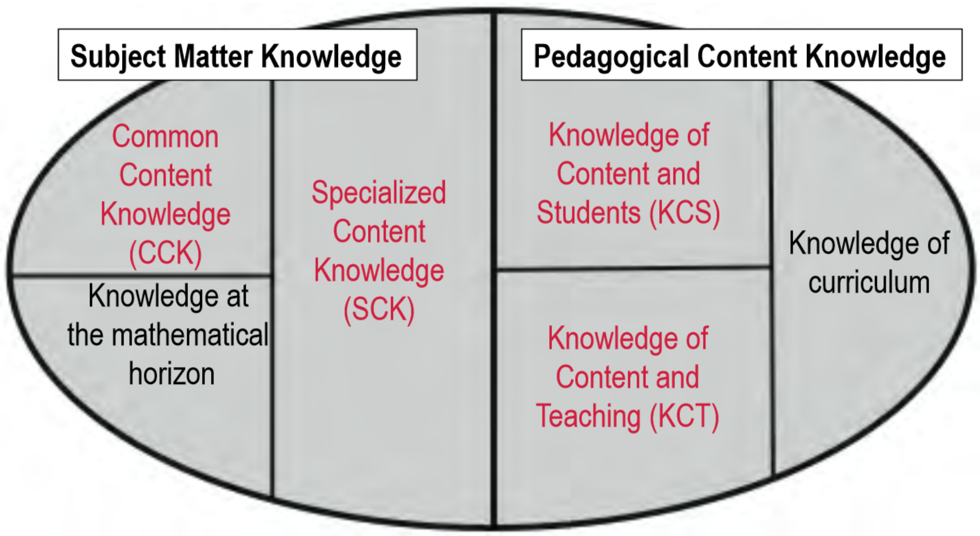
Mathematical knowledge for teaching (MKT) is a kind of professional knowledge of mathematics different from that demanded by other mathematically intensive occupations, such as engineering, physics, accounting, or carpentry. What distinguishes this sort of mathematical knowledge from other knowledge of mathematics is that it is subject matter knowledge needed by teachers for specific tasks of teaching, but still clearly subject matter knowledge. These tasks of teaching depend on mathematical knowledge, and, significantly, they have aspects that do not depend on knowledge of students or of teaching. These tasks require knowing how knowledge is generated and structured in the discipline and how such considerations matter in teaching, such as extending procedures and concepts of whole-number computation to the context of rational numbers in ways that preserve properties and meaning. These tasks also require a host of other mathematical knowledge and skill.

By "mathematical knowledge for teaching," we mean the mathematical knowledge needed to carry out the work of teaching mathematics. It is concerned with the tasks involved in teaching and the mathematical demands of these tasks. By "teaching," we mean everything that teachers must do to support the learning of their students. Clearly we mean the interactive work of teaching lessons in classrooms and all the tasks that arise in the course of that work. But we also mean planning for those lessons, evaluating students' work, writing and grading assessments, explaining the classwork to parents, making and managing homework, attending to concerns for equity, and dealing with the building principal who has strong views about the math curriculum. Each of these tasks, and many others as well, involve knowledge of mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency.

- Kilpatrick, Swafford, & Findell, 2000.

MKT is divided into two domains: subject matter knowledge and pedagogical content knowledge. Each domain includes three categories. Subject matter knowledge includes: common content knowledge, specialized content knowledge, and knowledge of the mathematical horizon. Pedagogical content knowledge includes: knowledge of content and students, knowledge of the curriculum, and knowledge of content and teaching.

Mathematical Knowledge for Teaching



Subject Matter Knowledge

1) **Common Content Knowledge (CCK):** the mathematical knowledge and skill used in settings other than teaching. Teachers need to know the material they teach; they must recognize when their students give wrong answers, or when the textbook gives an inaccurate definition. When teachers write on the board, they need to use terms and notation correctly. In short, they must be able to do the work that they assign their students. But some of this requires mathematical knowledge and skill that others have as well- thus, it is not special to the work of teaching. By "common," however, we do not mean to suggest that everyone has this knowledge. Rather, we mean to indicate that this is knowledge of a kind used in a wide variety of settings-in other words, not unique to teaching.

2) **Knowledge at the Mathematical Horizon:** Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas. Having this sort of knowledge of the mathematical horizon can help in making decisions about how, for example, to talk about the number line.

3) **Specialized Content Knowledge (SCK):** the mathematical knowledge and skill unique to teaching. Close examination reveals that SCK is mathematical knowledge not typically needed for purposes other than teaching. In looking for patterns in student error or in sizing up whether a nonstandard approach would work in general, teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed or even desirable in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work.

Pedagogical Content Knowledge

4) **Knowledge of Content and Students (KCS):** the knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing. When choosing an example, teachers need to predict what students will find interesting and motivating. When assigning a task, teachers need to anticipate what students are likely to do with it and whether they will find it easy or hard. Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. Each of these tasks requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking. Central to these tasks is knowledge of common student conceptions and misconception about particular mathematical content.

5) **Knowledge of Content and Teaching (KCT):** the knowledge that combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction. Teachers sequence particular content for instruction. They choose which examples to start with and which examples to use to take students deeper into the content. Teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally. Each of these tasks require an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning.

6) **Knowledge of Curriculum:** is "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (Shulman, p. 10)."

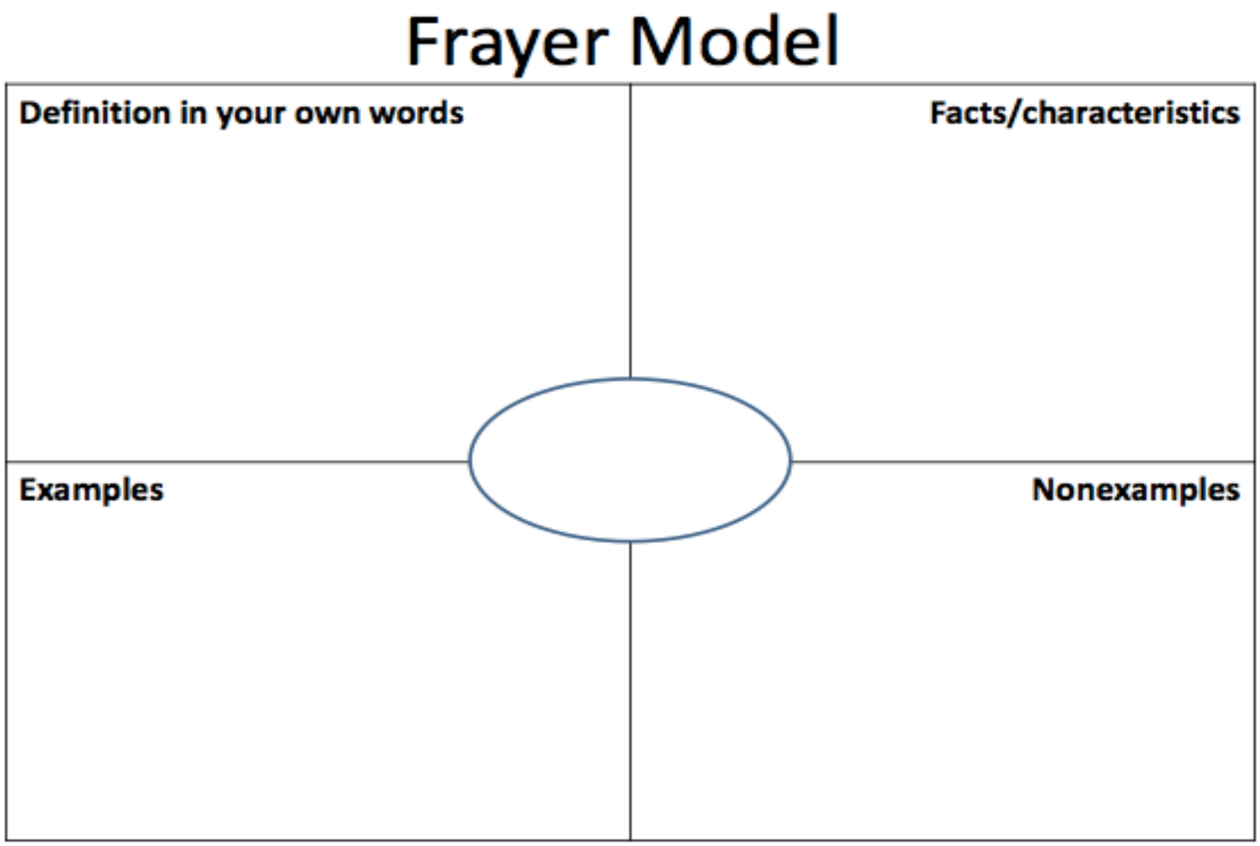
-Ball, Thames and Phelps: Content Knowledge for Teaching, What Makes It Special? (2008). (n.d.).

-Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough To Teach Third Grade, and How Can We Decide?

-Shulman, L. S. (1986).Those who understand: Knowledge growth in teaching. *Educational Researcher, l* 5(2), 4-14.

MKT Frayer Model Activity

- Discuss the particular domain of MKT assigned to your group.
- With your group complete Frayer Model for your particular domain on chart paper.
- Be prepared to share your model.



Small Group Discussion

- What resonates with you about the idea of MKT?
- What did you find new or interesting?
- How do you as a professional mathematics teacher engage in increasing your levels of MKT?
- What will you think about doing differently as a result?

"Teachers who score higher on...measures of mathematical knowledge for teaching produce better gains in student achievement."

- Ball, et al., 2005.

Mathematics Teaching Practices

“Eight Mathematics Teaching Practices provide a framework for strengthening the teaching and learning of mathematics. This research-informed framework of teaching and learning reflects... knowledge of mathematics teaching that has accumulated over the last two decades. The list [below and on the next page] identifies these eight Mathematics Teaching Practices, which represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics”
(Principles to Actions, 2014, p. 9).

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	
Connections to previous trainings	*Tools and resources*
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	
Connections to previous trainings	*Tools and resources*
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.	
Connections to previous trainings	*Tools and resources*
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	
Connections to previous trainings	*Tools and resources*

<table>
<tr><td colspan="2">Pose purposeful questions.
Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.</td></tr>
<tr><td>Connections to previous trainings</td><td>Tools and resources</td></tr>
<tr><td colspan="2">Build procedural fluency from conceptual understanding.
Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td></tr>
<tr><td>Connections to previous trainings</td><td>Tools and resources</td></tr>
<tr><td colspan="2">Support productive struggle in learning mathematics.
Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td></tr>
<tr><td>Connections to previous trainings</td><td>Tools and resources</td></tr>
<tr><td colspan="2">Elicit and use evidence of student thinking.
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td></tr>
<tr><td>Connections to previous trainings</td><td>Tools and resources</td></tr>
</table>

Small Group Discussion

- What did you find affirming?
- What do you need to refine?
- How do these practices connect to the idea of MKT?

Think about it…

"Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels."

-NCTM, Principles to Action Executive Summary.

Reflection

- Why is it necessary to consider productive vs. unproductive beliefs when designing lessons and implementing high-level tasks?

- How does having a deeper awareness of your own content and pedagogical knowledge impact your view of instruction?

- What will you do differently as a result?

- What are you still wondering about?

Module 2
Defining Mathematics Teaching and Learning Through Content Understanding

Rationale

"There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics."

- Lappan & Briars, 1995.

Goals

- Synthesize and refine our understanding of teaching and learning
- Consider the mathematical knowledge necessary to teach effectively
- Understand how deep understanding is necessary to support future learning

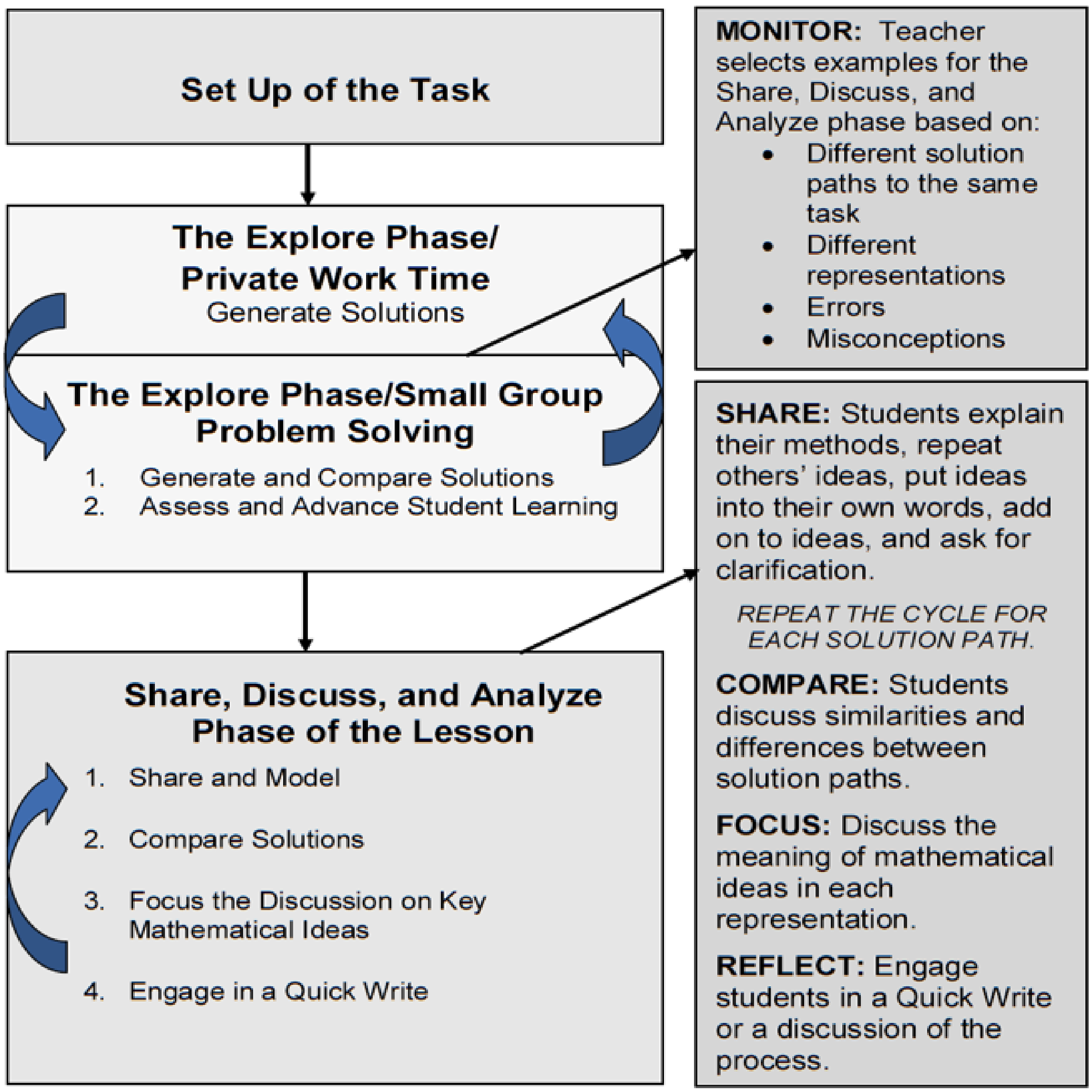
Session Activities

- Engage in a lesson
- Deepen our understanding of the mathematics in the NS domain in grades 6-8

Overview

In Module 2, participants will engage in multiple content-rich tasks designed to further develop their content knowledge and conceptual understanding as teachers of mathematics. Participants will analyze their learning, and reflect on the content through the lens of Mathematical Knowledge for Teaching (Ball, 2012), and the eight Mathematical Teaching Practices (NCTM, 2014).

Structures and Routines of a Lesson



Fun at the Ocean Task

On vacation last summer, the Baker family went to the beach. Tracey and her mom wanted to get a view of the coastline so they went parasailing 120 feet above the ocean. Jamie and his dad wanted to explore the fish and the reef so they went scuba diving 20 feet underwater.

a) For a moment in time, Tracey was directly above Jamie. Use a number line to show the location of Julie and Jamie at that moment.
b) Using your number line, write an equation to find the distance between Julie and Jamie.
c) They also observed more things while they were on vacation: a bird, a dolphin, a buoy and the bottom of the ocean. Plot each of these things on your number line where you think they may have been and explain your reasoning for each.
d) Find the distance between Jamie and the bottom of the ocean and the distance between Tracey and the bird. Write equations for each.

Independent Think Time

- Work on the task privately.

Small Group Time

- Compare your work with others in your group.
- Consider the different ways to solve the task.
- Place your group's thinking on chart paper.

Expectations for Whole Group Discussion

- Solution paths will be shared.
- Listen with the goals of:
  - putting the ideas into your own words;
  - adding on to the ideas of others;
  - making connections between solution paths; and
  - asking questions about the ideas shared.
- The main goal is to understand the mathematics and to make connections among the various solution paths and representations.

Reflecting on Your Learning

- What about this lesson made it possible for you to learn?

- Which Standards for Mathematical Practice did you have the opportunity to use?

- Where did you see any of the eight teaching practices?

Setting Up The Task

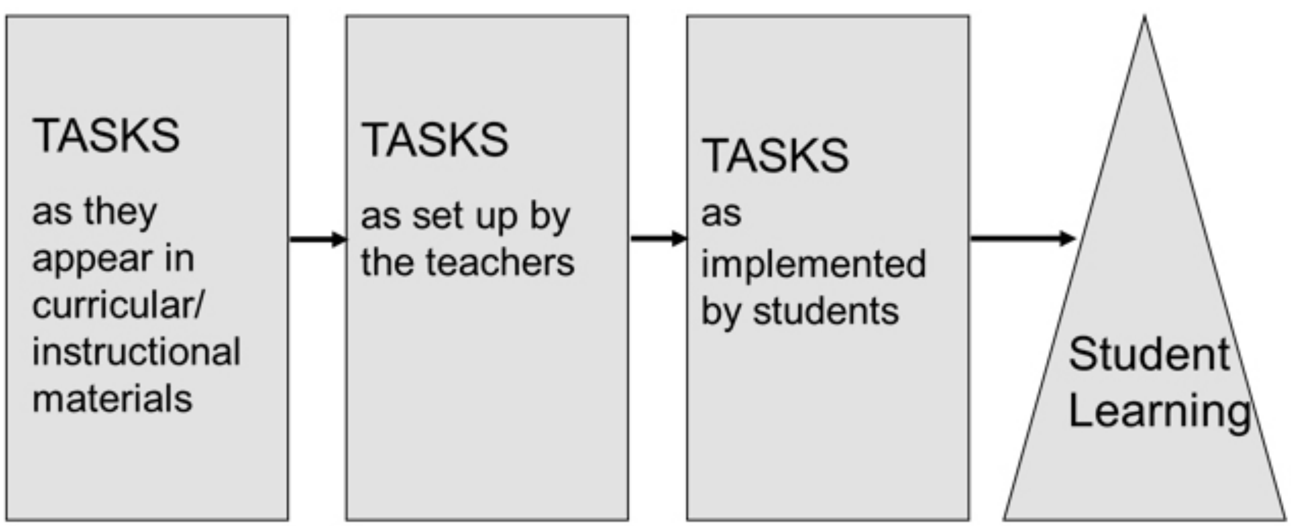
"A task's setup impacts both what students and the teacher are able to achieve during a lesson."

The set-up phase should orient the students to the task in order to ensure all students can engage productively. Here are some key features of the set-up phase:

1. Discuss contextual features.
2. Introduce key mathematical ideas.
3. Establish common language for discussing the mathematics in the task.
4. Maintain the demands of the task.

-Jackson, K., Shahan, E., Gibbons, L., & Cobb, P. (2012). Launching Complex Tasks*. Mathematics Teaching in the Middle School. 18*(1). 24-29.

The Mathematical Tasks Framework



-Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2000). Implementing standards-based mathematics instruction: A casebook for professional development (p. 4). New York, NY: Teachers College Press.

Factors Associated with Maintenance and Decline of High-Level Cognitive Demands

"Higher-achieving countries implemented a greater percentage of high-level tasks in ways that maintained the demands of the task."

-Stiegler & Hiebert, 2004.

Maintenance

- Scaffolds of student thinking and reasoning provided
- A means by which students can monitor their own progress is provided
- High-level performance is modeled
- A press for justifications, explanations through questioning and feedback
- Tasks build on students' prior knowledge
- Frequent conceptual connections are made
- Sufficient time to explore

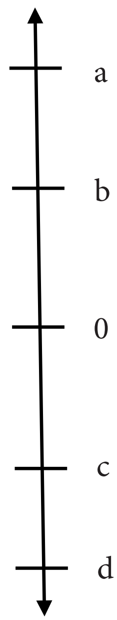
Decline

- Problematic aspects of the task become routinized
- Understanding shifts to correctness, completeness
- Insufficient time to wrestle with the demanding aspects of the task
- Classroom management problems
- Inappropriate task for a given group of students
- Accountability for high-level products or processes not expected

Thinking Deeply about Mathematics

"What does effective teaching require in terms of content understanding?"
Work on the Number Line Task privately for a few minutes before discussing with your group.

Number Line- Task A



1. Write two different expressions using absolute value to find the distance between:
   a. a and b

   b. b and c

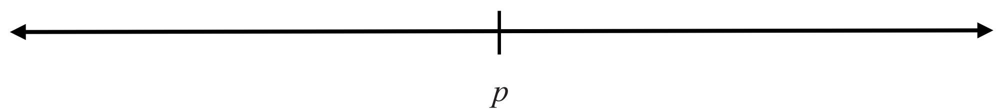
   c. c and d

2. Why is the location of the points with respect to 0 important when finding distances between them?

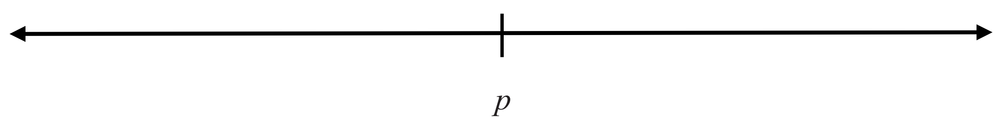
3. Given that $|b|=|c|$, what is $b+c$? Justify your reasoning.

Mind Your p's and q's- Task B

Complete numbers 1-4 for some fixed | q | ≠ 0



1. Place $p+q$ on the number line if $q>0$.
2. Place $p+q$ on the number line if $q<0$.

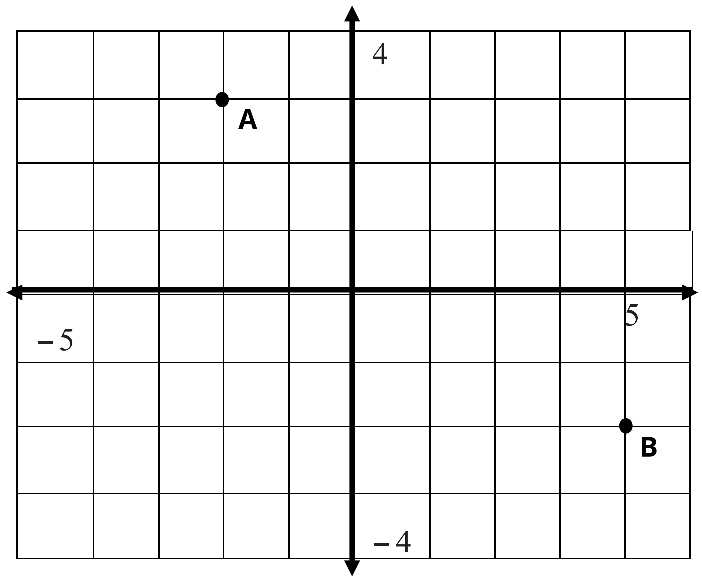


3. Place $p-q$ on the number line if $q>0$.
4. Place $p-q$ on the number line if $q<0$.

5. What do you notice about the points that name the same location on the two number lines? Explain your thinking mathematically. What does this tell you about adding and subtracting integers?

6. Explain why the distance between any two points on the number line p and q is defined by either $|p-q|$ or $|q-p|$.

Going the Distance- Task C



1. Find the distance between points A and B.

Task Reflection

What mathematical understandings from grades 6-8 did you need to have in order to solve this task? Be specific. (Use the 6^{th}-8^{th} Grade Number Systems (NS) Standards to help.)

6th Grade NS Standards

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm.

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.*

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.*

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*

6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

7th Grade NS Standards

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

8th Grade NS Standards

(plus 8.EE.A.2 and 8.G.B.8)

8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Reflection

- In what ways did you think differently about the teaching and learning of mathematics?

- What new insights about the NS Standards do you have?

- Which domain(s) of MKT did you feel increased and why is this important?

Module 3
Maintaining a Focus on Cognitive Rigor in Instruction

Rationale

Students who performed the best on project-based measures of reasoning and problem-solving were in classrooms in which tasks were more likely to be set up and enacted at high levels of cognitive demand.

- Stein & Lane, 1996; Stein, Lane, & Silver,1996.

The success of students was due in part to the high cognitive demand of the curriculum and the teachers' ability to maintain the level of demand during enactment through questioning.

- Boaler & Staples, 2008.

Goals

- Engage NCTM's Teaching Practices with an eye toward maintaining the cognitive rigor of the task
- Consider the importance of teaching as the intersection of content, pedagogy, and student learning

Session Activities

- Review tools that support rigorous task-based instruction
- Work on NCTM's Teaching Practices with student work
- Gallery walk and refine our ideas

Overview

Module 3 focuses participants back into the importance of intentional instruction through the analysis of a task using tools from previous summer trainings (structures and routines of a lesson, assessing and advancing questions, accountable talk, etc.). Student work will be analyzed, and teachers will react to the work using instructional strategies and effective teaching practices, all while working to maintain the cognitive demand of the task.

Tools to Support Task-Based Instruction

- On the next several pages are tools from previous summers that support selecting and enacting tasks at high levels of cognitive demand.
- In your small group, review your assigned tool and be prepared to give highlights to the whole group.

(Listen as other groups share for new insights if you are familiar or new learnings if you aren't familiar with these tools.)

Tools	New Insights or New Learnings
Task Analysis Guide	
Assessing and Advancing Questions	
Connections Between Representations	
Productive Discussions Through Selecting and Sequencing Student Work	
Accountable Talk Features and Indicators	

Task Analysis Guide

"If we want students to develop the capacity to think, reason, and problem-solve then we need to start with high-level, cognitively complex tasks."

-Stein, M. K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relatio-nship between teaching and learning in a reform mathematics project. Educational Research and Evaluation, 2 (4), 50-80.

Lower-Level Tasks	Higher-Level Tasks
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	Doing Mathematics Tasks • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Characteristics of Assessing and Advancing Questions

"Asking questions that assess student understanding of mathematical ideas, strategies or representations provides teachers with insights into what students know and can do. The insights gained from these questions prepare teachers to then ask questions that advance student understanding of mathematical concepts, strategies or connections between representations" (NCTM, 2000).

Assessing Questions

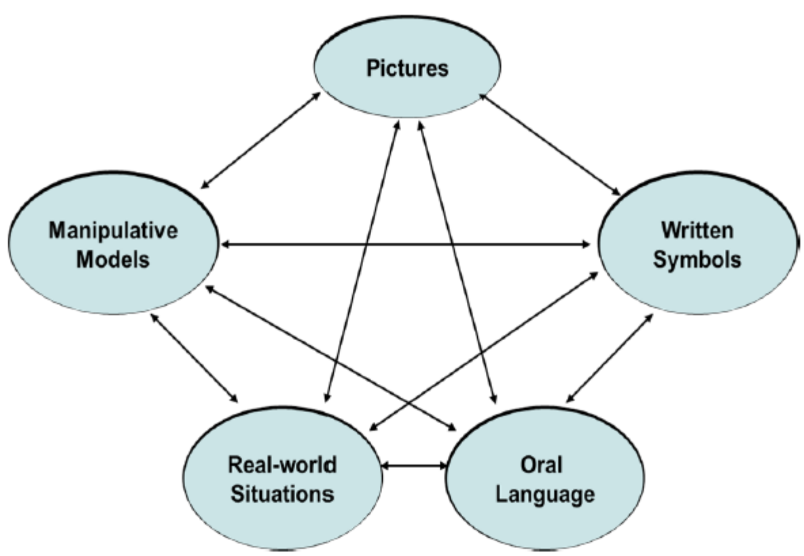
- Based closely on the work the student has produced.
- Clarify what the student has done and what the student understands about what s/he has done.
- Provide information to the teacher about what the student understands.

Advancing Questions

- Use what students have produced as a basis for making progress toward the target goal.
- Move students beyond their current thinking by pressing students to extend what they know to a new situation.
- Press students to think about something they are not currently thinking about.

Connections Between Representation

"Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling."



Adapted from Lesh, Post, & Behr, 1987

Orchestrating Productive Discussions

Teachers must "decide what aspects of a task to highlight, hot to organize and orchestrate the work of students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge" (NCTM, 2000, p. 19).

Rules of Thumb for Selecting and Sequencing Student Solutions

Based on level of student understanding:

- It is okay to begin by showing incomplete work or work that is not completely clear in order to engage the class in a discussion regarding what else needs to happen to complete or clarify the solution strategy.
- Arrange solutions in order of increasing difficulty with the most complex methods presented last; may be a movement from concrete to abstract.
- Select solutions that illustrate both efficient and inefficient methods so that you can discuss circumstances in which one may be preferable over the other.
- Share at least one completely correct response.
- Don't be afraid to address misconceptions if they are critical to the mathematics being discussed, but stay away from responses that show profound misunderstandings or that do not advance the mathematical discussion.
- Consider individual accomplishments of students (e.g., Is there a student who has not presented in a few days? Is there a student who has done something that is quite unique that would give that student a chance to shine in front of his/her peers?).

Based on the diversity of/similarity of the answers within the classroom:

- Show most frequently used solution methods first to provide entry to all (or the majority) of students.
- Present solutions that show a range of representations (e.g., graphs, tables, equations, diagrams).
- Order solutions (or pair them) so that each solution builds (to the extent possible) on the solution that preceded it.

Based on reaching your mathematical goals:

- Keep the goals and essential understandings in mind and build the discussion so that more students can access these concepts and leave with rich understandings.
- Know that there is more than one way to go about presenting the solutions—have a reason for what you are doing and a goal that your sequencing will target.
- Make sure you get to the generalization (if there is one).

Accountable Talk® Features and Indicators

"Mathematics reform calls for teachers to engage students in discussing, explaining, and justifying their ideas. Although teachers are asked to use students' ideas as the basis for instruction, they must also keep in mind the mathematics that the class is expected to explore" (Sherin, 2000, p. 125).

Accountability to the Learning Community

- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge

- Specific and accurate knowledge
- Appropriate evidence for claims and arguments
- Commitment to getting it right

Accountability to Rigorous Thinking

- Synthesize several sources of information
- Construct explanations and test understanding of concepts
- Formulate conjectures and hypotheses
- Employ generally accepted standards of reasoning
- Challenge the quality of evidence and reasoning

Think About It…

- Teacher A says, "My students work on problems so they know how to answer these types of problems."

- Teacher B says, "My students work on problems so they can learn the mathematics in these types of problems."

What is the difference between the goals of teacher A and teacher B and why is this significant?

Task-Based Instruction: True or False?

- All tasks must be high-level.
- Accountable talk is only used during a high-level task.
- All high level instructional tasks must have a context.
- Tennessee State Standards require task-based instruction.
- Students never need to engage in low-level tasks.
- Tasks are most effective when they are used to solidify learning.

Effective Uses of High Level Tasks	Ineffective Uses of High Level Tasks

Winter in Denmark Task

In Denmark, the average monthly temperature for February is 0° Celsius. The chart below shows the temperatures for 9 days during the month, taken at noon each day.

Temperature (Celsius)
2°
-8°
3°
7°
-10°
-6°
2°
7°
-3°

Write and evaluate two equivalent expressions that can be used to determine the difference between the highest and lowest temperatures.

Small Group Discussion

- Compare expressions with members in your group.
- What are all the different expressions your group came up with?

Analyzing Student Work

Independent Think Time:

- Carefully examine each student's work.
- What do the students know? Not know? What is the evidence?

Student A

Write and evaluate two equivalent expressions that can be used to determine the difference between the highest and lowest temperatures.

highest

$7°$

lowest

$-10°$

$-10-7=-17$ | $7-(-10)=17$

Student B

Write and evaluate two equivalent expressions that can be used to determine the difference between the highest and lowest temperatures.

Highest temp. = 7° / lowest temp. = -10

There is a 17 difference in the highest and lowest temperatures.

$y=7-10$ $y=$ temperature difference

Student C

Write and evaluate two equivalent expressions that can be used to determine the difference between the highest and lowest temperatures.

$7 - -10 = 17°$

$|7| + |-10| = 17°$

Student D

Write and evaluate two equivalent expressions that can be used to determine the difference between the highest and lowest temperatures.

My first exspressionis 2+3. 5
My second exspression is |-3-2|
This tells the diffrence
between the highest + lowest
temps wich is 5.

Student E

Write and evaluate two equivalent expressions that can be used to determine the difference between the highest and lowest temperatures.

$-10 + 7 = -3$

$7 + 10 = -3$

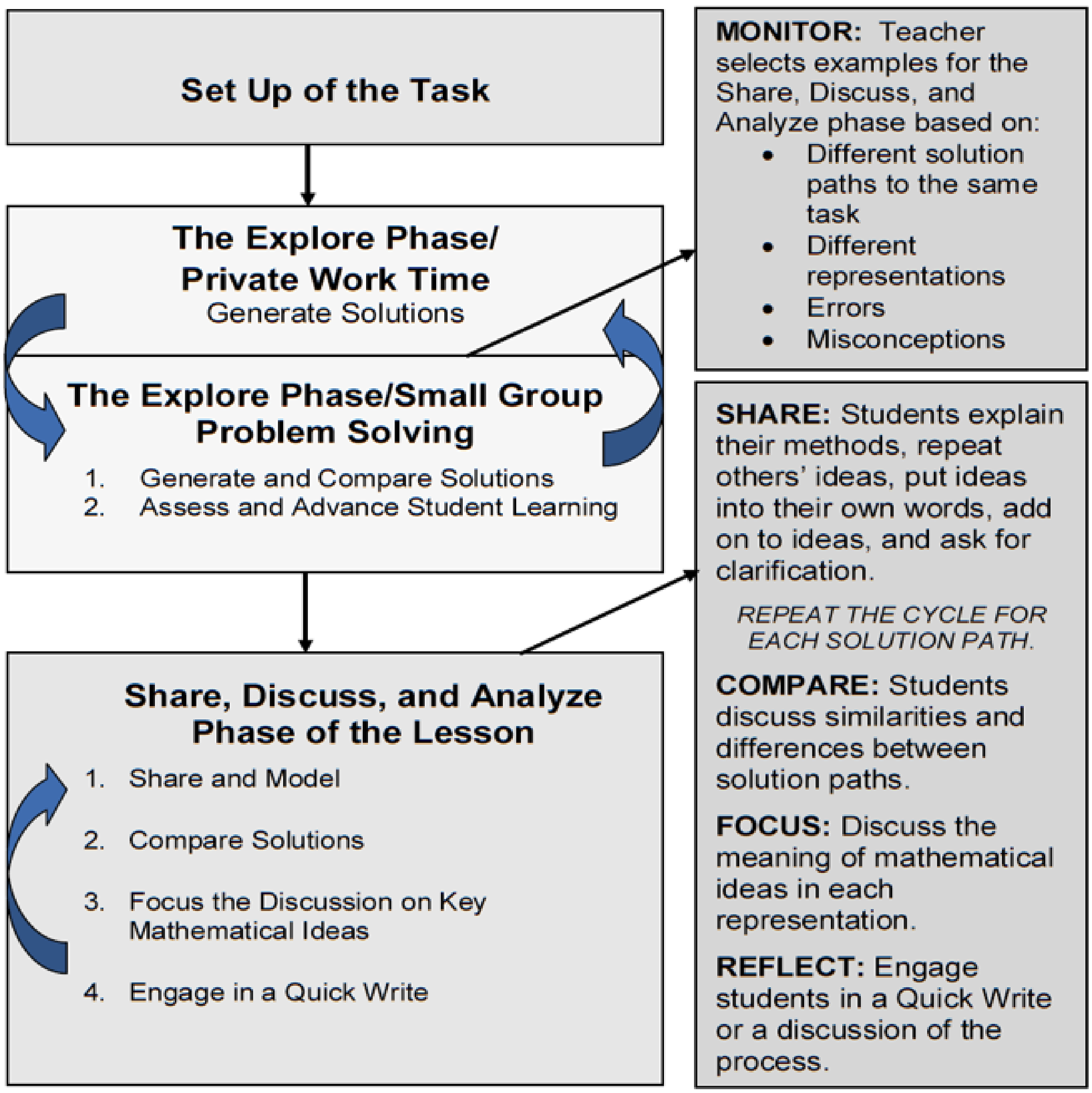
-10 in both represents the lowest degree celsius, +7 is the high in both. -3 is the difference in the high and low.

Engaging with the Teaching Practices

With your group, discuss the following questions to complete the chart on the next page.

- What would be your mathematical goal for students as they complete this task?
- For each piece of student work, write one assessing and one advancing question.
- What are the representations that you want to come out in the discussion to support the mathematics?
- Sequence the student work in the order you think it should be shared with the class. What questions might you ask to bring out and connect in support of your goal?
- Are there any other ways to support productive struggle? (i.e. scaffolds, manipulatives)

Structure and Routines of a Lesson



Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
What are the mathematical goals for this task?
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Write one assessing and advancing question for each student.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Sequence the student work for share-discuss-analyze phase of the lesson. *What questions might you ask to problematize the discussion?*
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Which representations are critical for understanding the mathematics? *How will you ask students to make connections among these representations?*
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Are there any other ways to support productive struggle? (i.e. scaffolds, manipulatives)

Establish mathematics goals to focus learning	
What are *teachers* doing?	**What are *students* doing?**
• Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. • Identifying how the goals fit within a mathematics learning progression. • Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. • Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	• Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) • Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. • Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. • Assessing and monitoring their own under-standing and progress toward the mathematics learning goals.

Pose purposeful questions	
What are *teachers* doing?	**What are *students* doing?**
• Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. • Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. • Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. • Allowing sufficient wait time so that more students can formulate and offer responses.	• Expecting to be asked to explain, clarify, and elaborate on their thinking. • Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. • Reflecting on and justifying their reasoning, not simply providing answers. • Listening to, commenting on, and questioning the contributions of their classmates.

Facilitate meaningful mathematical discourse	
What are *teachers* doing?	**What are *students* doing?**
• Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. • Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. • Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. • Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	• Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. • Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. • Seeking to understand the approach-es used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. • Identifying how different approaches to solving a task are the same and how they are different.

Use and connect mathematical representations	
What are *teachers* doing?	**What are *students* doing?**
• Selecting tasks that allow students to decide which representations to use in making sense of the problems. • Allocating substantial instructional time for students to use, discuss, and make connections among representations. • Introducing forms of representations that can be useful to students. • Asking students to make math drawings or use other visual supports to explain and justify their reasoning. • Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. • Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	• Using multiple forms of representations to make sense of and understand mathematics. • Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. • Making choices about which forms of representations to use as tools for solving problems. • Sketching diagrams to make sense of problem situations. • Contextualizing mathematical ideas by connecting them to real-world situations. • Considering the advantages or suitability of using various representations when solving problems.

Support productive struggle in learning mathematics	
What are *teachers* doing?	**What are *students* doing?**
• Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. • Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. • Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. • Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	• Struggling at times with mathematics tasks but knowing that breakthroughs of-ten emerge from confusion and struggle. • Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. • Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. • Helping one another without telling their classmates what the answer is or how to solve the problem.

Gallery Walk

- As a group, examine the work of the other groups.
- Leave targeted, actionable feedback.
- When you return to your group's work, consider revising your work based on the feedback.
- Be prepared to share your thinking with the whole group.

Reflection

- What are some new understandings you have regarding the tools to support task-based instruction?

- What do you believe to be the most critical considerations when reviewing student work? Why?

- Why is it important to consider the mathematical teaching practices when planning to implement a high-level task in your classroom?

- In what ways has this module increased your knowledge in any of the domains of MKT?

Module 4

Implications on Planning and Instruction

Rationale

"The move toward rigor places students squarely at the center of the classroom, where they will grapple with challenging content individually and collaboratively, and where they will be expected to actively demonstrate their learning."

-Marzano and Toth, 2014.

Goals

- Learn about TNReady design for mathematics and instructional implications
- Consider the importance of coherence for learning mathematics at the right grain size
- Examine how student learning is developed through a unit of study

Session Activities

- View blueprints, item types, and calculator policy of TNReady math assessment
- "Unpack" blueprints to consider instructional implications for 2015-2016
- Discuss how the concepts of coherence and grain size are connected to task arc creation and implementation
- Discuss planning units of study as they relate to coherence and the progression of student understanding

Overview

Participants in Module 4 will review the structure and design of the TNReady assessment. Activities in Module 4 engage participants in the process of focusing on the instructional and planning implications of the new assessment, while maintain a strong emphasis on intentionality and coherence and the impact of this focus on unit and lesson planning.

TNReady Note Table

	Noticings	Wonderings	Impact on Planning and Instruction
Math Priorities			
Calculator Policy			
Item Types			
Fluency			
Blueprints			

TNReady Overview

Beginning with the 2015-16 school year, TNReady will provide students, teachers, and parents with more detailed, accurate, and authentic information about each student's progress and achievement in the classroom.

TNReady is more than just a new "TCAP." It is a new way to assess what our students know and what we can do to help them succeed in the future.

TNReady Math Priorities

- Grades 3-8: Focus on fewer concepts – assess those topics in a range of ways
- High School: Strengthen coherence – assess topics in connected ways
- Include authentic assessment of real-life situations
- Support alignment with ACT
- Include calculator-permitted and calculator-prohibited sections at every grade level

Calculator Policy

Two central beliefs:

- Calculators are important tools for college and career readiness.
- Students must be able to demonstrate many skills without reliance on calculators.

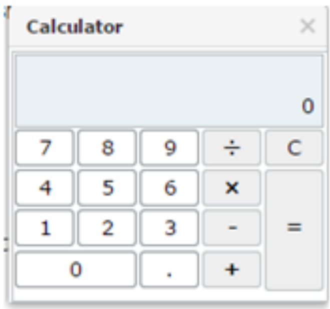
At all grade levels and in all courses, **TNReady will include both calculator permitted and calculator prohibited sections.**

Examples of permitted and non-permitted calculators, consistent with ACT and other benchmark assessments.

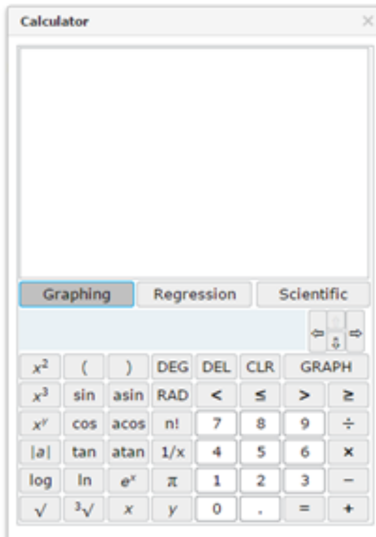
Handhelds are permitted with online testing.

Calculator Types

Basic



Graphing



Scientific

Calculator Policy – Think About it…

- How do the changes in calculator policy impact classroom instruction?

- What considerations will have to be made to ensure students are prepared for this transition?

Math Item Types Overview

There are 6 types of items in Mathematics:

1. Equation
2. Graphic
3. Multiple Choice
4. Multiple Select
5. Performance Tasks (for grade 3-8 only)
6. Technology Enhanced Items

Fluency

Grade	Standard	Expected Fluency
3	**3.**OA.C.7 **3.**NBT.A.2	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4	Add/Subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations

Grades 3-6 Fluency standards will only be assessed on Part II of the TNReady assessment. Calculators will not be allowed on fluency items.

Math Blueprints

Blueprint Summary Includes:

- Range of number of items for each part
- Range of percentage for each part
- Total range of number of items
- Total range of percentage for each part
- Percentage of test derived from each cluster

Each blueprint also includes a table that shows:

- What standards are assessed on Part I
- What standards are assessed on Part II
- An overall table for both Part I and Part II

Each of these tables also includes a range of number of items and a range of score points

Grades 3-8 Math Blueprints

- 100% of the content on Part I of the math section will be drawn from the major work of the grade
- 40-60% of the content on Part II of the math section will be drawn from the major work of the grade
- Across both Part I and Part II, 65-75% of the content of the math section will be drawn from the major work of the grade

Content drawn from the major work of the grade is bolded.

High School Math Blueprints

- Clusters have been grouped by category
- Part I includes items that are:
    - Best assessed through equation, graphic and performance tasks
    - Topics that are widely recognized prerequisites for college readiness
    - Topics that need to be treated in a coherent way
- Part II includes all standards with continued focus on questions that draw on the coherence of the standards
- Standards information is available by part

Blueprint Activity

Small Group

- Review the blueprint for your grade level or course.
- Look for the following ideas:
    - Which standards are only assessed in either Part I or Part II?
    - Which standards are assessed on both Part I and Part II?
    - Which sections are assessed most heavily? Least?

Whole Group Discussion

- How will this inform or modify instructional decisions for 2015-2016?
- What activities should we work to engage in to best prepare our students for success?

Grain Size

"Each discipline has a granularity at which its truth is clearest, most coherent. To depart from this grain size in either direction is to depart from the truth."

-Aristole, Ethics.

"[When considering mathematics] proper grain size is the unit at which it makes most sense to organize mathematics for learning."

-Daro, 2013.

Small Group Discussion

- Discuss your noticings and wonderings from the video with your small group.

Grain Size

- Mathematics is simplest at the right grain size
- "Strands" are too big, vague e.g. "numbers"
- Lessons are too small: too many small pieces scattered over the floor, what if some are missing or broken?
- Units are the right size (8-12 a year)
- Stop managing lessons
- Start managing units

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.

-CCSSM K-8 Publishers Criteria, 2012.

Small Group Discussion

- How are the concepts of coherence and grain size connected?

Whole Group Discussion

- Why is it important to consider coherence across standards and concepts when applying the ideas of “grain size” to our planning?

Think About It…

- How do we provide coherent structures around mathematical concepts to ensure students make connections?

Task Arc

[Materials] are coherent if they are: articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential and hierarchical nature of the disciplinary content from which the subject matter derives.

- Schmidt & Houang, 2012, "Curricular Coherence and the Common Core State Standards for Mathematics," Educational Researcher, http://edr.sagepub.com/content/41/8/294, p. 295.

Small Group Discussion

Review the task arc located in your handouts, specifically the arc preview and standards alignment pages. Discuss the following ideas with your small group:

- How does the structure of the task arc support instruction and student learning?
- How do the task arcs support the idea of grain size discussed in the video?
- How could task arcs be utilized in the planning of a unit of study?

Record your group take-aways on chart paper.

Private Think Time

- How do task arcs support the instruction of a group of standards?

Small Group Discussion

- How does the progression of standards within the task arc support student learning? Why is this progression important?

Unit Planning

It is the nature of mathematics that much new learning is about extending knowledge from prior learning to new situations. For this reason, teachers need to understand the progressions in the standards so they can see where individual students and groups of students are coming from, and where they are heading.

- Daro, McCallum, Zimba, 2012.

Private Think Time

- How do you, or teachers in your building, currently work to plan mathematic units of study?

- Do these practices align with the ideas of coherence, grain size, and learning progressions? How or how not?

Small Group Discussion

- How can you utilize the ideas of coherence and grain size to support your own unit planning?
- What resources and tools do you have to support unit planning?
- How can you utilize these tools and resources to ensure student learning develops throughout the unit?
- How do the teaching practices and MKT domains impact your ability to provide coherence and support student learning in your unit plan?

Record your groups thinking on chart paper.

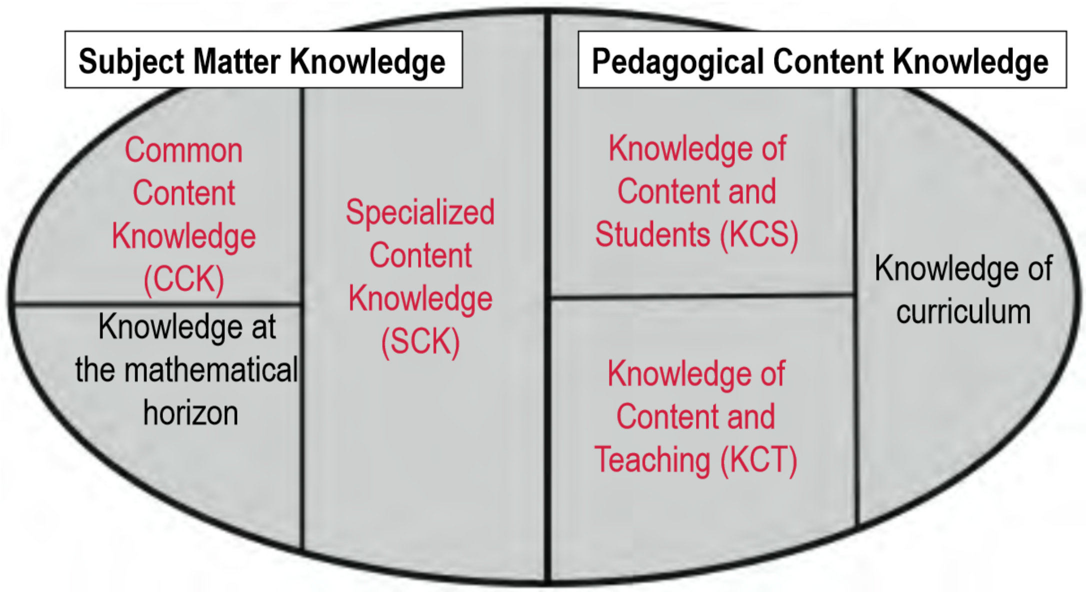
Gallery Walk

- Review other groups ideas on unit planning. Record noticings and wonderings as you walk.

Whole Group Discussion

- How do we work to ensure units of study support student progression through the content in a coherent manner?

Mathematical Knowledge for Teaching



Think About it…

- How does increasing the six domains of subject matter and pedagogical content knowledge areas of MKT support your implementation of the 8 teaching practices as you think about planning, instruction, and assessment?

- Why is it important to consider all six domains when planning a unit of study?

Reflection

- How has this module changed the way you think about planning and implementing units of study?

- What connections can you make between the four modules we have completed?

- What are you still wondering about?

Appendix

Appendix

Tennessee State Standards

TNCore tools

TNReady Blueprint

Task Resources

Progressions

Tennessee State Standards

Appendix A
Standards for Mathematical Practice

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Appendix B
Academic Standards

Tennessee's State Mathematics Standards | Grade 6

In Grade 6, instructional time should focus on four critical areas:

(1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;
(2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers;
(3) writing, interpreting, and using expressions and equations; and
(4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Domain	Cluster	Standard
Ratios and Proportional Relationships	**Understand ratio concepts and use ratio reasoning to solve problems.**	1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
		2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (Expectations for unit rates in this grade are limited to non-complex fractions.)
		3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
The Number System	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**	1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
	Compute fluently with multi-digit numbers and find common factors and multiples.	2. Fluently divide multi-digit numbers using the standard algorithm.
		3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
		4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Domain	Cluster	Standard
The Number System	**Apply and extend previous understandings of numbers to the system of rational numbers.**	5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
		6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
		7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3\,^{o}C > -7\,^{o}C$ to express the fact that $-3\,^{o}C$ is warmer than $-7\,^{o}C$.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $\|-30\| = 30$ to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*
		8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Domain	Cluster	Standard
Expressions and Equations	Apply and extend previous understandings of arithmetic to algebraic expressions.	1. Write and evaluate numerical expressions involving whole-number exponents.
		2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\,s^2$ *to find the volume and surface area of a cube with sides of length s = 1/2.*
		3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
		4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
	Reason about and solve one-variable equations and inequalities.	5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
		6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
		7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
		8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
	Represent and analyze quantitative relationships between dependent and independent variables.	9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Domain	Cluster	Standard
Geometry	**Solve real-world and mathematical problems involving area, surface area, and volume.**	1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
		2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
		3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world an mathematical problems.
		4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Statistics and Probability	**Develop understanding of statistical variability.**	1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
		2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
		3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
	Summarize and describe distributions.	4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
		5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Major Content	**Supporting Content**	**Additional Content**

Tennessee's State Mathematics Standards | Grade 7

In Grade 7, instructional time should focus on four critical areas:

(1) developing understanding of and applying proportional relationships;
(2) developing understanding of operations with rational numbers and working with expressions and linear equations;
(3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and
(4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Adopted by the Tennessee State Board of Education, July 2010

Domain	Cluster	Standard
Ratios and Proportional Relationships	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**	1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
		2. Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
		3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
The Number System	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**	1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.
		2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
		3. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Domain	Cluster	Standard
Expressions and Equations	Use properties of operations to generate equivalent expressions.	1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
		2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
		4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Geometry	Draw, construct, and describe geometrical figures and describe the relationships between them.	1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
		2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
		3. Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
		5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in the figure.
		6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Domain	Cluster	Standard
Statistics and Probability	**Use random sampling to draw inferences about a population.**	1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
		2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
	Draw informal comparative inferences about two populations.	3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
		4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
	Investigate chance processes and develop, use, and evaluate probability models.	5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
		6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
		7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
		8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood.*

	Major Content		**Supporting Content**		**Additional Content**

Tennessee's State Mathematics Standards | Grade 8

In Grade 8, instructional time should focus on three critical areas:

(1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;
(2) grasping the concept of a function and using functions to describe quantitative relationships;
(3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Domain	Cluster	Standard
The Number System	Know that there are numbers that are not rational, and approximate them by rational numbers.	1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
		2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
Expressions and Equations	**Work with radicals and integer exponents.**	1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*
		2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
		3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*
		4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
	Understand the connections between proportional relationships, lines, and linear equations.	5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
		6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
	Analyze and solve linear equations and pairs of simultaneous linear equations.	7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
		8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Domain	Cluster	Standard
Functions	**Define, evaluate, and compare functions.**	1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
		2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
		3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
	Use functions to model relationships between quantities.	4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
		5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Geometry	**Understand congruence and similarity using physical models, transparencies, or geometry software.**	1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
		2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
		3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
		4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.
		5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
	Understand and apply the Pythagorean Theorem.	6. Explain a proof of the Pythagorean Theorem and its converse.
		7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
		8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Domain	Cluster	Standard
Statistics and Probability	**Investigate patterns of association in bivariate data.**	1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
		2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
		3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
		4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

	Major Content		**Supporting Content**		**Additional Content**

TNCore Tools

Appendix C
Structure and Routines of a Lesson

The Structure and Routines of a Lesson

Set Up of the Task

The Explore Phase/ Private Work Time

Generate Solutions

The Explore Phase/Small Group Problem Solving

1. Generate and Compare Solutions
2. Assess and Advance Student Learning

Share, Discuss, and Analyze Phase of the Lesson

1. Share and Model
2. Compare Solutions
3. Focus the Discussion on Key Mathematical Ideas
4. Engage in a Quick Write

MONITOR: Teacher selects examples for the Share, Discuss, and Analyze phase based on:

- Different solution paths to the same task
- Different representations
- Errors
- Misconceptions

SHARE: Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification.

REPEAT THE CYCLE FOR EACH SOLUTION PATH.

COMPARE: Students discuss similarities and differences between solution paths.

FOCUS: Discuss the meaning of mathematical ideas in each representation.

REFLECT: Engage students in a Quick Write or a discussion of the process.

Appendix D
Math Accountable Talk®
Academic Discussion

institute for learning

TNCore

Accountable Talk® Features and Indicators

Accountability to the Learning Community

- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge

- Specific and accurate knowledge
- Appropriate evidence for claims and arguments

Accountable Talk® Moves

Talk Move	Function	Example
To Ensure Purposeful, Coherent, and Productive Group Discussion		
Marking	Direct attention to the value and importance of a student's contribution.	That's an important point.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	S: 4 + 4 + 4. You said three groups of four.
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?

institute for learning

TNCore

Accountable Talk® Moves

Talk Move	Function	Example
	To Ensure Purposeful, Coherent, and Productive Group Discussion	
Marking	Direct **attention** to the value and importance of a student's contribution.	It is important to say describe to compare the size of the pieces and then to look at how many pieces of that size.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	You said 3, yes there are three columns and each column is 1/3 of the whole
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?
	To Support Accountability to Community	
Keeping the Channels Open	Ensure that students can hear each other, and remind them that they must hear what others have said.	Say that again and louder. Can someone repeat what was just said?
Keeping Everyone Together	Ensure that everyone not only heard, but also understood, what a speaker said.	Can someone add on to what was said? Did everyone hear that?
Linking Contributions	Make explicit the relationship between a new contribution and what has gone before.	Does anyone have a similar idea? Do you agree or disagree with what was said? Your idea sounds similar to his idea.
Verifying and Clarifying	Revoice a student's contribution, thereby helping both speakers and listeners to engage more profitably in the conversation.	So are you saying..? Can you say more? Who understood what was said?
	To Support Accountability to Knowledge	
Pressing for Accuracy	Hold students accountable for the accuracy, credibility, and clarity of their contributions.	Why does that happen? Someone give me the term for that.
Building on Prior Knowledge	Tie a current contribution back to knowledge accumulated by the class at a previous time.	What have we learned in the past that links with this?
	To Support Accountability to Rigorous Thinking	
Pressing for Reasoning	Elicit evidence to establish what contribution a student's utterance is intended to make within the group's larger enterprise.	Say why this works. What does this mean? Who can make a claim and then tell us what their claim means?
Expanding Reasoning	Open up extra time and space in the conversation for student reasoning.	Does the idea work if I change the context? Use bigger numbers?

Appendix E
Task Analysis Guide

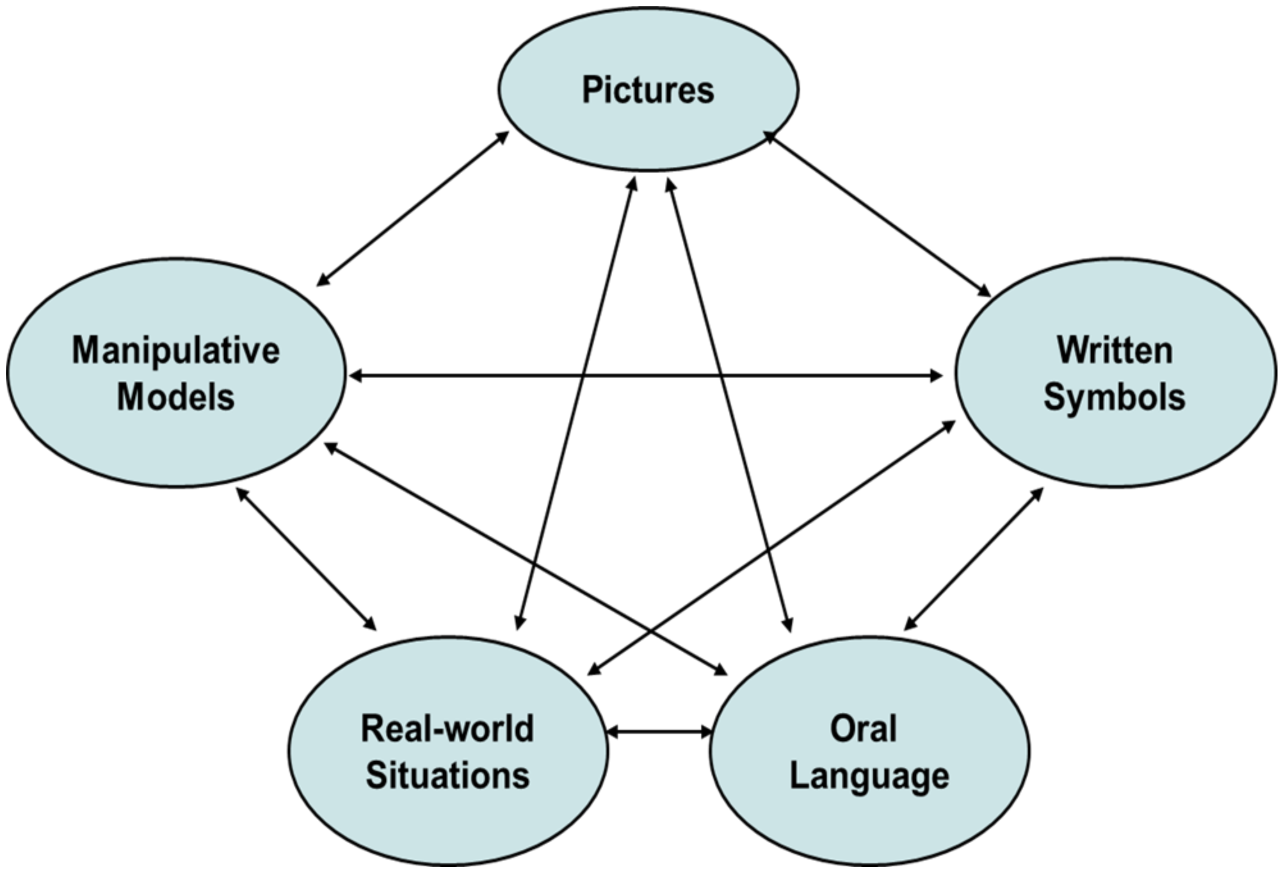
The Mathematical Task Analysis Guide

Lower-Level Demands	*Higher-Level Demands*
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	Doing Mathematics Tasks • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Appendix F
Connections between Representations

Linking to Research/Literature
Connections between Representations



Adapted from Lesh, Post, & Behr, 1987

Appendix G
Strategies for Modifying Textbook Tasks

Strategies for Modifying Textbook Tasks

Compare your list of task modifications with the list of task modification strategies identified by others. How is your list similar? Different?

- Ask students to create real-world stories for “naked number” problems.
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context).
- Include a prompt that requires students to make a generalization.
- Use a task “out of sequence” before students have memorized a rule or have practiced a procedure that can be routinely applied.
- Eliminate components of the task that provide too much scaffolding.
- Adapt a task so as to provide more opportunities for students to think and reason – let students figure things out for themselves.
- Create a prompt that asks students to write about the meaning of the mathematics concept.
- Add a prompt that asks students to make note of a pattern or to make a mathematical conjecture and to test their conjecture.
- Include a prompt that requires students to compare solution paths or mathematical relationships and write about the relationship between strategies or concepts.
- Select numbers carefully so students are more inclined to note relationships between quantities (e.g., two tables can be used to think about the solutions to the four, six or eight tables).

TNReady Blueprint

Appendix H
TNReady Blueprints

TNReady 6th Grade Math Blueprint

	Part I		Part II		Total # of Items	Total # of score points	% of Test
	# of items	% of PT 1	# of items	% of PT 2			
Major Work of the Grade	**25-29**	**100%**	**16-20**	**41-53%**	**41-49**	**41-53**	**65-70%**
• **Understand ratios**	**8-10**	**32-34%**	**3-5**	**9-11%**	**11-15**	**11-17**	**18-20%**
• **Multiply and divide fractions**	**2-4**	**10-12%**	**1-3**	**4-6%**	**3-7**	**3-9**	**6-8%**
• **Apply system of rational numbers**	**4-6**	**18-20%**	**2-4**	**7-9%**	**6-10**	**6-12**	**11-13%**
• **Arithmetic with algebraic expressions**	**3-5**	**14-16%**	**2-4**	**7-9%**	**5-9**	**5-11**	**9-11%**
• **Solve one-variable equations and inequalities**	**3-5**	**14-16%**	**2-4**	**7-9%**	**5-9**	**5-11**	**9-11%**
• **Represent relationships between variables**	**1-3**	**3-5%**	**2-4**	**7-9%**	**3-7**	**3-9**	**5-7%**
Additional and Supporting	0	0%	20-22	53-61%	20-22	20-33	30-35%
• Solve area, surface area, and volume problems	0	0%	6-8	17-19%	6-8	6-10	9-11%
• Compute fluently with multi-digit numbers	0	0%	4-6	12-14%	4-6	4-8	6-8%
• Understand statistical variability	0	0%	4-6	12-14%	4-6	4-8	6-8%
• Describe distributions	0	0%	4-6	12-14%	4-6	4-8	6-8%
Total	**25-29**	**100%**	**36-42**	**100%**	**61-71**	**61-86**	**100%**

Additional Notes:

*****On Part I, 100% of the content in grades 3-8 mathematics will be drawn from the clusters designated as major work of the grade. The math standards in grades 3-8 are coherent and the connections between major work and the additional and supporting clusters are important throughout the year. Part II questions measure major, additional, and supporting topics, including the integration of the key ideas at each grade level.

*****The total number of score points does not match the total number of items. This is because some items may be worth more than one point.

*****Clusters drawn from the major work of the grade are bolded throughout this document.

Part I – Calculator Allowed

100% of the content in Part I is drawn from the major work

Cluster	Standards		# of Items
6.RP.A - Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	8-10
	6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.	
	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	
6.NS.A - Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	2-4
6.NS.C - Apply and extend previous understandings of numbers to the system of rational numbers.	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	4-6
	6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	
	6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or	

<table>
<tr><td rowspan="2"></td><td></td><td>negative quantity in a real-world situation.
d. Distinguish comparisons of absolute value from statements about order.</td><td rowspan="2"></td></tr>
<tr><td>6.NS.C.8</td><td>Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</td></tr>
<tr><td rowspan="4">6.EE.A - Apply and extend previous understandings of arithmetic to algebraic expressions.</td><td>6.EE.A.1</td><td>Write and evaluate numerical expressions involving whole-number exponents.</td><td rowspan="4">3-5</td></tr>
<tr><td>6.EE.A.2</td><td>Write, read, and evaluate expressions in which letters stand for numbers.
a. Write expressions that record operations with numbers and with letters standing for numbers.
b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</td></tr>
<tr><td>6.EE.A.3</td><td>Apply the properties of operations to generate equivalent expressions.</td></tr>
<tr><td>6.EE.A.4</td><td>Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).</td></tr>
<tr><td rowspan="4">6.EE.B - Reason about and solve one-variable equations and inequalities.</td><td>6.EE.B.5</td><td>Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</td><td rowspan="4">3-5</td></tr>
<tr><td>6.EE.B.6</td><td>Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</td></tr>
<tr><td>6.EE.B.7</td><td>Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.</td></tr>
<tr><td>6.EE.B.8</td><td>Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</td></tr>
<tr><td>6.EE.C - Represent and analyze quantitative relationships between dependent and independent variables.</td><td>6.EE.C.9</td><td>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</td><td>1-3</td></tr>
</table>

Part II – Calculator and Non-Calculator Portions

45-50% of the content in Part II is drawn from the major work

<table>
<tr><th>Cluster</th><th colspan="2">Standards</th><th># of Items</th></tr>
<tr><td rowspan="3">6.RP.A - Understand ratio concepts and use ratio reasoning to solve problems.</td><td>6.RP.A.1</td><td>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</td><td rowspan="3">3-5</td></tr>
<tr><td>6.RP.A.2</td><td>Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.</td></tr>
<tr><td>6.RP.A.3</td><td>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed.
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td></tr>
<tr><td>6.NS.A - Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</td><td>6.NS.A.1</td><td>Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</td><td>1-3</td></tr>
<tr><td rowspan="3">6.NS.C - Apply and extend previous understandings of numbers to the system of rational numbers.</td><td>6.NS.C.5</td><td>Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</td><td rowspan="3">2-4</td></tr>
<tr><td>6.NS.C.6</td><td>Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</td></tr>
<tr><td>6.NS.C.7</td><td>Understand ordering and absolute value of rational numbers.
a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.</td></tr>
</table>

		c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order.	
	6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	
6.EE.A - Apply and extend previous understandings of arithmetic to algebraic expressions.	6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	2-4
	6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	
	6.EE.A.3	Apply the properties of operations to generate equivalent expressions.	
	6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).	
6.EE.B - Reason about and solve one-variable equations and inequalities.	6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	2-4
	6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
	6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	
	6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	
6.EE.C - Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	2-4
6.G.A - Solve real-world and mathematical problems involving area, surface area, and volume.	6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	6-8
	6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying	

		the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	
	6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	
	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	
6.NS.B - Compute fluently with multi-digit numbers and find common factors and multiples.	6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.	4-6
	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	
	6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	
6.SP.A - Develop understanding of statistical variability.	6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	4-6
	6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	
	6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	
6.SP.B - Summarize and describe distributions.	6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	4-6
	6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

Overall Blueprint (includes Part I and Part II)

Cluster	Standards		# of Items	% of Test
6.RP.A - Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	11-15	18-20%
	6.RP.A.2	Understand the concept of a unit rate *a*/*b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship.		
	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.		
6.NS.A - Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	3-7	6-8%
6.NS.C - Apply and extend previous understandings of numbers to the system of rational numbers.	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	7-11	11-13%
	6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.		
	6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a		

		positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order.		
	6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.		
6.EE.A - Apply and extend previous understandings of arithmetic to algebraic expressions.	6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	5-9	9-11%
	6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).		
	6.EE.A.3	Apply the properties of operations to generate equivalent expressions.		
	6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).		
6.EE.B - Reason about and solve one-variable equations and inequalities.	6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	5-9	9-11%
	6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.		
	6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.		
	6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.		
6.EE.C - Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	3-7	5-7%
6.G.A - Solve real-world and mathematical problems involving area, surface area, and volume.	6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	6-8	9-11%
	6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional		

		edge lengths in the context of solving real-world and mathematical problems.		
	6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.		
	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.		
6.NS.B - Compute fluently with multi-digit numbers and find common factors and multiples.	6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.	4-6	6-8%
	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.		
	6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.		
6.SP.A - Develop understanding of statistical variability.	6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	4-6	6-8%
	6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.		
	6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.		
6.SP.B - Summarize and describe distributions.	6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	4-6	6-8%
	6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.		

TNReady 7th Grade Math Blueprint

	Part I		Part II		Total # of items	Total # of score points	% of Test
	# of items	% of PT I	# of items	% of PT II			
Major Work of the Grade	**25-29**	**100%**	**16-18**	**40-45%**	**44-46**	**44-59**	**65-70%**
• **Analyze proportional relationships**	**10-12**	**39-41%**	**5-7**	**14-16%**	**15-19**	**15-21**	**25-27%**
• **Operate with rational numbers**	**6-8**	**24-26%**	**4-6**	**12-14%**	**10-14**	**10-16**	**17-19%**
• **Generate equivalent expressions**	**4-6**	**17-19%**	**2-4**	**7-9%**	**6-10**	**6-12**	**11-13%**
• **Solve problems using expressions and equations**	**4-6**	**17-19%**	**2-4**	**7-9%**	**6-10**	**6-12**	**11-13%**
Additional and Supporting Work of the Grade	0	0%	20-24	55-60%	20-24	20-32	30-35%
• Geometrical figures	0	0	2-4	7-9%	2-4	2-6	4-6%
• Angle measure, area, surface area	0	0	3-5	9-11%	3-5	3-7	5-7%
• Statistics: Random sampling	0	0	4-6	13-15%	4-6	4-8	7-9%
• Statistics: Population comparisons	0	0	3-5	9-11%	3-5	3-7	5-7%
• Chance and probability models	0	0	4-6	11-13%	4-6	4-8	7-9%
Total	**25-29**	**100%**	**36-42**	**100%**	**64-70**	**64-82**	**100%**

Additional Notes:

*On Part I, 100% of the content in grades 3-8 mathematics will be drawn from the clusters designated as major work of the grade. The math standards in grades 3-8 are coherent and the connections between major work and the additional and supporting clusters are important throughout the year. Part II questions measure major, additional, and supporting topics, including the integration of the key ideas at each grade level.

*The total number of score points does not match the total number of items. This is because some items may be worth more than one point.

*Clusters drawn from the major work of the grade are bolded throughout this document.

Part I-Calculator Allowed

100% of the content in Part I is drawn from the major work of the grade

Cluster	Standards		# of Items
7.RP.A - Analyze proportional relationships and use them to solve real-world and mathematical problems	7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	10-12
	7.RP.A.2	Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	
	7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems.	
7.NS.A – Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers.	6-8
	7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	
7.EE.A – Use properties of operations to generate equivalent	7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	4-6
	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	

expressions			
7.EE.B - Solve real-life and mathematical problems using numerical and algebraic expressions and equations	7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	4-6
	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	

Part II – Calculator and Non-Calculator Portions

40-45% of the content in Part II is drawn from the major work of the grade

Cluster	Standards		# of Items
7.RP.A- Analyze proportional relationships and use them to solve real-world and mathematical problems	7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	5-7
	7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.	
	7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems.	
7.NS.A- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	4-6
	7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	
7.EE.A- Use properties of operations to generate equivalent expressions	7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	2-4
	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	

7.EE.B- Solve real-life and mathematical problems using numerical and algebraic expressions and equations	**7.EE.B.3**	**Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.**	**2-4**
	7.EE.B.4	**Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.** **a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.** **b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.**	
7.G.A – Draw, construct, and describe geometrical figures and describe the relationships between them	7.G.A.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	2-4
	7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	
	7.G.A.3	Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	
7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume	7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	3-5
	7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in the figure.	
	7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	
7.SP.A – Use random sampling to draw inferences about a population	7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	4-6
	7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions	
7.SP.B – Draw informal comparative inferences about two populations	7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	3-5
	7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	
7.SP.C – Investigate chance processes and develop, use, and evaluate probability models	7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	4-6
	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	

	7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	
	7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items	% of Test
7.RP.A - Analyze proportional relationships and use them to solve real-world and mathematical problems	7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	15-19	25-27%
	7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.		
	7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems.		
7.NS.A - Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	10-14	17-19%
	7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.		
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the		

		rules for manipulating fractions to complex fractions.)		
7.EE.A - Use properties of operations to generate equivalent expressions	7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	6-10	11-13%
	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.		
7.EE.B - Solve real-life and mathematical problems using numerical and algebraic expressions and equations	7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	6-10	11-13%
	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.		
7.G.A - Draw, construct, and describe geometrical figures and describe the relationships between them	7.G.A.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	2-4	4-6%
	7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.		
	7.G.A.3	Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.		
7.G.B - Solve real-life and mathematical problems involving angle measure, area, surface area, and volume	7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	3-5	5-7%
	7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in the figure.		
	7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.		
7.SP.A - Use random sampling to draw inferences about a population	7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	4-6	7-9%
	7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions		
7.SP.B - Draw informal comparative inferences about	7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	3-5	5-7%
	7.SP.B.4	Use measures of center and measures of variability for numerical data		

two populations		from random samples to draw informal comparative inferences about two populations.		
7.SP.C - Investigate chance processes and develop, use, and evaluate probability models	7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	4-6	7-9%
	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.		
	7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?		
	7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.		

TNReady 8th Grade Math Blueprint

	Part I		Part II		Total # of Items	Total # of score points	% of Test
	# of items	% of PT 1	# of items	% of PT 2			
Major Work of the Grade	**25-29**	**100%**	**19-23**	**50-55%**	**46-50**	**46-56**	**65-70%**
• **Work with radicals and integer exponents.**	**2-4**	**12-14%**	**2-4**	**6-8%**	**4-8**	**4-10**	**8-10%**
• **Understand connections between proportional relationships and linearity**	**3-5**	**14-16%**	**2-4**	**6-8%**	**5-9**	**5-11**	**9-11%**
• **Solve linear equations and systems of linear equations.**	**3-5**	**14-16%**	**2-4**	**6-8%**	**5-9**	**5-11**	**9-11%**
• **Define, evaluate, and compare functions.**	**4-6**	**17-19%**	**2-4**	**7-9%**	**6-10**	**6-12**	**11-13%**
• **Use functions to model relationships between quantities.**	**4-6**	**17-19%**	**2-4**	**7-9%**	**6-10**	**6-12**	**11-13%**
• **Understand congruence and similarity**	**2-4**	**12-14%**	**2-4**	**7-9%**	**4-8**	**4-10**	**8-10%**
• **Understand and apply the Pythagorean Theorem.**	**2-4**	**9-11%**	**2-4**	**6-8%**	**4-8**	**4-10**	**8-10%**
Additional and Supporting	0	0%	18-22	45-50%	18-22	18-28	30-35%
• Know that there are numbers that are not rational	0	0%	6-8	16-18%	6-8	6-10	9-11%
• Solve problems involving volume of cylinders, cones, and spheres	0	0%	4-6	11-13%	4-6	4-8	6-8%
• Investigate patterns of association in bivariate data	0	0%	7-9	19-21%	7-9	7-11	11-13%
Total	**25-29**	**100%**	**37-45**	**100%**	**64-72**	**64-84**	**100%**

Additional Notes:

*****On Part I, 100% of the content in grades 3-8 mathematics will be drawn from the clusters designated as major work of the grade. The math standards in grades 3-8 are coherent and the connections between major work and the additional and supporting clusters are important throughout the year. Part II questions measure major, additional, and supporting topics, including the integration of the key ideas at each grade level.

*****The total number of score points does not match the total number of items. This is because some items may be worth more than one point.

*****Clusters drawn from the major work of the grade are bolded throughout this document.

Part I – Calculator Allowed

100% of the content in Part I is drawn from the major work

Cluster	Standards		# of Items
8.EE.A – Work with radicals and integer exponents.	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	2-4
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	
	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	
	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
8.EE.B – Understand the connections between proportional relationships, lines, and linear equations.	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	3-5
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	
8.EE.C – Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.C.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	3-5
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.	

<table>
<tr><td rowspan="3">8.F.A.–
Define, evaluate, and compare functions.</td><td>8.F.A.1</td><td>Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)</td><td rowspan="3">4-6</td></tr>
<tr><td>8.F.A.2</td><td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td></tr>
<tr><td>8.F.A.3</td><td>Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</td></tr>
<tr><td rowspan="2">8.F.B –
Use functions to model relationships between quantities.</td><td>8.F.B.4</td><td>Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td><td rowspan="2">4-6</td></tr>
<tr><td>8.F.B.5</td><td>Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</td></tr>
<tr><td rowspan="5">8.G.A –
Understand congruence and similarity using physical models, transparencies, or geometry software.</td><td>8.G.A.1</td><td>Verify experimentally the properties of rotations, reflections, and translations:
a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.</td><td rowspan="5">2-4</td></tr>
<tr><td>8.G.A.2</td><td>Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</td></tr>
<tr><td>8.G.A.3</td><td>Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</td></tr>
<tr><td>8.G.A.4</td><td>Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.</td></tr>
<tr><td>8.G.A.5</td><td>Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.</td></tr>
<tr><td rowspan="3">8.G.B –
Understand and apply the Pythagorean Theorem.</td><td>8.G.B.6</td><td>Explain a proof of the Pythagorean Theorem and its converse.</td><td rowspan="3">2-4</td></tr>
<tr><td>8.G.B.7</td><td>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</td></tr>
<tr><td>8.G.B.8</td><td>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td></tr>
</table>

Part II – Calculator and Non-Calculator Portions

50-55% of the content in Part II is drawn from the major work

Cluster	Standards		# of items
8.EE.A – Work with radicals and integer exponents.	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	2-4
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	
	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	
	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
8.EE.B – Understand the connections between proportional relationships, lines, and linear equations.	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	2-4
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	
8.EE.C – Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.C.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	2-4
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.	

8.F.A.– Define, evaluate, and compare functions.	8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)	2-4
	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	
8.F.B – Use functions to model relationships between quantities.	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	2-4
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	
8.G.A – Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	2-4
	8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
	8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	
	8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.	
	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	
8.G.B – Understand and apply the Pythagorean Theorem.	8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.	2-4
	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
	8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
8.NS.A – Know that there are numbers that are not rational,	8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	6-8

and approximate them by rational numbers.	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).	
8.G.C – Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	4-6
8.SP.A – Investigate patterns of association in bivariate data.	8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	7-9
	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	
	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	

Overall Blueprint (includes Part I and Part II)

Cluster	Standards		# of Items	% of Test
8.EE.A – Work with radicals and integer exponents.	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	4-8	8-10%
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.		
	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.		
	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.		
8.EE.B – Understand the connections between proportional relationships, lines, and linear equations.	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	5-9	9-11%
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.		
8.EE.C – Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.C.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	5-9	9-11%

	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.		
8.F.A.– Define, evaluate, and compare functions.	8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)	6-10	11-13%
	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).		
	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.		
8.F.B – Use functions to model relationships between quantities.	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	6-10	11-13%
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.		
8.G.A – Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	4-8	8-10%
	8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.		
	8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.		
	8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.		

	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.		
8.G.B – Understand and apply the Pythagorean Theorem.	8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.	4-8	8-10%
	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.		
	8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		
8.NS.A – Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	6-8	9-11%
	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).		
8.G.C – Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	4-6	6-8%
8.SP.A – Investigate patterns of association in bivariate data.	8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	7-9	11-13%
	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.		
	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.		
	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.		

Appendix I
Fluency Document

Tennessee State Standards, Fluency in Mathematics

Fluency is not meant to come at the expense of conceptual understanding. Rather, it should be an outcome resulting from a progression of learning and thoughtful practice. It is important to provide the conceptual building blocks that develop understanding along with skill toward developing fluency; the roots of this conceptual understanding often occur one or more grades earlier in the standards than the grade when fluency is expected.

"Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently*. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships."

NCTM, Principles and Standards for School Mathematics, p. 152 (2000)

Fluency Expectations, K-8

Grade	Standard	Expected Fluency
K	K.OA.A.5	Add/Subtract within 5
1	1.OA.C.6	Add/Subtract within 10
2	2.OA.B.2 2.NBT.B.5	Add/Subtract within 20 (Know single digit sums from memory) Add/Subtract within 100
3	3.OA.C.7* 3.NBT.A.2*	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4*	Add/Subtract within 1,000,000
5	5.NBT.B.5*	Multi-digit multiplication
6	6.NS.B.2* 6.NS.B.3*	Multi-digit division Multi-digit decimal operations
7	7.NS.A.1,2 7.EE.B.3 7.EE.B.4	Fluency with rational number arithmetic Solve multistep problems with positive and negative rational numbers in any form Solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$ fluently
8	8.EE.C.7 8.G.C.9	Solve one-variable linear equations, including cases with infinitely many solutions or no solutions Solve problems involving volumes of cones, cylinders, and spheres together with previous geometry work, proportional reasoning and multi-step problem solving in grade 7

*These fluency standards will be assessed on TNReady. Students will not have access to a calculator for fluency items on TNReady.

Tennessee State Standards, Fluency in Mathematics

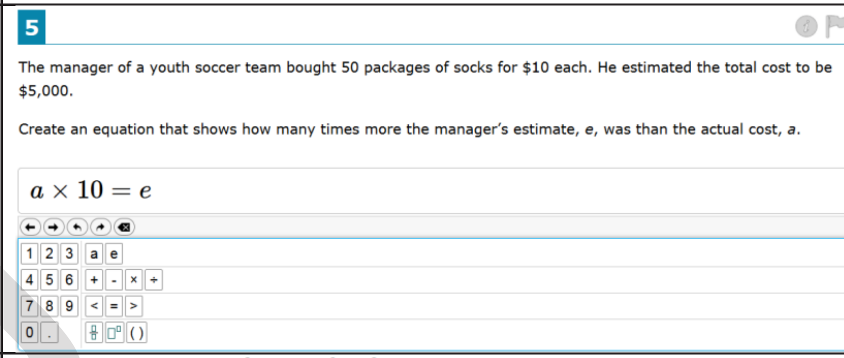
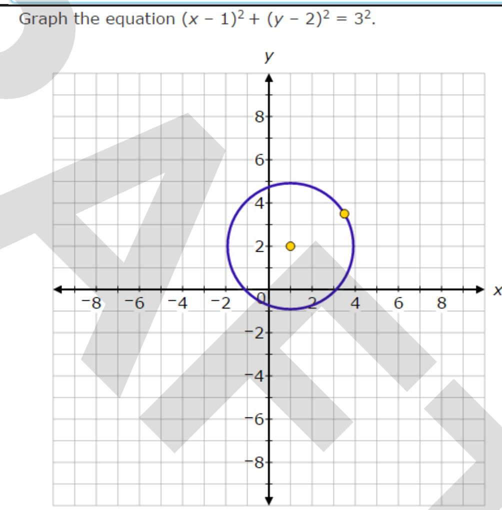
The high school standards do not set explicit expectations for fluency, but fluency is important in high school mathematics. Fluency in algebra can help students get past the need to manage computational and algebraic manipulation details so that they can observe structure and patterns in problems. Such fluency can also allow for smooth progress toward readiness for further study/careers in science, technology, engineering, and mathematics (STEM) fields. These fluencies are highlighted to stress the need to provide sufficient supports and opportunities for practice to help students gain fluency. Fluency is not meant to come at the expense of conceptual understanding. Rather, it should be an outcome resulting from a progression of learning and thoughtful practice. It is important to provide the conceptual building blocks that develop understanding along with skill toward developing fluency

Fluency Recommendations, High School

Course	Standard	Recommended Fluency
Algebra I	A/G A-APR.A.1 A-SSE.A.1b	Solving characteristic problems involving the analytic geometry of lines Fluency in adding, subtracting, and multiplying polynomials Fluency in transforming expressions and seeing parts of an expression as a single object
Geometry	G-SRT.B.5 G-GPE.B.4, 5, 7 C-CO.D.12	Fluency with the triangle congruence and similarity criteria Fluency with the use of coordinates Fluency with the use of construction tools
Algebra II	A-APR.D.6 A-SSE.A.2 F.IF.A.3	Divide polynomials with remainder by inspection in simple cases See structure in expressions and use this structure to rewrite expressions Fluency in translating between recursive definitions and closed forms

Appendix J
Item Types

Math Item Type

Item Type	Examples	Illustration
Equation: Students generate response. On both part I and part II. Typically worth one point (one right answer). 10-15% of questions.	**Equation Editor:** Students type in numeric answers from a palette of options.	
Graphic: Students depict graphically. On both part I and part II but more often on part I. Usually one point. 10-25% of questions, more in high school.	**Circles:** Students graph a circle by plotting the center point first and then the radius.	

Graphic (continued)

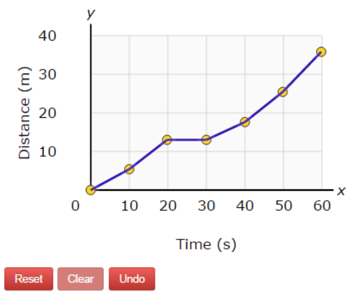
Line Graph: Students click to add a point. Adding another point will automatically connect the points to form a line graph**.**

The table below shows the speed of a bicyclist.

Using the data in the table, create a line on the graph showing the bicyclist's speed.

Bicyclist Speed

Time (s)	Distance (m)
0	0
10	5
20	12
30	12
40	18
50	25
60	36



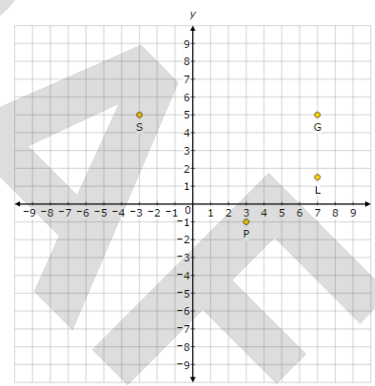
Placing Points: Students click to add a point.

The data table shows the locations of different buildings in a town.

Town Buildings

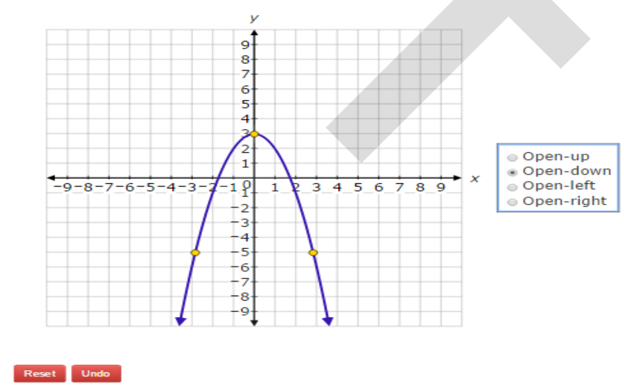
Buildings	Location
Grocery Store (G)	(7, 5)
Library (L)	(7, 1.5)
Park (P)	(3, −1)
School (S)	(−3, 5)

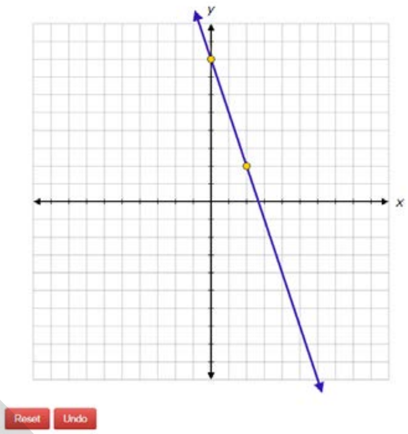
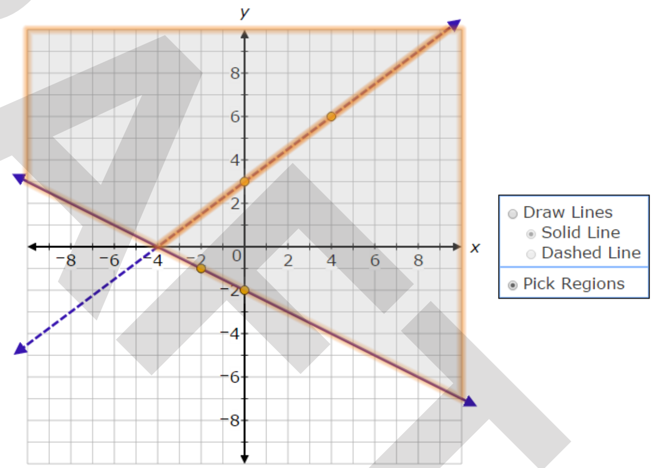
The grid shown represents a town. Plot each point on the coordinate grid. Label each point with the name of the building.

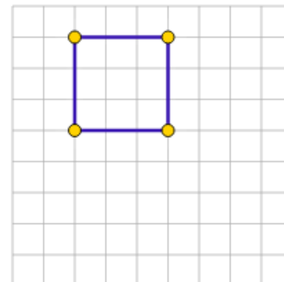


Single Parabola: Students select open-up, open-down, open-left, open-right to place a parabola on the grid. The points or the entire line can then be moved.

Graph the equation $y = -x^2 + 3$ on the coordinate grid.



Graphic (continued)	**Straight Lines:** Students click to place the first point then clicks again to place the second point, which creates the line. It can be either a line segment or a line.	Graph the line $y = -3x + 8$. 
	Straight lines then select: Students plot lines on a grid similar to "Single Line and Straight Lines" and select dotted or solid. Then students select "Pick Regions" to shade a region.	Graph the solution to the inequalities. $-3x + 4y > 12$ $x + 2y \geq -4$ 

<table>
<tr><td></td><td>Vertex based Quadrilaterals: Students plot points to form different polygons. The option can be set to close the shape after four points are plotted.</td><td>Create a shape on the grid with the following:
• 4 sides of equal length
• 2 pairs of parallel sides
• 4 right angles
</td></tr>
<tr><td rowspan="2">Multiple Choice: Students select one answer, only one answer is correct.

On both part I and part II.

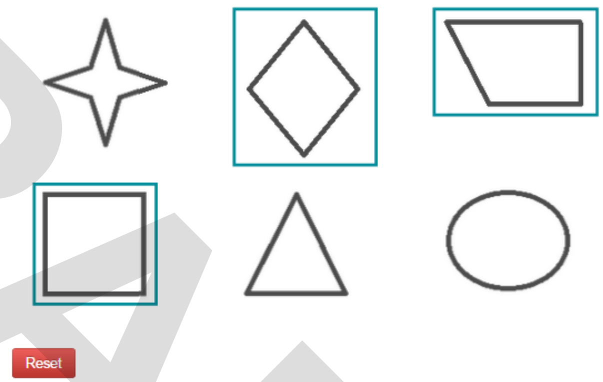
Worth one point.

60-75% of questions.</td><td>Dropdowns: Students choose from a dropdown list.</td><td>Use this number to create a true sentence.
684.425
The value of the 4 in the tenths place is [one-tenth of ▾] the value of the 4 in the ones place.</td></tr>
<tr><td>Multiple Choice: Student selects one correct answer from many options.</td><td>1
What is 78 rounded to the nearest ten?
Ⓐ 70
Ⓑ 75
● 80
Ⓓ 100</td></tr>
<tr><td>Multiple select: Students select multiple options, multiple correct.

On both part I and part II but more on part II.

Can be worth one to two points, depending on the</td><td>Check Box: Students select multiple correct answer choices.</td><td>3
Consider the family of quadrilaterals that includes parallelograms, rectangles, squares, and rhombuses.
Select all the statements about these quadrilaterals that are true.
☑ Squares are always rectangles.
☐ Rectangles are always squares.
☐ Rhombuses are always squares.
☑ Squares are always rhombuses.
☑ Rhombuses are always parallelograms.
☑ Rhombuses are sometimes rectangles.</td></tr>
</table>

<table>
<tr>
<td>wording of the question.

Up to 10% of questions.</td>
<td>Matching Table: Students select multiple correct answer choices.</td>
<td>Select one phrase that describes the value of each expression.
<table>
<tr><th></th><th>Greater than 3</th><th>Equal to 3</th><th>Less than 3</th></tr>
<tr><td>$3\times\frac{1}{2}$</td><td>☐</td><td>☐</td><td>☑</td></tr>
<tr><td>$3\times1\frac{1}{4}$</td><td>☑</td><td>☐</td><td>☐</td></tr>
<tr><td>$3\times\frac{6}{6}$</td><td>☐</td><td>☑</td><td>☐</td></tr>
<tr><td>$3\times\frac{3}{2}$</td><td>☑</td><td>☐</td><td>☐</td></tr>
</table></td>
</tr>
<tr>
<td></td>
<td>Select Objects: Students click on objects to select. A select border color appears around the objects when selected.</td>
<td>Click on the shapes that are quadrilaterals.
</td>
</tr>
<tr>
<td>Performance Tasks: Solve multistep problems and demonstrate how solution is achieved.

1 task on part I only in grades 3-8 only.

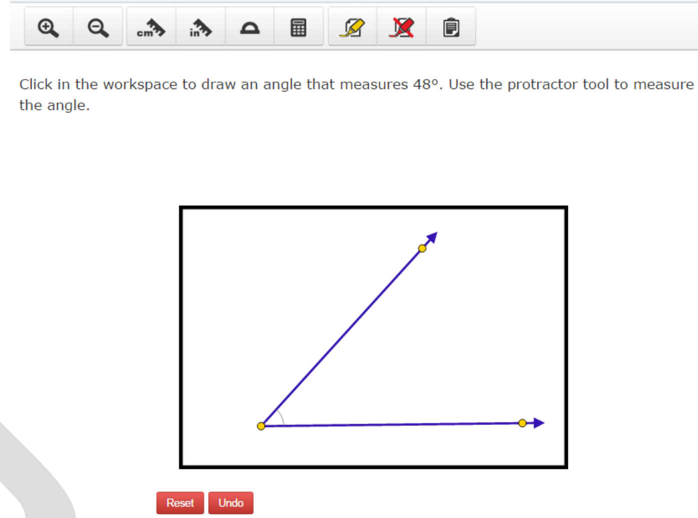
Worth 10-15 points with partial credit.</td>
<td>These will not mimic the previous CRA tasks but will require multistep problem solving and will require students to explain the problem solving approach.

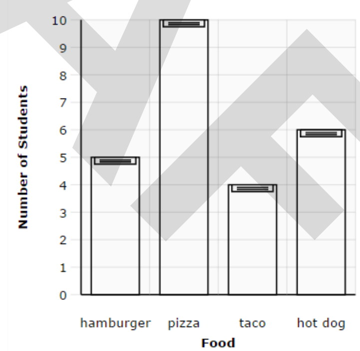
Partial credit will be available.</td>
<td>We are waiting on copyright clearance to release an example performance task. A depiction of a similar type of problem will be shared in the power point.</td>
</tr>
</table>

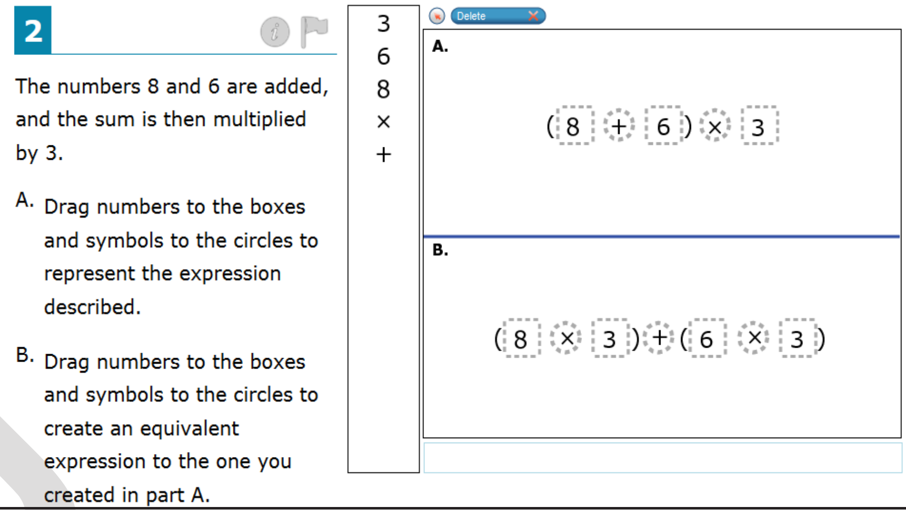
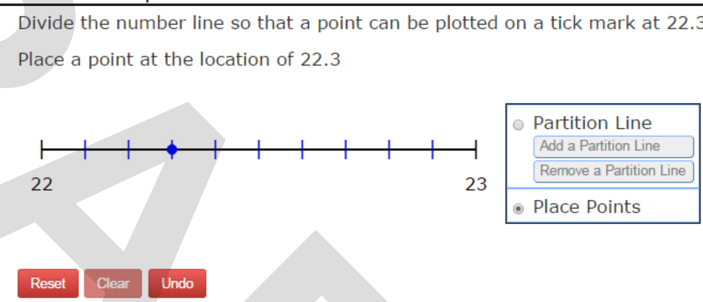
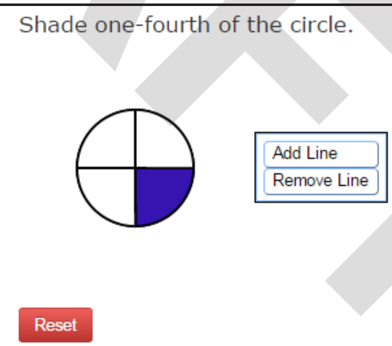
<table>
<tr>
<td rowspan="2">TEI: Students perform an interaction to respond to the question.

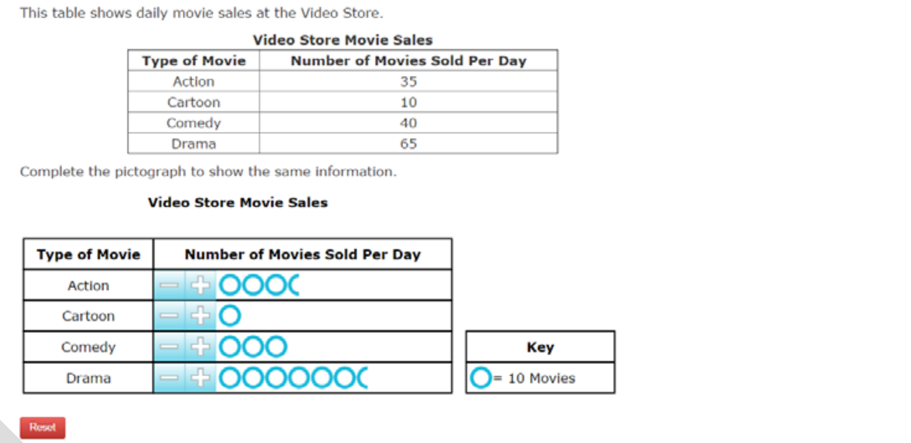
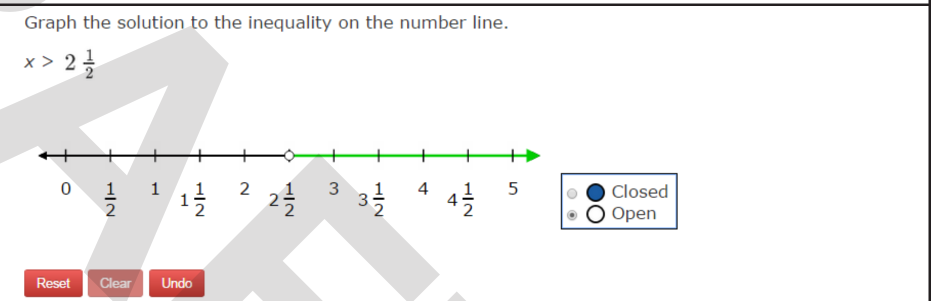
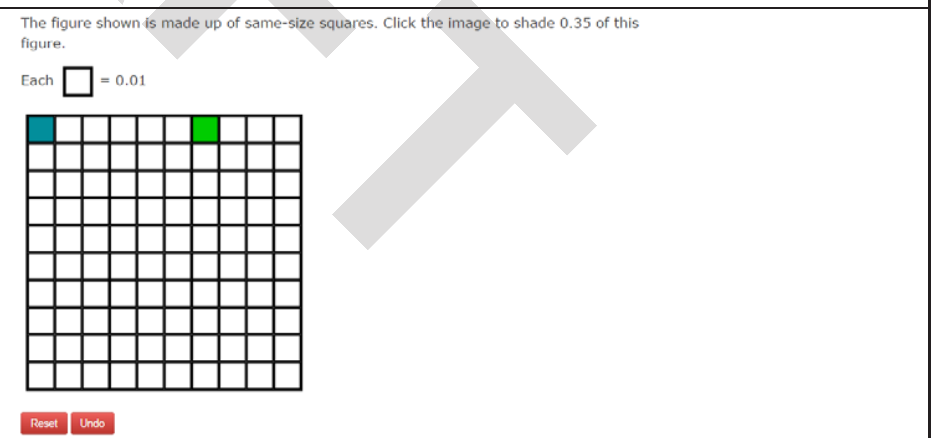
Usually worth one point. Occasionally involves two parts. With two parts, the scoring can either be worth one point or two points (with partial credit) depending on the wording of the question.

On both Part I and Part II.

5-15%</td>
<td>Angles: Students plot 3 points to form an angle. The first point plotted is the vertex.</td>
<td>
</td>
</tr>
<tr>
<td>Bar Graph: Students drag bars up or down to place.</td>
<td>The table shows the favorite foods of students in a class.

Favorite Food
<table><tr><th>Food</th><th>Number of Students</th></tr><tr><td>hamburger</td><td>5</td></tr><tr><td>pizza</td><td>10</td></tr><tr><td>taco</td><td>4</td></tr><tr><td>hot dog</td><td>6</td></tr></table>
Complete the bar graph to show the same information.

</td>
</tr>
</table>

<table>
<tr><td>TEI (continued)</td><td>Classification: Students drag and drop objects to different regions.</td><td>
</td></tr>
<tr><td></td><td>Partition Number Lines then Place Points: Students click a button to partition a line into equal parts. They can then select "Place Points" to be able to click on the line to place a point. With Student Label, the student will add partitions to the line instead of using a button to partition equally.</td><td>
</td></tr>
<tr><td></td><td>Partition Object then Select: Students click a button to partition an object into equal parts. They can then select "Shade Regions" to shade sections. In Select Defined partitions, a student will not be partitioning.</td><td>
</td></tr>
</table>

<table>
<tr>
<td>TEI (continued)</td>
<td>Pictograph: Students click a plus or minus to add shapes to the pictograph.</td>
<td>
</td>
</tr>
<tr>
<td></td>
<td>Rays: Students plot 2 points to form a ray. The first point is the end point. The second point has the arrow at the end.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Select Points And Ranges on Number Lines: Students click on a point on the line to place an open/closed marker. After markers are placed, students can click in between points to select a region. Regions extend between two points or one point and the end of the number line.</td>
<td>
</td>
</tr>
<tr>
<td></td>
<td>Shade Regions: Students click to shade regions of a rectangle. Selected regions are either outlined or filled with a solid color, symbol, or custom graphic.</td>
<td>
</td>
</tr>
</table>

Appendix K
Calculator Policies

TNReady Calculator Policy for Mathematics

The TNReady Calculator Policy is based on two central beliefs:

1) Calculators are important tools and, in order to be ready for career and college, students need to understand how to use calculators effectively, and
2) In order to demonstrate mastery of the mathematics standards, students must demonstrate many skills without reliance on calculators.

Therefore, at all grade levels and in all courses, TNReady will include both calculator permitted sections and calculator prohibited sections.

- Part I will allow calculator use at all grade levels.
- Part II will include a calculator permitted section and a calculator prohibited section at all grade levels.

The following considerations will shape how items are assigned to each section:

- Questions based on standards that require students to perform calculations in order to arrive at an answer will appear on the non-calculator-permitted section of the assessment. For example, 5.NF.A.1 expects students to add/subtract fractions with unlike denominators.
- Other questions may be based on standards where a calculation is a means to demonstrating other understanding. In this case, a student's error could be based on a misconception or a miscalculation, which would color the evidence of what is intended by the assessment. For example, 6.G.A.1 expects students to find area of composite figures and the calculations performed should not be a barrier for students demonstrating understanding of how to determine the area.
- Questions based on standards like 3.G.A.1 which ask students to recognize examples of quadrilaterals may appear on either the calculator or non-calculator section.

Calculator Specifics

- It is the responsibility of the Test Administrator to ensure the regulations outlined in this policy pertaining to calculator use are followed.
- All memory and user-entered programs and documents must be cleared or removed before and after the test.
- A student may use any permitted calculator at any grade level.
- For calculator-permitted sections of TNReady, students may use the online calculator or a handheld calculator provided by the school/district or one owned personally. Students may use either or both during the test.

- Students should have access to no more than one handheld calculator device for calculator-permitted sections of TNReady.
- Students will have access to practice with the same functionalities that will be available on the operational assessment on the item sampler and the practice tests.

Calculator Types

Below are examples of calculator functionalities and calculators that are permitted on TNReady. (Note: this is not an exhaustive list and students should be familiar with particular functions at the appropriate grade level.)

Examples of Permitted Functionalities:

- Square root ($\sqrt{}$)/Square key (x^2 and/or x^y)
- Pi(π)
- Graphing capability
- Data entry
- Matrices
- Regression
- Trigonometric functions (sine, cosine, tangent)
- Logarithm (log and/or ln) and exponential functions (a^x and/or e^x)

Examples of permitted calculators:

- TI-30
- Casio FX260
- Sharp EL344RB
- TI-84 plus family
- TI-NSpire (non-CAS) and TI-NSpire-CX (non-CAS)

Below are calculator functionalities and examples of calculators that are not allowed on TNReady. (Students may use any four- function, scientific, or graphing calculator, which does not include any of the prohibited functionalities.)

Calculator functionalities that are prohibited:

- Any calculator with CAS (computer algebra system) capabilities (including any programs or applications)
- Wireless communication capability
- QWERTY keyboard
- Cell phones, tablets, iPods, etc.

Examples of prohibited calculators:

- TI-89
- TI-NSpire (CAS version)
- HP-40G
- Casio CFX-9970

Appendix L
Practice Tools

TNReady Practice Tools

The TDOE will make two optional tools available to educators and districts.

<table>
<tr><th rowspan="2"></th><th colspan="2">TNReady Item Sampler</th><th rowspan="2">TNReady Practice Test</th></tr>
<tr><th>Phase 1</th><th>Phase 2</th></tr>
<tr><td>Purpose</td><td colspan="2">• In Phase 1, to give educators access to questions that are reflective of the rigor and the format of questions that will be on TNReady.
• In Phase 2, to give educators access to additional items and provide students a chance to practice with the same tools they will have on TNReady in an instructional setting.</td><td>• To simulate a short-form of each part of the TNReady test (Part I and Part II).
• To allow students to experience a practice test with the same features as the operational assessment.
• To allow teachers and systems to practice set up and administration.</td></tr>
<tr><td>Limitations</td><td colspan="2">• The TNReady Item Sampler will not serve as a full set of interim or formative assessments.
• The items will not be secure. (All teachers will have access at the same time.)
• The items will be comparable to the items on TNReady but the test forms will not be comparable, as they are teacher-created.
• The results will not necessarily be comparable to results in other classrooms because the user experience cannot be controlled.
• The ability to add customized items specific to teacher, school, or district will not be available at this time.</td><td>• Results on the TNReady Practice Test will not necessarily be predictive of student performance on TNReady.
• The Practice test will not reflect a full form, but it will include all major item types for each part.</td></tr>
<tr><td>Timeline</td><td>• Launch May 2015
• Continuously available
• 8-12 items per grade per subject
• Full range of item types
• Access for teachers only</td><td>• Launch September 2015
• Continuously available
• 25-40 additional items per grade per subject
• Full range of standards
• Access for teachers and students</td><td>• <u>Window 1:</u> September 28 – October 30, 2015 (All grades Part I & II)
• <u>Window 2:</u> January 4 – February 6, 2016 (All grades Part I & II)
• <u>Window 3:</u> March 7 – April 8, 2016 (All grades Part II)</td></tr>
<tr><td>User Set-Up</td><td>• All teachers will be set up to get access on May 2015.
• More info about set up will be shared with testing coordinators by April 15, 2015.</td><td>• Students will be set up based on August 14 EIS pull, provided scheduling data is available. Regular EIS updates thereafter.
• More info will be shared with testing coordinators by August 1, 2015.</td><td>• Same upload process as MIST practice test has been in the past.</td></tr>
<tr><td>Training</td><td colspan="2">• Training on MICA will be incorporated in summer training for teachers and principals.
• Web-based video training will be available.
• Additional support from CORE analysts.</td><td>• Web-based video training will be available.</td></tr>
<tr><td>Reporting</td><td>• Teacher Reports
  ○ Student, Assessment, Class
  ○ Content strand summary</td><td>• Teacher Reports (same as Phase I)
• Administrator Reports
  ○ Utilization by teacher and content strand
  ○ “Impersonation” at teacher level</td><td>• Student Report and Roster Report for schools and districts including:
  ○ Overall Score and By Standard
  ○ Raw Score (number correct)
  ○ % correct</td></tr>
<tr><td>Platform</td><td colspan="2">• MICA (Measurement Inc. Classroom Assessments) which is built to reflect MIST.</td><td>• MIST (the same platform as the operational assessment).</td></tr>
<tr><td>Accessibility Features</td><td colspan="2">• Supports common web-browser text reader tools. Not integrated in MICA.</td><td>• Text reader availability by Windows 2 and 3.</td></tr>
<tr><td>Scoring</td><td colspan="3">• All machine scorable items will be automatically scored within platform.
• Teachers will be able to go into the MICA system to score student answers to open-response items using the same tools and scoring guides that will be used to score TNReady.</td></tr>
<tr><td>Cost</td><td colspan="3">• Provided to all Tennessee districts at no additional charge.</td></tr>
</table>

TN Task Resources

Appendix M
Task Packet

Tennessee Department of Education:

Task: Fun at the ocean **6th Grade**

On vacation last summer, the Baker family went to the beach. Julie and her mom wanted to get a view of the coastline, so they went parasailing 120 feet above the ocean. Jamie and his dad wanted to explore the fish and the reef, so they went scuba diving twenty feet underwater.

a) For a moment in time, Julie was directly above Jamie. Use a number line to show the location of Julie and Jamie at that moment.

b) Using your number line, write an equation to find the distance between Julie and Jamie.

c) They also observed more things while they were on vacation: a bird, a dolphin, a buoy and the bottom of the ocean. Plot each of these things on your number line where you think they may have been and explain your reasoning for each.

Teacher Notes

- This is an instructional task where the teacher should be pressing to hear students discover the importance of zero on a number line and what object represents zero at the beach. Hopefully, students will discover how to create a vertical number line to illustrate real world problems of elevation. We should also hear students discover that the distance between two points on a number line is additive, and which refers to the absolute value of each number.
- It may be necessary for the teacher to discuss and show pictures of some of the vocabulary terms, such as parasailing and buoy.
- There are numerous answers that are acceptable for Part C. Answers are acceptable, as long as the student can give a rational justification for each point they plotted on the number line. The intent is to place the buoy at sea level, the bird above sea level, the ocean floor a little below Jamie and the dolphin could be either right above sea level or below sea level depending on the student reasoning.

Tennessee State Standards	Tennessee State Standards for Mathematical Practice
6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.C.7c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars,*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

write $|-30| = 30$ to describe the size of the debt in dollars.

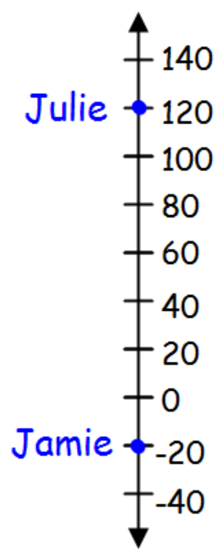
Essential Understandings

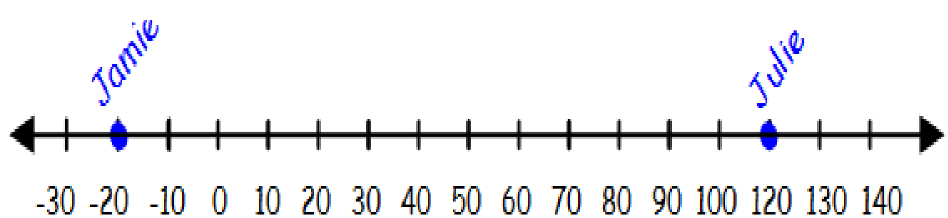
- Real world situations can be represented using positive and negative numbers.
- Each integer can be associated with a unique point on the number line.
- Number lines can be both horizontal and vertical.
- Distances between two points or objects will always have a positive result.
- Absolute value is a measure of a number's distance from zero.

Explore Phase

Possible Solution Paths

Part A)





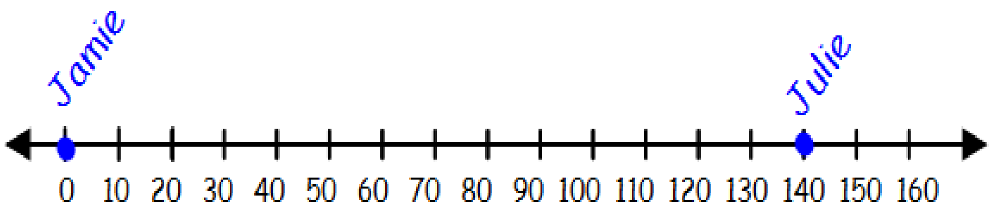
Assessing and Advancing Questions

Assessing Questions:
- Why did you decide to draw a vertical number line?
- Can you explain what the -20 means for Jamie?
- Can you explain why you chose the increments for your number line?

Advancing Questions:
- What does the zero on your number line represent?
- What do the negative numbers on your number line represent? The positive ones?

Assessing Questions:
- Why did you decide to draw a horizontal number line?
- Can you explain why you plotted Jamie at negative 20?
- I notice that you plotted Julie at 120. What does this represent?

Advancing Questions:
- Can you think of another way to draw a number line for this situation?
- What do the negative numbers on your number line represent? The positive ones?

<table>
<tr>
<td>
</td>
<td>Assessing Questions:
• What is a number line?
• How does your number line relate to the question?
• I notice that you plotted Julie at 140. Why did you decide to plot Julie there?

Advancing Questions:
• Can you show me where sea level would be represented on your number line?
• If Jamie is scuba diving here, where might the ocean floor be on your number line? What number will you use to represent the ocean floor?</td>
</tr>
<tr>
<td>Part B)

|-20| + |120| = 140

20 + 120 = 140

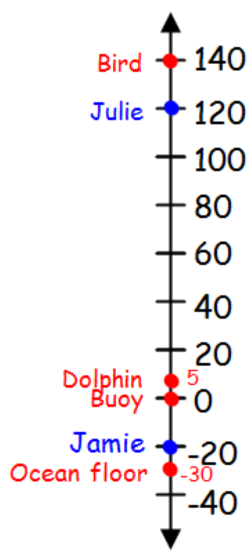
120 – (-20) = 140</td>
<td>Assessing Questions:
• Can you explain to me how you came up with your equation?
• Why did you add? Why did you subtract?
• Why did you choose to use absolute value?
• What does the 140 represent?

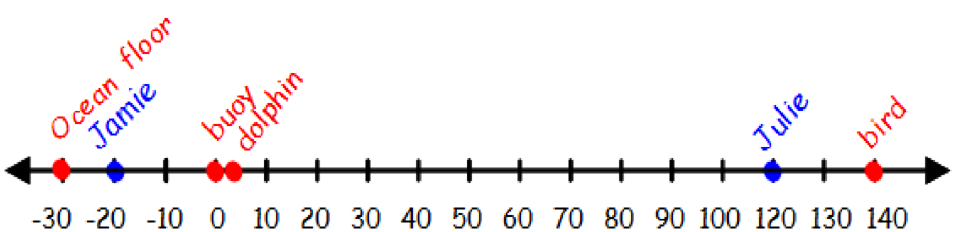
Advancing Questions:
• How does your equation relate to your number line?
• Does your answer make sense? Explain why.
• You represented Jamie with -20. Can we really have a negative distance? What does the negative represent?
• Can you write a different equation that represents the same problem?</td>
</tr>
</table>

Part C) Refer to *teacher notes* section above.

Possible student responses for the points plotted on each of the number lines:

I thought the bird would be flying even higher in the sky than Julie. I imagined seeing a dolphin as it jumped out of the water. The buoy would be floating on the surface of the water. Since Jamie was looking at the reef on the bottom of the ocean, I thought the ocean floor would be just a little bit lower than where he was scuba diving.





Assessing Questions:

- What were some of the things you had to consider when you plotted these things seen at the ocean?
- What does the zero on your number line represent at the beach?
- Why did you label some of your new plots with numbers, yet some of them you didn't? Was this necessary? Why?

Advancing Questions:

- What is the distance between your highest and lowest points on your number line?
- Could any of these items have been placed in their opposite position on the number line? Explain.

Assessing Questions:

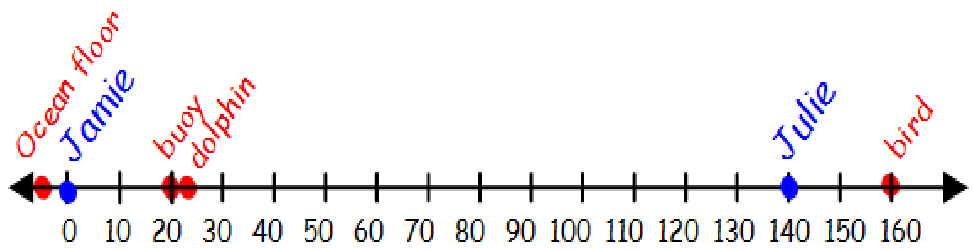
- What were some of the things you had to consider when you plotted these things seen at the ocean?
- What does the zero on your number line represent at the beach?
- Why did you decide not to identify the point for the dolphin?

Advancing Questions:

- What is the distance between your highest and lowest points on your number line?
- Could any of these items have been placed in their opposite position on the number line? Explain.
- Did you attend to precision?

Assessing Questions:

- Which item did you plot first? Why?
- How did you decide to plot the buoy at 20?
- Why did you decide to not identify the points for the



ocean floor and the dolphin with an integer?

Advancing Questions:
- What integer would represent the ocean floor on your number line?
- What is the distance between your highest and lowest points on your number line?

Possible Student Misconceptions

Students may not relate to a beach setting or be familiar with vocabulary terms in the task such as parasailing and buoy.

Assessing Questions:
- Have you ever been to or seen in pictures or on TV people at the beach?
- Are there any words in the problem that you don't know, or need some help clarifying?

Advancing Questions:
- What are some of the things you have seen people doing at the beach?
- Can you draw a picture that shows Julie and Jamie doing their activities at the beach?

Students may have the misconception that Jamie and Julie are the extremes in this problem and not rationalize that the ocean floor would be plotted on the number line.

Assessing Questions:
- What is Jamie doing in this problem?
- What are some of the things Jamie is going to see while scuba diving? Where are those things in relation to where Jamie is?
- Can you draw me a picture to help illustrate the problem?

Advancing Questions:
- Where would Jamie see the ocean floor? Where will you represent that on your number line?

Entry/Extensions

Assessing and Advancing Questions

If students can't get started….

Assessing Questions:
- What can you tell me about the problem?
- What is a number line?

	• Can you draw me a picture to help illustrate the problem? **Advancing Questions:** • On your picture you have this line for the top of the water. Where do you think that water line should be represented on your number line? Explain why.
If students finish early….	• Using your work, write a generalization for finding the distance between two points on a number line. • Which point on your line could have been represented as either a positive or negative number? Why? • Think of some other objects that may be seen at the beach and place them on your number line. Explain your reasoning for each. • Draw your number line in the opposite direction and use it to plot the same locations. Compare the two solution paths, and be ready to share with the class which one is easier for you to understand, and why.

Discuss/Analyze

Whole Group Questions

- Can someone explain which number line (horizontal or vertical) seemed to work best for this situation?
- Were there any key words in the problem that may have suggested using a vertical number line? Which ones?
- What is the only object that everyone plotted in the same location in relation to Julie and Jamie? Why is that?
- Can anyone generalize how to find the distance between any two points on a number line?
- What does absolute value of a number mean?

Appendix N
Task Arc

Mathematics Task Arcs

Overview of Mathematics Task Arcs:

A task arc is a set of related lessons which consists of eight tasks and their associated lesson guides. The lessons are focused on a small number of standards within a domain of the Tennessee State Standards for Mathematics. In some cases, a small number of related standards from more than one domain may be addressed.

A unique aspect of the task arc is the identification of essential understandings of mathematics. An essential understanding is the underlying mathematical truth in the lesson. The essential understandings are critical later in the lesson guides, because of the solution paths and the discussion questions outlined in the share, discuss, and analyze phase of the lesson are driven by the essential understandings.

The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress; however the context or representations change.

Bias and sensitivity:

Social, ethnic, racial, religious, and gender bias is best determined at the local level where educators have in-depth knowledge of the culture and values of the community in which students live. The TDOE asks local districts to review these curricular units for social, ethnic, racial, religious, and gender bias before use in local schools.

Copyright:

institute for learning

Grade 6

Locating, Ordering, and Finding Distance Between Positive and Negative Numbers

A SET OF RELATED LESSONS

UNIVERSITY OF PITTSBURGH

Table of Contents

Introduction

Locating, Ordering, and Finding Distance Between Positive and Negative Numbers

A SET OF RELATED LESSONS

Overview

In this set of related lessons, students learn strategies for locating and ordering positive and negative numbers, as well as strategies for determining the distance between positive and negative numbers. The related lessons address the Tennessee State Standards 6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c, 6.NS.C.7a, and 6.NS.C.7c and require students to make use of all of the Mathematical Practice Standards.

There are a total of eight tasks in this set of related lessons. Six of the tasks are developing understanding tasks and two are solidifying understanding tasks. In Task 1, students place values on a number line, exploring the relationship between positive values, negative values, and zero. In Tasks 2 and 3, students write inequalities and order numbers using the number line. In Task 4, students solidify understanding of how to place and order values using a number line.

Tasks 5 - 8 develop and solidify understanding of absolute value and methods of calculating distance between two points on a number line. In Task 5, students develop a definition of absolute value. Tasks 6 and 7 explore strategies for calculating the distance between any two points on a number line and develop the understanding that the distance between a positive and a negative value is equal to the sum of their absolute values. In Task 8, students solidify these understandings.

The prerequisite knowledge necessary to enter these lessons is a conceptual understanding of the relative size and location on a number line of positive values.

Through engaging in the lessons in this set of related tasks, students will:

- order positive and negative numbers on a number line;
- compare the relative size of positive and negative numbers; and
- calculate distance between positive and negative numbers.

By the end of these lessons, students will be able to answer the following overarching questions:

- What strategies can be used to order and compare positive and negative numbers?
- What strategies can be used to determine the distance between positive and negative numbers?

The questions provided in the guide will make it possible for students to work in ways consistent with the Standards for Mathematical Practice. It is not the Institute for Learning's expectation that students will name the Standards for Mathematical Practice. Instead, the teacher can mark agreement and disagreement of mathematical reasoning or identify characteristics of a good explanation (MP3). The teacher can note and mark times when students independently provide an equation and then re-contextualize the equation in the context of the situational problem (MP2). The teacher might also ask students to reflect on the benefit of using repeated reasoning, as this may help them understand the value of this mathematical practice in helping them see patterns and relationships (MP8). In study groups, topics such as these should be discussed regularly because the lesson guides have been designed with these ideas in mind. You and your colleagues may consider labeling the questions in the guide with the Standards for Mathematical Practice.

Identified CCSSM and Essential Understandings

CCSS for Mathematical Content: The Number System		**Essential Understandings**
Apply and extend previous understandings of numbers to the system of rational numbers.		
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0).
6.NS.C.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.	Rational numbers can be located on a number line with opposite numbers on opposite sides of 0.
6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point.

CCSS for Mathematical Content: The Number System		**Essential Understandings**
6.NS.C.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*	The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.
6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars*	The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero. The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from zero.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Common Core State Standards, 2010, NGA Center/CCSSO*

Tasks' CCSSM Alignment

Task	6.NS.C.5	6.NS.C.6a	6.NS.C.6c	6.NS.C.7a	6.NS.C.7c
Task 1 Straight Line Board Game Developing Understanding	✓	✓			
Task 2 Straight Line Board Game Continued Developing Understanding			✓	✓	
Task 3 Temperature in Winter Developing Understanding	✓		✓	✓	
Task 4 Order Matters Solidifying Understanding				✓	
Task 5 Sunnyside Office Building Developing Understanding	✓		✓		✓
Task 6 Hiking Developing Understanding	✓		✓		✓
Task 7 Comparing Temperatures Developing Understanding	✓		✓	✓	✓
Task 8 Beanbag Toss Solidifying Understanding	✓		✓		✓

Tasks' CCSSM Alignment

Task	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 Straight Line Board Game Developing Understanding	✓	✓		✓		✓		
Task 2 Straight Line Board Game Continued Developing Understanding	✓	✓		✓		✓		
Task 3 Temperature in Winter Developing Understanding	✓	✓	✓	✓		✓		✓
Task 4 Order Matters Solidifying Understanding	✓			✓		✓		✓
Task 5 Sunnyside Office Building Developing Understanding	✓	✓		✓		✓		
Task 6 Hiking Developing Understanding	✓	✓		✓	✓	✓		
Task 7 Comparing Temperatures Developing Understanding	✓	✓		✓		✓		✓
Task 8 Beanbag Toss Solidifying Understanding	✓	✓	✓	✓	✓	✓	✓	✓

Lesson Progression Chart

Overarching Questions

- What strategies can be used to order and compare positive and negative numbers?
- What strategies can be used to determine the distance between positive and negative numbers?

	Task 1 **Straight Line Board Game** *Developing Understanding*	**Task 2** **Straight Line Board Game Continued** *Developing Understanding*	**Task 3** **Temperature in Winter** *Developing Understanding*	**Task 4** **Order Matters** *Solidifying Understanding*
Content	Develop understanding of the placement of positive and negative position on the number line. Consider what it means for numbers to be opposites.	Develop understanding of the relationship between inequality statements and relative position on the number line.	Develop understanding of the placement and ordering of positive and negative numbers.	Solidify understanding of ordering and comparing positive and negative numbers.
Strategy	Using a number line diagram to position integers and identify opposites.	Using a number line diagram to position and compare integers.	Using diagrams and number lines to locate points on a number line and write inequalities comparing temperatures.	Using absolute value, horizontal and vertical number lines.
Representations	Starts with context. Students use words and number lines to describe positive and negative positions.	Starts with context and table. Students use words and number lines to describe and order positive and negative values with respect to their position on the number line.	Starts with a context and vertical number line.	Presented out of context, students use various strategies to order and compare the numbers.

	Task 5 **Sunnyside Office Building** *Developing Understanding*	Task 6 **Hiking** *Developing Understanding*	Task 7 **Comparing Temperatures** *Developing Understanding*	Task 8 **Beanbag Toss** *Solidifying Understanding*
Content	Develop understanding of absolute value.	Develop understanding of distance between positive and negative numbers.	Develop understanding of distance between positive and negative numbers.	Solidify understanding of distance between positive and negative numbers through repeated reasoning.
Strategy	Constructing a number line to model a context provides opportunity for students to explore the concept of absolute value.	Using counting or algebraic method from diagrams, horizontal number lines, and absolute value equations.	Using counting or algebraic method from a vertical number line and absolute value equations.	Using number lines, counting, and absolute value equations.
Representations	Starts with context and moves to number line diagrams and numeric expressions and equations.	Starts with context and moves to algebraic representations.	Starts with context and requires students to model the problem using a vertical number line, diagram, or equation.	Starts with a context and asks students to create number lines or equations to represent their thinking.

Grade 6

Tasks and Lesson Guides

Locating, Ordering, and Finding Distance Between Positive and Negative Numbers

A SET OF RELATED LESSONS

Name____

TASK 1

Straight Line Board Game

Malik, Sydney, and Kyaralind create a board game. The board is made of 21 rectangles in a straight line. "Start" is located in the center as shown. According to the rules of the game, each player answers a trivia question and then flicks a spinner numbered 1 - 8. If the player answers the trivia question correctly, the player moves forward that number of spaces. If the player answers incorrectly, the player moves backwards that number of spaces.

Below is a description of plays made during the game.

- Malik spins a 4 and answers a question correctly.
- Kyaralind spins a 1 and answers correctly.
- Sydney spins a 4 and answers incorrectly.

										Start										

1. Locate each player's position on the board. Explain how you decided where to place each player. What reasoning did you use in your decision making?

2. Sydney says: "It doesn't make any sense to represent both Malik's and my score with 4 when I am on the opposite side of 'Start.' I think the number that represents my position is -4." Sydney is correct. Explain what the *negative sign (-)* means in -4.

TASK
1

3. What number can you use to describe Malik's position? Kyaralind's position? Sydney's position?

4. Label the game board using positive and negative numbers. Explain your strategy for labeling the board.

TASK 1

5. If two players have opposite scores, what might their scores be?

6. At some point in the game, Carolyn's score is the opposite of -8. What is her score? Explain your reasoning. Use an equation to describe your reasoning.

LESSON GUIDE 1

Straight Line Board Game

Rationale for Lesson: Students are familiar with placing positive numbers on the number line. This lesson will introduce placement and movement of negative integers on a quasi-number line and recognize the relationship between opposite numbers positioned on a number line.

Task 1: Straight Line Board Game

Malik, Sydney, and Kyaralind create a board game. The board is made of 21 rectangles in a straight line. "Start" is located in the center as shown. According to the rules of the game, each player answers a trivia question and then flicks a spinner numbered 1 - 8. If the player answers the trivia question correctly, the player moves forward that number of spaces. If the player answers incorrectly, the player moves backwards that number of spaces.

Below is a description of plays made during the game.

- Malik spins a 4 and answers a question correctly.
- Kyaralind spins a 1 and answers correctly.
- Sydney spins a 4 and answers incorrectly.

										Start										

1. Locate each player's position on the board. Explain how you decided where to place each player. What reasoning did you use in your decision making?
2. Sydney says: "It doesn't make any sense to represent both Malik's and my score with 4 when I am on the opposite side of 'Start.' I think the number that represents my position is -4." Sydney is correct. Explain what the *negative sign (-)* means in -4.

See student paper for complete task.

Tennessee State Standards	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision.

LESSON GUIDE 1

Essential Understandings	• Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0). • Rational numbers can be located on a number line with opposite numbers on opposite sides of 0.
Materials Needed	• Task sheet.

SET-UP PHASE

Can I have a volunteer read the beginning of the Straight Line Board Game? Can somebody summarize how the game is played? Now take 5 minutes to work on the problem individually before you begin working in your groups.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Uses number line strategy incorrectly. **Sydney** 	What does the "Start" represent numerically?	You have worked with number lines before. If the "Start" is zero, where do you place Malik and why? What does that suggest about where to place Sydney?
Represents opposites numerically, but does not relate that to the number line model.	Why do you say 4 and -4 are opposites?	Where are 4 and -4 located on the number line? What do you notice about their positions with respect to zero?
Argues that negative numbers don't have opposites, because opposite means to make the number negative.	Can you tell me about your understanding of "opposite?"	What are some examples of opposites that are not numeric? How does the meaning of opposite in this case compare to the meaning of opposite with respect to numbers?
Finishes early.	Can you summarize your thinking? What does it mean to be an opposite? An opposite of an opposite?	How can you calculate the distance between a number and its opposite?

LESSON GUIDE 1

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0).

- Describe how your group placed each player on the board.
- Who can come up and show us how this group placed Malik's position on the board?
- How does their strategy relate to placing any negative number on a number line? Any positive number?
- So, on a horizontal number line, positive numbers are always to the right of zero and negative numbers are always to the left of zero. **(Marking)**

EU: Rational numbers can be located on a number line with opposite numbers on opposite sides of 0.

- What do you notice about Malik's and Sydney's positions on the board?
- What if they were at positions 5 and -5? 9 and -9? Come up and show where this would be on the game board.
- Does everybody agree with these placements? Why or why not?
- How do you know where to place numbers on a number line? Where are the greater numbers? How do you know?
- From these examples, describe the position of any pair of opposite numbers on a number line.
- When Carolyn's score was the opposite of -8, what was her score? How do you know? So can we agree that the opposite of negative 8 can be written as -(-8) and that this value is positive? **(Marking)**

Application	• Kyaralind is 5 spaces to the left of the "Start" and Sydney is 8 spaces to the left of the "Start." What number represents their positions on the board? • Which number is ***greater*** and why?
Summary	• Describe how a number line with positive and negative numbers is created. • Where are the ***greater*** values located on a number line?
Quick Write	Describe the placement of any pair of opposite numbers on a number line.

Support for students who are English Learners (EL):

1. When introducing the game board, hold it up and demonstrate the words found in the context such as "forward," "backwards," "spaces," etc. This helps students who are English Learners make the connection between the words and the concrete representation.
2. Have game boards available and ask students who are identified as English Learners to physically point to the board as they explain their answers.
3. Slow down discussions for students who are English Learners by asking other students to repeat ideas, to put ideas in their own words, and to continually point to the game board model simultaneously.

TASK 2

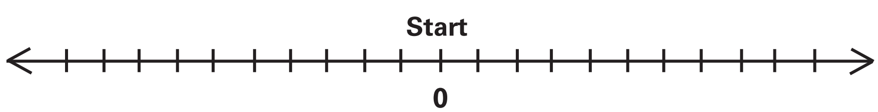
Name____

Straight Line Board Game Continued

After testing their game, Sydney, Malik, and Kyaralind decide to ask a group of friends to join them. Their scores are shown on the table below.

Name	Sydney	Malik	Kyaralind	Craig	Tim	Gina	Roosevelt	Sonia	Carl	Lin
Score	1	-5	-4	-3	-1	-2	3	-1	-6	4

1. Place these players with their scores on the number line below:



2. Determine which player has the higher score in the players below:
   A. Sydney or Lin
   B. Gina or Kyaralind
   C. Carl or Tim

3. Use >, <, or = to describe the relationship between the players' scores.
   A. Tim's score ____ Malik's score
   B. Carl's score ____ Sonia's score
   C. Lin's score ____ Roosevelt's score
   D. Tim's score ____ Sonia's score
   E. Describe the strategy you used for choosing >, <, or = for the relationships above.

4. If the game allowed for $\frac{1}{2}$ square positions, is -3 less than or greater than -3.5? Explain your reasoning.

LESSON GUIDE 2

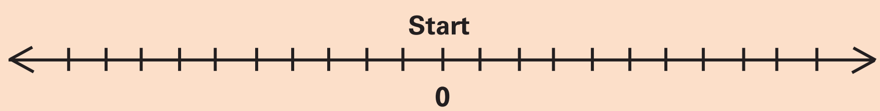
Straight Line Board Game Continued

Rationale for Lesson: Continue developing understanding of placement of positive and negative values and interpret inequality statements as indicators of relative position on a number line.

Task 2: Straight Line Board Game Continued
After testing their game, Sydney, Malik, and Kyaralind decide to ask a group of friends to join them. Their scores are shown on the table below. Below is a description of plays made during the game.

Name	Sydney	Malik	Kyaralind	Craig	Tim	Gina	Roosevelt	Sonia	Carl	Lin
Score	1	-5	-4	-3	-1	-2	3	-1	-6	4

1. Place these players with their scores on the number line below:



2. Determine which player has the higher score in the players below:
   A. Sydney or Lin
   B. Gina or Kyaralind
   C. Carl or Tim

See student paper for complete task.

Tennessee State Standards	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
Standards for Mathematical Practice		MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision.

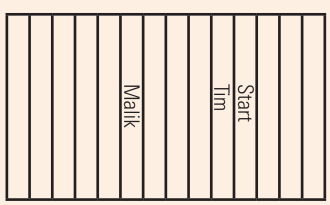
Essential Understandings	• Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point. • The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.
Materials Needed	• Task sheet. • Additional number lines.

LESSON GUIDE 2

SET-UP PHASE

Silently read the beginning of the task. Then, take 5 minutes to work on the problem individually before you begin working in your groups.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	Can you explain to me what information is in the table? Who has the highest score?	I want you to rank the players according to what place they're in. Who is in first? Second? Third? After that, I want you to arrange them on this number line with zero right here (pointing).
Uses numerical reasoning. Tim > Malik, since -1 > -5.	Can you explain how you decided that -1 is greater than -5?	How can you use this reasoning to order any integers on a number line?
Uses number line strategy through game board. Tim > Malik  	Explain how your representation shows who has a greater score.	How does this relate to a number line?
Uses number line with explanation. Malik (-5) Tim (-1) 0	What was your strategy for positioning them on the number line?	So, who's winning? How can you use a number's distance from zero to determine which number is greater?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point.

- Describe how your group determined each player's position on the number line.
- Who heard what (student name) said about positive and negative direction and can restate it in their own words?
- How far did we move from zero? How do we determine that?
- So, are you saying that the sign tells you the direction to move and the magnitude of the number tells you how far to move from zero and that in that way, you can place any number on a number line? **(Revoicing)**
- Will this work for numbers that aren't integers? What about fractions and decimals? **(Challenging)**

EU: The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.

- When comparing numbers, this group used the example of 5 pencils is more than 1 pencil. What do other people think about this example? How does it help you write the inequalities in this task?
- Can you come up with a context like this for comparing two negative numbers? A negative and a positive?
- This group used the number line to determine who had a greater score. Tell us about how the number line helped you think about the order of the values.
- Who heard what (student name) said about the numbers increasing and can add on to that?
- How can -1 be greater than -5? Isn't 1 less than 5? **(Challenging)**
- So, if a number is farther to the right, it is greater. **(Marking)** Help us all understand why that is true.
- Interesting - (student name) said 0 is greater than any negative value. Do you agree or disagree? Why? *(Because it is further to the right on the number line.)*
- So, how does that help us understand why -1 > -5? *(Negative one is further to the right on the number line.)*
- When comparing negative numbers, the one closer to 0 is greater. **(Revoicing)** Is that true of positive numbers as well?
- Let me see if I can sum up this discussion. We started by simply comparing numbers. We know, for example, that 5 is greater than 1 because we can see that if we have 5 pencils, that is more than 1 pencil. Then we looked at the number line and noticed that 5 is further to the right than 1. We saw, too, that this was true when we compared two negative numbers. The number further to the right is closer to zero and therefore greater. So, we have a general rule that the greater number is farther to the right on the number line. **(Recapping)**

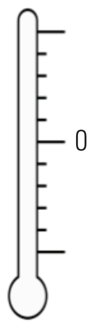
Application	Position 2, -1, 1.25, and -4 on a number line. Write inequalities comparing the numbers. Explain how you determined their positions and how you compared their values.
Summary	How can you use a number line to place positive and negative values? How can you use inequality statements to indicate their position on the number line?
Quick Write	No quick write for students.

TASK
3

Name____

Temperature in Winter

1. Place the following temperatures on the thermometer below. Then explain the strategy you used to place them on the thermometer.

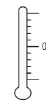
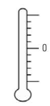


5°C
-3°C
-1°C
2°C

2. List the numbers from least to greatest. Explain your reasoning.

3. On Wednesday night, the temperature was -3.5°C. The temperature Thursday night changed slightly to -3.75°C. Steve says that this makes Thursday night slightly warmer since 3.75 is a little larger than 3.5. Is Steve's reasoning correct? Use an equation and a thermometer to help explain your thinking.

4. Complete the table below by filling in the missing starting temperature, changes in temperature, or ending temperature and a statement comparing the starting and ending temperatures. If a temperature is missing, illustrate the correct temperature on the thermometer provided.

Day of Week	Starting Temperature	Change in Temperature	Ending Temperature	Insert <, >, or = Starting Temp___ Ending Temp
Tuesday	3°C	Fell 5°C		
Wednesday	-2.5°C	Rose 4°C		
Thursday		Fell 3°C	-2°C	
Friday	-5.2°C		-1°C	
Saturday	-4.6°C		4°C	
Sunday			-3°C	

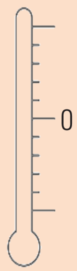
LESSON GUIDE 3

Temperature in Winter

Rationale for Lesson: Continue developing understanding of position of positive and negative numbers, but in the context of a vertical number line. Order and compare rational number values using a vertical number line. Begin to explore the idea of movement on a number line as a precursor to modeling arithmetic operations on a number line.

Task 3: Temperature in Winter

1. Place the following temperatures on the thermometer below. Then explain the strategy you used to place them on the thermometer.



5ºC

-3ºC

-1ºC

2ºC

2. List the numbers from least to greatest. Explain your reasoning.
3. On Wednesday night, the temperature was -3.5°C. The temperature Thursday night changed slightly to -3.75°C. Steve says that this makes Thursday night slightly warmer since 3.75 is a little larger than 3.5. Is Steve's reasoning correct? Use an equation and a thermometer to help explain your thinking.

See student paper for complete task.

Tennessee State Standards	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP8 Look for and express regularity in repeated reasoning.
Essential Understandings	• Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0). • Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point. • The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.
Materials Needed	• Task sheet. • Additional number lines.

SET-UP PHASE

Read the task to yourselves before we read it aloud. As you think about this problem, you may want to reference the artifacts from the previous two tasks concerning distance with positive and negative integers. Work through the task individually for 5 minutes. Then, get into your groups and begin collaborating.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Uses numerical reasoning. -3.5 is greater than -3.75 because 3.75 > 3.5 and the greater the negative, the less the value.	Tell me about your thinking. Why does this work for any two negative rational numbers?	Is there a way to show or explain your reasoning on a vertical number line?
Writes numeric equations to represent temperature changes (may include incorrect calculations).	How do the equations you wrote represent the problem situation?	What does addition look like on the vertical number line? What does subtraction look like?
Uses vertical number line.  -3 -3.25 -3.5 -3.75	How did you decide where to position the numbers?	How can this help generalize positioning and value on a vertical number line? How does this compare to a horizontal number line?
Counts up or counts down to find unknown temperature.	How did you determine the starting temperature? Show me how you counted.	How can your counting method help you determine which value is greater?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0).

- Tell us about how you placed the points on the thermometer in part one.
- Who heard what (student name) just said about temperatures above and below zero and can restate it in their own words?
- How do we represent numbers above zero? Who can show us a temperature above zero on the thermometer and write the number that represents that point? *(I can place the number above zero by counting the degrees above.)*
- Who can show us a temperature below zero on the thermometer and write the number that represents that point? *(I can place the number below zero by counting the degrees below.)*
- Numbers greater than zero are positive and are positioned above the zero on a vertical number line. Numbers less than zero are negative and are positioned below the zero on a vertical number line. **(Revoicing)**

EU: Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point.

- The previous group helped us understand which direction to go from zero when plotting a point to represent a positive or negative value. Can we get more specific? This group used number lines to find the starting and ending temperatures. Tell us how you determined the location of -4.6.
- Who can restate in their own words how this group located points on the number line?
- The sign of the number indicated the direction and the magnitude indicated how far they counted away from zero. **(Marking)**
- Is it okay to estimate? **(Challenging)** Is it important that everyone plots -5.2 in the exact same spot? How precise do we need to be for this problem?
- Once we have the starting temperature plotted, how can we determine the ending temperature?
- Who can explain the method they used to find the unknown temperature and refer to the thermometer on the board?
- This group used equations to find the unknown temperature. Can you explain to us how you used equations? How is this the same/different as using a number line?
- How are the sign and magnitude shown?

EU: The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.

- Describe how your group decided which symbol (<, >, or =) to place between the starting and ending temperatures.
- Someone else say this in their own words before we try to make connections to Steve's thinking.
- So I hear that the number furthest to the right on the number line is greater than and the symbol looks like this. **(Marking)**
- How does this relate to Steve's thinking about -3.5 and -3.75? How would you explain the situation so that Steve could see his error? *(I could plot them on a number line and I see that the smaller negative value is closer to zero.)*
- In general, how do we know from a vertical number line which values are greater? *(The greater numbers are further up on the vertical number line.)*
- You are saying the greater the value, the higher it will be placed on a vertical number line. **(Marking)** How does this compare to how we order numbers on a horizontal number line?
- Can we make a general statement about positioning on both types of number lines?

Application	One winter day in Minneapolis, the low temperature was -20°F. The high temperature was 12°F higher. On a number line, model the low and high temperatures and the difference between them.
Summary	• How do we know where a number is located on a vertical number line? • How do we compare values of numbers using their positions on the number line?
Quick Write	Explain how to locate the points 4.5 and -3.2 on a vertical number line.

Support for students who are English Learners (EL):

1. When introducing the task, model the use of a thermometer to determine temperature. Discuss the difference between Celsius and Fahrenheit.

TASK
4

Name____

Order Matters

1. Order the following sets of numbers from least to greatest.
   A. 4, 7, -3, -4, -3, 0, 6, 1

   B. -99, 100, -100, -50, -1, 40

   C. -3.75, -4, 0.75, 0, 0.04, -0.99, -1

   Describe your strategy for ordering numbers.

2. Complete each statement with >, <, or =.

A.	-5_____4	B.	-19_____-20
C.	-19.5_____-20	D.	-0.5_____0
E.	0.50_____0.60	F.	-0.50_____-0.60

3. Which number is further from 0 on the number line: -19.1 or 20? Explain your reasoning.

4. Given any two negative numbers *a* and *b*, summarize using words and a number line how to determine which number is greater.

Order Matters

LESSON GUIDE 4

Rationale for Lesson: Solidify understanding of placement and order of positive and negative rational numbers. In this lesson, students use distance from zero and relative position on the number line.

Task 4: Order Matters

1. Order the following sets of numbers from least to greatest.
   A. 4, 7, -3, -4, -3, 0, 6, 1
   B. -99, 100, -100, -50, -1, 40
   C. -3.75, -4, 0.75, 0, 0.04, -0.99, -1
   Describe your strategy for ordering numbers.
2. Complete each statement with >, <, or =.

A.	-5____4	B.	-19____-20
C.	-19.5____-20	D.	-0.5____0
E.	0.50____0.60	F.	-0.50____-0.60

3. Which number is further from 0 on the number line: -19.1 or 20? Explain your reasoning.
4. Given any two negative numbers *a* and *b*, summarize using words and a number line how to determine which number is greater.

Tennessee State Standards	6.NS.C.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP4 Model with mathematics. MP6 Attend to precision. MP8 Look for and express regularity in repeated reasoning.
Essential Understandings	• The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.
Materials Needed	• Task sheet. • Additional number lines.

LESSON GUIDE 4

SET-UP PHASE

Read the task to yourselves before we read it aloud. As you consider strategies to order and compare the numbers in this task, you may want to reference the artifacts from the previous three tasks. Work through the task individually and then you'll get a chance to collaborate.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
When comparing numbers, uses distance from zero.	Tell me about how you use the distance from zero to compare the value of two numbers.	Does your method always work? For two positive values? Two negative? A negative and a positive? Explain.
When ordering numbers, plots the values on a number line.	Can you tell me about your strategy? How does the number line help you think about the order?	How can you decide without using a number line?
Finishes early.	Can you describe your strategy?	Can you summarize a rule using our classroom artifacts that can be used to compare any two rational numbers?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The value of two or more numbers can be compared using their positions on a number line relative to each other because movement right (up) on a number line signifies a positive movement and movement left (down) signifies a negative movement.

- This group considered the distance of each value from zero. Tell us about how you used that distance to think about ordering and comparing numbers.
- Can somebody summarize this method in your own words? What do they mean by "how big it is?" *(They mean that if you ignore the sign, the number that is greater - the number that is further from zero.)*
- Are there instances when considering distance from zero is not enough information? Explain.
- So, if the numbers are both positive, what does the distance from zero tell you? If they are both negative? What if we have one positive and one negative?
- This group used a number line. Let's see how their method compares to the first method.
- What is different about their method? *(They are plotting the values and comparing them to see which is further to the right.)*
- So I hear you saying that they need to consider not only the size of the number, but the direction. **(Revoicing)**
- In order to compare the numbers we must consider direction and magnitude. **(Marking)**
- So when we have two negative numbers, how can we determine which is greater?
- What do we know about their position on the number line?
- What do we know about their distance from zero on the number line?
- Does it matter if we use a horizontal or vertical number line? **(Challenging)** Why? Can you show me an example of each one?
- What is significant about zero? *(Positive and negative values are the same distance in opposite directions from zero. For example, 5 and –5 are the same distance in opposite directions.)*
- We learned today that when putting values in order, we have to consider both the sign and the magnitude of the number. We also reinforced our understanding that on a horizontal number line, values increase to the right and on a vertical number line, values increase as you go up. We also recalled that zero is the dividing line between negative values and positive values, so all positive values are greater than zero and all negative values are less than zero. **(Recapping)**

Application	Put the following numbers in order. Explain how you determined their order. -3, 1.5, 2.1, -1.8, 2, 0
Summary	How can we use position on the number line and distance from zero to explain why -4 is greater than -5?
Quick Write	If you have two numbers, how can you determine which number is greater using a number line?

TASK
5

Name____

Sunnyside Office Building

In the town of Sunnyside, there is an office building with floors above and below ground. The building has 9 floors. 4 of the floors are below ground.

Visitors enter the building at ground level and then use the elevators or stairs to go up or down.

1. Draw and label a number line to represent all 9 floors of the office building.

2. Karyn and Colin enter the ground level of the building. Karyn goes up to her office on the third floor. Colin goes down to his office on the bottom floor.

   A. Represent Karyn's and Colin's locations on the number line.

   B. Who has traveled farther from the ground floor? Explain.

3. Manuela's office is on the top floor. Is Manuela or Colin farther from the ground floor? Explain.

Sunnyside Office Building

LESSON GUIDE 5

Rationale for Lesson: In the previous tasks, students placed positive and negative numbers on a number line and determined distance from zero. In this task, they will move forward in their study of positive and negative integers to develop an understanding of the distance between positive and negative integers.

Task 4: Sunnyside Office Building
In the town of Sunnyside, there is an office building with floors above and below ground. The building has 9 floors. 4 of the floors are below ground.
Visitors enter the building at ground level and then use the elevators or stairs to go up or down.

1. Draw and label a number line to represent all 9 floors of the office building.
2. Karyn and Colin enter the ground level of the building. Karyn goes up to her office on the third floor. Colin goes down to his office on the bottom floor.
   A. Represent Karyn's and Colin's locations on the number line.
   B. Who has traveled farther from the ground floor? Explain.
3. Manuela's office is on the top floor. Is Manuela or Colin farther from the ground floor? Explain.

Common Core Content Standards	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write* $\lvert -30 \rvert = 30$ *to describe the size of the debt in dollars.*

LESSON GUIDE 5

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision.
Essential Understandings	• Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0). • The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.
Materials Needed	• Task sheet. • Additional number lines.

SET-UP PHASE

Please read the task silently while (student name) reads it aloud. Do you have a house with a basement? Have you ever been in a building with more than one floor underground? In this task, we will consider a building with 4 floors underground. Start on the task independently. I will let you know when it is time to work in your groups.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	Can you describe what the building looks like? Can we draw a picture of a building with floors above and below ground?	How can we represent the stories of this building using a number line?
Counts tic marks to compare distance.	Tell me about your method. What are you counting? How do you know which person is farther from the ground floor?	Can you come up with a way to determine the distance without counting?
Writes a conjecture that opposites are the same distance from 0.	What numbers did you use to represent Karyn and Manuela's distances from the ground floor?	How can you calculate their distance from zero? What do you notice about their distances?
Represents ground floor with the number 1.	Tell me about your number line. Where is the ground floor? What number are you using to represent the floor below the ground floor? The one below that?	Now that you know the ground floor is zero, show the distance between each person and the ground floor.

LESSON GUIDE 5

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0).

- Tell us about how you designed your number line model.
- Does it matter if the number line is horizontal or vertical? Why?
- Why did this group choose to represent the ground floor with the number 0?
- Which floors are represented by negative values? Which floors are represented by positive values? Why? *(Floors above ground level are represented by positive numbers, while floors below are represented by negative numbers.)*
- If we use the ground floor as the baseline value, or 0, then we can measure positions above and below the baseline using positive and negative numbers. **(Recapping)**

EU: The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.

- Explain to the class how you determined each person's distance from the ground floor.
- What do you notice about the distances this group determined by counting?
- What did (student name) mean when s/he said "they are the same as the number without the negative?"
- Do you think this will be true for much larger numbers as well? Is -5,235 positioned 5,235 units away from 0? Why or why not? *(Yes, this will be true. They are the same distance from zero.)*
- A number's distance from zero is its absolute value. We use this notation to represent absolute value. **(Revoicing)**
- This group made a conjecture about opposites. Tell us about your conjecture. *(Opposites have the same absolute value.)*
- Who heard this group's conjecture and can restate it in their own words? Indicate placement in the diagram in your response.
- Why do opposites have the same absolute value? *(They are the same distance from zero, just in opposite directions.)*
- So, opposite numbers are equidistant from zero and therefore have the same absolute value. **(Marking)**

Application	Which of the following numbers are equidistant from 0? 2, 7, -3, -7, 1.4, -14
Summary	What is absolute value?
Quick Write	Why do opposite numbers have the same absolute value?

Support for students who are English Learners (EL):

1. Discuss what it means for a building to extend above and below ground. Ask students to give examples of buildings they have been in that have more than one floor below ground.

Name____

TASK
6

Hiking

Dia'Monique and Yanely picnicked together. After the picnic, Dia'Monique hiked 17 miles. Yanely hiked 14 miles in the opposite direction.

1. What is Dia'Monique and Yanely's distance from each other? Draw a picture to show their distance from each other.

2. Write an equation representing the distance between the two girls. Explain your reasoning.

LESSON GUIDE 6

Hiking

Rationale for Lesson: Develop understanding of distance between positive and negative integers in a context that is open for multiple representations of the problem. In this lesson, counting methods are no longer efficient and students are pressed to develop a method using addition.

Task 6: Hiking
Dia'Monique and Yanely picnicked together. After the picnic, Dia'Monique hiked 17 miles. Yanely hiked 14 miles in the opposite direction.

1. What is Dia'Monique and Yanely's distance from each other? Draw a picture to show their distance from each other.
2. Write an equation representing the distance between the two girls. Explain your reasoning.

Tennessee State Standards	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write* $\lvert -30 \rvert = 30$ *to describe the size of the debt in dollars.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision.	

Essential Understandings	• Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0). • Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point. • The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero. • The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.
Materials Needed	• Task sheet. • Calculators (as needed).

LESSON GUIDE 6

SET-UP PHASE

Can I have a volunteer read the Hiking Task? Now take 5 minutes to work on the problem individually before you begin working in your groups.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Students draw diagram that is not a number line.	Can you show me how your diagram represents the girls' distances?	How can you represent this using a number line?
Students subtract the distances.	Why did you subtract the distances?	Can you draw a diagram showing the girls' distances from the camp? Does your subtraction problem model the relationship in your diagram?
Students use an absolute value equation to determine the sum.	Why did you add the numbers? Why are both numbers positive?	How can you represent this on a number line? Where do you see addition?
Students plot the points on a number line.	How did you determine the location on the number line?	How can you use the points you plotted to determine the distance between the two girls?
Students draw an incorrect number line using positive numbers on both sides of zero.	Tell me about your number line. How did you decide on the intervals and how to label each tic mark?	Where is zero on your number line? Where would -1 appear on your number line?

LESSON GUIDE 6

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Positive numbers represent values greater than 0 and negative numbers represent values less than 0. Many real-world situations can be modeled with both positive and negative values because it is possible to measure above and below a baseline value (often 0).
EU: Any rational number can be modeled using a point on the number line because the real number line extends infinitely in the positive and negative directions. The sign and the magnitude of the number determine the location of the point.

- Let's compare these number lines that groups used. How does each of them represent the problem?
- Who can come up and show us how each of these models shows the hikers' distances from the camp?
- Why did this group use a negative number to represent Yanely's distance?
- Where are they in relation to zero? What does zero represent?
- I'm hearing you say that since the girls are moving in opposite directions, one can be represented with a positive number while the other is represented with a negative number. **(Marking)**
- Does it matter which hiker is on the positive side and which is on the negative side? **(Challenging)** Will the distance between them remain the same?

EU: The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.

- This group did not use a negative value to represent either hiker's position. Tell us about why you used positive values.
- Who heard and can say in their own words what (student name) said about distance? *(Distance is a measure of how far you are from zero. It can't be negative. For example, you don't drive a positive distance north and then a negative distance south.)*
- So, distance is always positive. **(Marking)** In this case, what are we measuring distance from? We are measuring the distance from zero.
- Distance from zero is called absolute value. The magnitude changes depending on how far away it is from zero, but it is always positive. **(Revoicing)**

LESSON GUIDE 6

EU: The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero.

- Who can show us on the number line how far apart the girls are?
- (Student's name) determined the distance by counting the tic marks on the number line while (student's name) added the distances together. How are these methods the same? How are they different?
- The distance between the girls is 31 miles. So why does the equation -14 + 17 not represent this situation?
- How would you change the equation to reflect what's going on in this problem?
- What operation do you use? Why?
- How do the numbers you are adding relate to each girl's position?
- That is interesting. (Student name) said we are adding the absolute values together. **(Marking)**
- Why would we add the absolute values together? **(Challenging)** *(One girl is a distance from zero in one direction and the other girl is a distance in the other direction.)*
- I hear you saying that we will have to add the absolute values together to get the total distance from zero. **(Revoicing)**
- Can we always add the absolute values to find the distance between two numbers? Turn and talk with a partner about when this method works and when it does not.
- Several of you shared your thoughts and examples. After hearing several of you share out, I am hearing you say that since the absolute value is the distance from zero, adding the absolute value of two numbers will tell you how far apart they are from each other if they are on opposite sides of zero. **(Recapping)**

Application	Determine the distance between -3 and 4. Describe your method.
Summary	Describe two different ways to determine the distance between two points on the number line.
Quick Write	No quick write for students.

Support for students who are English Learners (EL):

1. Have students act out their movement of walking in different directions so that the context of the problem is more concrete for students who are English Learners.
2. Co-create a list of key words and definitions (along with visual representations) such as absolute value.

TASK
7

Name____

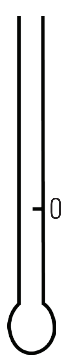
Comparing Temperatures

Mr. Winkle gave the chart below to his students:

City	Temperature
Charleston, South Carolina	35°
Amherst, Massachusetts	-5°
Madison, Wisconsin	-10°
Reading, Pennsylvania	0°
Eugene, Oregon	20°
Los Angeles, California	45°

This chart shows the temperatures in different cities on a cold day. Mr. Winkle asked Bill, "What is the difference between the highest temperature and the lowest temperature?"
Bill did not know how to answer Mr. Winkle's question.

1. Locate the highest and lowest temperatures in the chart. Label them on the thermometer below. Determine the difference between the highest and lowest temperatures. Explain your reasoning.



2. For what temperatures in the table above is the difference between the temperatures 30°? Explain how you made your decision.

LESSON GUIDE 7

Comparing Temperatures

Rationale for Lesson: Continue developing an understanding of distance and absolute value. In this lesson, students work with absolute value equations as they compare positive and negative temperatures on a vertical number line.

Task 7: Comparing Temperatures
Mr. Winkle gave the chart below to his students:

City	Temperature
Charleston, South Carolina	35º
Amherst, Massachusetts	-5º
Madison, Wisconsin	-10º
Reading, Pennsylvania	0º
Eugene, Oregon	20º
Los Angeles, California	45º

This chart shows the temperatures in different cities on a cold day. Mr. Winkle asked Bill, "What is the difference between the highest temperature and the lowest temperature?"
Bill did not know how to answer Mr. Winkle's question.

1. Locate the highest and lowest temperatures in the chart. Label them on the thermometer below. Determine the difference between the highest and lowest temperatures. Explain your reasoning.

2. For what temperatures in the table above is the difference between the temperatures 30°? Explain how you made your decision.

Tennessee State Standards	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

	6.NS.C.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
	6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision. MP8 Look for and express regularity in repeated reasoning.	
Essential Understandings	• The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero. • The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.	
Materials Needed	• Task sheet. • Additional number lines. • Calculators as needed.	

LESSON GUIDE 7

SET-UP PHASE

Read the Comparing Temperatures Task to yourselves silently. What is the temperature in each of the cities given? What are you being asked to calculate? Now take 5 minutes to work on the problem individually before you begin working in your groups. Number lines are available at your tables to use as needed.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Students determine the highest/lowest values but cannot determine the difference between them.	Can you show me the highest and lowest temperatures on the thermometer?	Show on your paper how you can use the thermometer to determine the difference.
Students add the temperatures 45 + (-10).	Why did you add the numbers? What is the sum?	The answer to this addition problem is 35. Is the temperature difference 35? What changes must you make to your equation and why?
Students plot the points on the thermometer.	How did you determine the location on the thermometer? How is this like a number line?	How can you use the points you plotted to determine the difference?
Students finish early.	How did you determine the difference in temperatures?	Can you summarize a rule that you can use to calculate the difference between a positive and negative number?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.

- This group plotted all of the temperatures on a thermometer. Tell us how this helped you identify the highest and lowest temperature.
- I don't understand…10 is more than 5, how can -10 be the coldest temperature? Who can explain this to us?
- Who heard and can restate what (student name) just said about distance from zero? What is the mathematical term for the measurement of how far a number is from zero?
- Who can come up and show the temperatures and their relationship to zero?
- Why does the magnitude of the negative numbers increase as the thermometer goes down?
- Since -10 is the coldest temperature, does that mean -11 will be colder or warmer? Why?
- Let's explore this idea of absolute value a little more. What is the absolute value of 17? -23? 4? How do you know?
- Can two different numbers have the same absolute value? Why or why not?
- (Student name) says that opposites have the same absolute value. This is an important idea that we will explore further. **(Marking)**

EU: The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero.

- This group found the difference between the high and low temperatures by counting the tic marks. Can you come up and explain your thinking to the class?
- Who can show us on the thermometer how far apart the temperatures are?
- Does this method work to find the distance between two positive numbers? Two negative numbers? Why or why not?
- Is there a shortcut we can use? What if the numbers are very far apart and it will take too long to count them? Show us your addition equation and tell us how you used it to find the difference between the lowest and highest temperatures.
- Who can explain this group's work? How can you *add* to find the difference? *(I can figure out the distance from zero in each direction. This is always positive. Then I can add them together to calculate the total distance.)*
- So we agree that the difference is 55 degrees. How is this reflected in the equations that several groups wrote?
- Where do you see 45 + 10 on the thermometer? Since the temperature is -10, what is wrong with adding 45 and -10?
- How is this represented on the number line?
- In the past we subtracted when calculating a difference. Why are we now adding? *(That worked for the distance between two positive numbers because the distance is the difference between the two numbers. Here we are considering a positive and a negative.)*
- Why is 45 + 10 the same as 45 - (-10)? Can you show this on the number line?
- Can you summarize a method for finding the difference between a positive and negative number? How is this different than if the two numbers were positive? *(The difference between two positive numbers is their difference. When you have a positive and a negative value, you are subtracting a negative, or adding their distances from zero.)*
- How is this task the same as the Hiking Task? How is it different? *(It's similar in that distances were in opposite directions of zero and in that problem we had to construct our own number line to represent the problem. Here we are given a vertical number line.)*
- So, we found that when we want to calculate the difference or the distance between a positive and a negative number on a number line, we have two options. The first method is counting the spaces between them. Some groups used a skip counting method, while others counted each space. This method will always work, but it may be difficult if the numbers are very far apart or if they are not integer values. The other method is to add their absolute values. You told me that this works because the numbers are on opposite sides of zero. I heard you tell me that the total distance between them is the sum of their distances from zero if one of the values is positive and one is negative. **(Recapping)**

LESSON GUIDE 7

Application	On a given day, the low temperature in Anchorage, Alaska was -22° while the low temperature in Miami, Florida was 72°. Determine the difference in the temperature.
Summary	How can you determine the absolute value of a number? What does absolute value mean? How can you use absolute value to determine the difference between a positive and negative number?
Quick Write	Why is absolute value always positive?

Support for students who are English Learners (EL):

1. Discuss the context of the problem with visual representations of a thermometer so that the context of the problem is more concrete for students who are English Learners.
2. In the set-up phase, translate key words such as "temperature" when describing the context.

Name____

TASK 8

Beanbag Toss

In the game Beanbag Toss, a beanbag is thrown onto the game board shown below. Two players are on a team. Each player on the team throws the beanbag once and the team score is calculated by determining the difference between the two players' scores. The team with the greatest difference between the two scores wins the game.

-2	8	3	-6
2	-1	6	-8
4	5	1	-5
-3	7	-4	-7

1. Four teams are competing and their beanbag tosses are described below. Calculate each team's score and then decide which team won the game.
   A. **Team 1:** Jillian's beanbag lands on -2 and Marcus' beanbag lands on 8.
   B. **Team 2:** Joseph's beanbag lands on -5 and Fernando's beanbag lands on 4.
   C. **Team 3:** Jasmine's beanbag lands on 3 and Lynn's beanbag lands on -7.
   D. **Team 4:** Michael's beanbag lands on -8 and Chu's beanbag lands on -1.

Summarize your strategy for calculating each team's score.

TASK
8

2. In a second game, Jillian's beanbag lands on a 4 and Marcus's beanbag lands on -4. Since they are opposite numbers, Jillian and Marcus write down their team score as 0. Is this correct? Explain why or why not.

3. Do you agree or disagree with the following statement: "All team scores in the beanbag toss will be positive"? Explain why or why not.

Beanbag Toss

LESSON GUIDE
8

Rationale for Lesson: Solidify understanding of absolute value and difference between positive and negative integers. Students will determine the sum of the absolute values or recognize that subtracting a negative value is the same as adding the values.

Task 8: Beanbag Toss
In the game Beanbag Toss, a beanbag is thrown onto the game board shown below. Two players are on a team. Each player on the team throws the beanbag once and the team score is calculated by determining the difference between the two players' scores. The team with the greatest difference between the two scores wins the game.

-2	8	3	-6
2	-1	6	-8
4	5	1	-5
-3	7	-4	-7

See student paper for complete task.

Tennessee State Standards	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

LESSON GUIDE 8

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.
Essential Understandings	• The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero. • The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.
Materials Needed	• Task sheet. • Additional number lines. • Calculators (as needed).

LESSON GUIDE 8

SET-UP PHASE

Read the Beanbag Toss Task to yourselves silently. Can somebody explain in just a sentence or two how the game is played? How is score kept? I'm going to have you take five minutes to work on the problem individually before you begin working in your groups. Number lines are available at your tables. You may also want to refer to classroom artifacts from the previous tasks.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Student incorrectly adds the values. For example, 8 + (-5) = 3.	Why did you add the numbers? What is the sum?	The answer to this addition problem is 3. Plot the numbers on the number line. Is the distance between them 3 units? How can you determine the difference?
Student plots the points on a number line and counts.	How did you determine the location on the number line? How can you use this to calculate distance?	How can you represent this with an equation? How can this help you determine the difference?
Student writes an absolute value equation.	How did you write your equation? What does each of the values represent in the problem?	How can you use an absolute value equation to calculate the difference between two negative numbers?
Student writes a subtraction equation, recognizing that subtracting a negative is the same as adding a positive. For example: 5 - (-1) = 6.	How did you write your equation? Why did you add?	Can you represent this problem with an absolute value equation?
Students finish early.	How did you determine the difference?	Can you summarize a rule that you can use to calculate the difference between a positive and negative number?

LESSON GUIDE 8

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The absolute value of a number is the number's magnitude or distance from 0. If two rational numbers differ only by their signs, they have the same absolute value because they are the same distance from 0.
EU: The distance between a positive and negative value on a number line is equal to the sum of their absolute values because they are located on opposite sides of zero.

- I noticed while circulating that some groups used an absolute value equation, others used a numeric equation, and others used a counting method. Let's compare how different groups represented the difference between the scores.
- Who can come up and show how you counted the difference on the number line?
- How is the distance from zero on the number line determined?
- Would this method be any different if both numbers were positive or negative? Why might we want other methods besides counting?
- It looks like some groups used absolute value. Who can explain this method?
- What does absolute value mean and how does it relate to this problem? Can you point out the values on the number line while you explain? *(The absolute value represents this distance and this distance [pointing to each].)*
- So I hear you saying and pointing out that absolute value is the distance from zero and since we have two distances from zero, one in the positive direction and one in the negative direction, then the total distance will be their sum. **(Marking)**
- How does this relate to subtracting a negative number? For example, why is 5 - (-1) the same as 5 + 1?
- Can you explain this on the number line? Why is this the same as the sum of the absolute values?
- I also heard most people explaining that Marcus and Jillian were incorrect in their reasoning. How so? I'd like somebody to first explain their misconception and then explain how to correct their reasoning. *(Marcus and Jillian were thinking of the numbers being the same distance from zero. Maybe they added them to get zero, but their total distance is the sum of their distances from zero, which is 8.)*
- So, Marcus and Jillian recognize that opposite numbers have the same distance from zero, but didn't recognize that the distance between two different numbers is always a positive value. **(Marking)**
- Why is the distance between two different numbers always positive?

Application	Write a problem situation that could be represented by the equation: $\lvert -10\rvert+\lvert 12\rvert=22$.
Summary	What does absolute value tell you about a number? How do we determine the difference between two numbers? When is absolute value useful in determining that difference?
Quick Write	No quick write for students.

Support for students who are English Learners (EL):

1. Discuss the context of the problem with visual representations of the game board so that the context of the problem is more concrete for students who are English Learners.

Institute for Learning
Learning Research and Development Center

University of Pittsburgh
3939 O'Hara Street
LRDC Suite 310
Pittsburgh, PA 15260
T: 412-624-8319

www.instituteforlearning.org

institute for learning

Grade 7

Adding and Subtracting Positive and Negative Rational Numbers

A SET OF RELATED LESSONS

UNIVERSITY OF PITTSBURGH

Table of Contents

Introduction

Tasks and Lesson Guides

Grade 7

Introduction

Adding and Subtracting Positive and Negative Rational Numbers

A SET OF RELATED LESSONS

Overview

In this set of related lessons, students extend their previous understanding of addition and subtraction of positive numbers to include negative numbers.

In the first two tasks, students explore opposite numbers and the idea that, for the sum of numbers to be zero, their magnitude, or relative position from zero, must be equal. Students use this understanding to determine whether the sum of positive and negative integers will be positive or negative or zero. In Task 3, students continue to explore decomposing and regrouping numbers using chips to model the zero-sum property. In Task 4, students solidify their understanding of adding positive and negative rational numbers.

In Tasks 5 and 6, students extend use of number lines and manipulatives to subtract integers. They start by using a chip model to represent subtraction of integers and then move to a number line and consider rational numbers that are not integer values. In Task 7, students continue developing their understanding of subtraction in the context of a thermometer vertical number line. Students grapple with the question of why addition is commutative while subtraction is not. In Task 8, students solidify their understanding of subtracting positive and negative rational numbers. The tasks are aligned to the 7.NS.A.1, 7.NS.A.1a, 7.NS.A.1b, 7.NS.A.1c, 7.NS.A.1d, and 7.NS.A.3 Content Standards of the CCSSM.

The prerequisite knowledge necessary to enter these lessons is an understanding of the placement of positive and negative numbers on a number line, as well as knowledge of the notation used when describing operations with negative numbers (i.e., use of parentheses in equations with integers).

Through engaging in the lessons in this set of related tasks, students will:

- add opposite numbers and determine that their sum is always 0;
- make conjectures about sums and differences of positive and negative numbers based on observed patterns;
- develop an understanding of addition and subtraction as movement along the number line; and
- articulate the algorithm for adding and subtracting integers.

By the end of these lessons, students will be able to answer the following overarching questions:

- How are addition and subtraction of positive and negative rational numbers used to solve problems?
- What strategies can we use to add and subtract positive and negative rational numbers?

The questions provided in the guide will make it possible for students to work in ways consistent with the Standards for Mathematical Practice. It is not the Institute for Learning's expectation that students will name the Standards for Mathematical Practice. Instead, the teacher can mark agreement and disagreement of mathematical reasoning or identify characteristics of a good explanation (MP3). The teacher can note and mark times when students independently provide an equation and then re-contextualize the equation in the context of the situational problem (MP2). The teacher might also ask students to reflect on the benefit of using repeated reasoning, as this may help them understand the value of this mathematical practice in helping them see patterns and relationships (MP8). In study groups, topics such as these should be discussed regularly because the lesson guides have been designed with these ideas in mind. You and your colleagues may consider labeling the questions in the guide with the Standards for Mathematical Practice.

Identified CCSSM and Essential Understandings

CCSS for Mathematical Content: The Number System		Essential Understandings
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.		
7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	Addition and subtraction of rational numbers can be represented by movement on a number line, because the sum (or difference) is another rational number whose location is determined by its magnitude and sign.
7.NS.A.1a	Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*	Two opposite numbers combine to make zero because they represent the same distance from zero on the number line.
7.NS.A.1b	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	The sum of two numbers p and q is located $\|q\|$ units from p on the number line, because addition can be modeled by movement along the number line. When q is a positive number, $p + q$ is to the right of p. When q is a negative number, $p + q$ is to the left of p. The sum of a number and its opposite, $p + -p$, is equal to zero because p and $-p$ are the same distance from 0 in opposite directions.
7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	The difference of two numbers $p - q$ is equal to $p + (-q)$ because both can be modeled by the same movement on the number line. That is, if q is positive, $p - q$ and $p + (-q)$ are both located at the point q units to the left of p and if q is negative, both are located at a point q units to the right of p. The distance between a positive number p and negative number q is the sum of their absolute values because this represents each of their distances from zero and therefore their total distance from each other.

CCSS for Mathematical Content: The Number System		**Essential Understandings**
7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.	Rational numbers can be decomposed and regrouped to efficiently add and subtract positive and negative integers. The differences $b - a$ and $a - b$ are opposites because they represent the same distance between two points on the number line. Subtraction of a lesser number minus a greater number will result in a negative difference, while subtraction of a greater number minus a lesser number will result in a positive difference. The order of the values being added does not affect the sum, but the order of the values being subtracted does affect the difference because $a + b$ models the same movement on the number line as $b + a$, while $a - b$ and $b - a$ model movement in opposite directions on the number line.
7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.	Many real-world situations can be modeled and solved using operations with positive and negative rational numbers.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Common Core State Standards, 2010, NGA Center/CCSSO*

Tasks' CCSSM Alignment

Task	7.NS.A.1	7.NS.A.1a	7.NS.A.1b	7.NS.A.1c	7.NS.A.1d	7.NS.A.3
Task 1 Football Developing Understanding	✓		✓			✓
Task 2 Football Part 2 Developing Understanding			✓		✓	✓
Task 3 Fundraiser Developing Understanding			✓		✓	
Task 4 If A then B Solidifying Understanding			✓		✓	
Task 5 Choose Your Chips Wisely Developing Understanding	✓	✓	✓	✓	✓	✓
Task 6 Traveling on the Number Line Developing Understanding	✓			✓	✓	✓
Task 7 Cold Weather Developing Understanding	✓			✓	✓	✓
Task 8 Keeping it Real Solidifying Understanding	✓			✓	✓	✓

Task	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 Football Developing Understanding	✓	✓		✓	✓	✓	✓	✓
Task 2 Football Part 2 Developing Understanding	✓			✓	✓	✓	✓	✓
Task 3 Fundraiser Developing Understanding	✓	✓	✓	✓	✓	✓	✓	✓
Task 4 If A then B Solidifying Understanding	✓		✓	✓	✓	✓	✓	✓
Task 5 Choose Your Chips Wisely Developing Understanding	✓		✓	✓	✓	✓		✓
Task 6 Traveling on the Number Line Developing Understanding	✓		✓		✓	✓	✓	✓
Task 7 Cold We ather Developing Understanding	✓	✓		✓		✓	✓	
Task 8 Keeping it Real Solidifying Understanding	✓		✓			✓	✓	✓

Lesson Progression Chart

Overarching Questions

- How are addition and subtraction of positive and negative rational numbers used to solve problems?
- What strategies can we use to add and subtract positive and negative rational numbers?

	TASK 1 **Football** *Developing Understanding*	TASK 2 **Football Part 2** *Developing Understanding*	TASK 3 **Fundraiser** *Developing Understanding*	TASK 4 **If A then B** *Solidifying Understanding*
Content	Develop understanding of integer properties and calculating sums. Show that a number and its opposite have a sum of zero.	Recognize whether a sum will be positive or negative based on the sign and magnitude of the numbers.	Develop an algorithm for adding integers using context of expenses, income, and profit.	Solidify a strategy to determine sums of positive and negative integers; determine whether an expression has a sum that is positive, negative, or 0 without calculating exact values.
Strategy	Use real-world context of running in opposite directions on a quasi-number line to understand positive and negative numbers that sum to zero.	Compare sets of similar equations to find patterns in adding numbers with different signs. Develop a zero-sum strategy.	Develop a zero-sum strategy for adding integers and compare to chip and number line models.	Apply a zero-sum strategy to determine information about the sum.
Representations	Use arrows on a quasi-number line to represent magnitude and direction.	Use number line model and arrows on the number line.	Use number line and chip models to solidify concepts of quantity and opposite.	Use number line and chips to support use of numeric strategies for adding integers.

	TASK 5 **Choose Your Chips Wisely** *Developing Understanding*	TASK 6 **Traveling on the Number Line** *Developing Understanding*	TASK 7 **Cold Weather** *Developing Understanding*	TASK 8 **Keeping it Real** *Solidifying Understanding*
Content	Extend understanding of sums to analyze differences of positive and negative numbers.	Understand subtraction as the opposite operation of addition.	Develop understanding that subtraction is not communicative.	Solidify an algorithm for subtracting integers.
Strategy	Represent the same value with different amounts of positive and negative chips using zero-sums.	Compare movement on the number line in order to generalize.	Write equations to represent differences in temperature. Compare positive and negative changes.	Apply conceptual understandings of quantity and opposite values when subtracting integers.
Representations	Represent a context with a chip model. Use a number line to support this model.	Start with an equation and construct number lines and chip representations.	Use a number line to compare two equations where the order of the integers has been switched.	Use number line and chips to support use of numeric strategies for subtracting integers.

Tasks and Lesson Guides

Adding and Subtracting Positive and Negative Rational Numbers

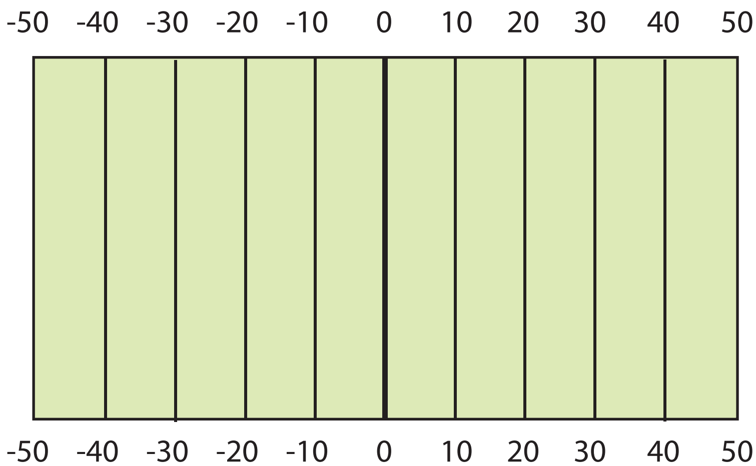
A SET OF RELATED LESSONS

Name____

TASK
1

Football

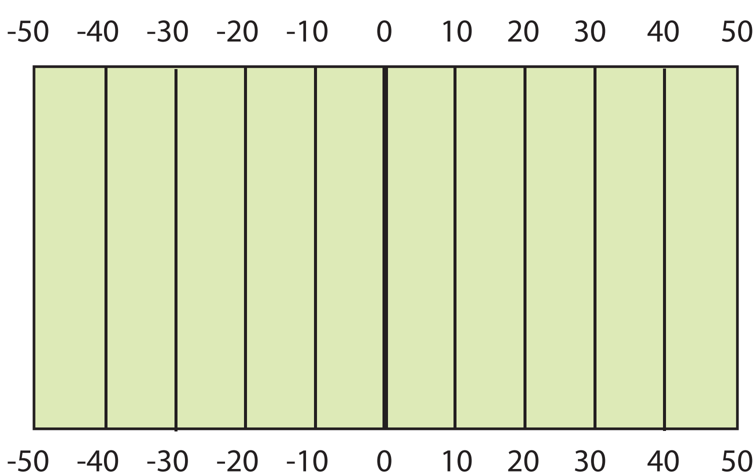
The Mathletes have developed a new system for the yardage on a football field. The middle of their field is "zero" and the teams score touchdowns by crossing the 50 or the -50 yard line.



The teams that play on the field are named the "Positives" and the "Negatives." The Positives always move to the right to score in their end zone (50 yards) and the Negatives always move to the left to score in their end zone (-50 yards). **Assume that when the first team starts in each problem, the ball is at zero.**

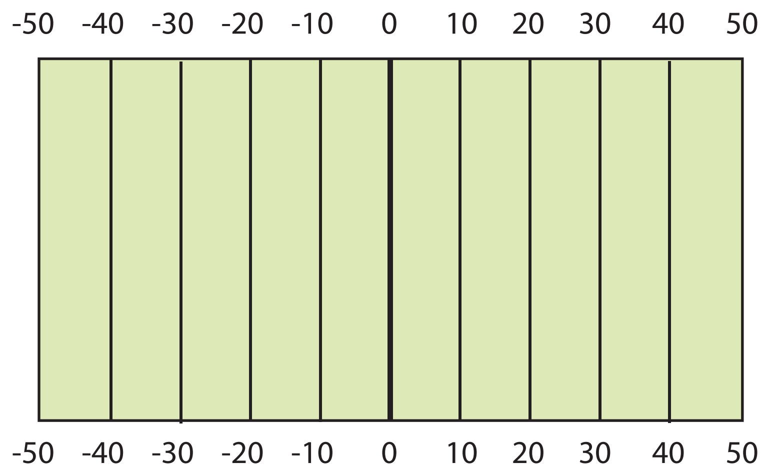
1. Draw each scenario on the football field number line with arrows and determine the ball's final location.

    A. The Positives run 40 yards and then the Negatives get the ball and run 40 yards.



TASK
1

B. The Negatives run the ball 12 yards and then another 8 yards. The Positives get the ball and run 20 yards.



2. The scorekeeper recorded the three plays represented by the expression shown below. The plus sign indicates that a new play has been added.

$$(-18) + (-7) + 25$$

Describe the three plays and then determine the location of the ball at the end of the three plays. Justify your answer using a number line.

3. The scorekeeper records the following mathematical equations for a number of plays. Use the number line to determine the missing values.

   A. 40 + (-30) + ______ = 0

   B. - 25 + (-10) + ______= 0

   C. -20 + 60 +_____ + _____ = 0

   D. 30 + (-65) + _____+______+_______ = 0

Summarize your strategy for determining the missing values

Football

LESSON GUIDE 1

Rationale for Lesson: Develop an understanding of how to use equations and the number line to calculate zero-sums. In this lesson, students explore the idea that opposite numbers have a zero-sum because they represent the same distance from zero on the number line.

Task: Football
The Mathletes have developed a new system for the yardage on a football field. The middle of their field is "zero" and the teams score touchdowns by crossing the 50 or the -50 yard line.
The teams that play on the field are named the "Positives" and the "Negatives." The Positives always move to the right to score in their end zone (50 yards) and the Negatives always move to the left to score in their end zone (-50 yards). **Assume that when the first team starts in each problem, the ball is at zero.**
See student paper for the complete task.

Common Core Content Standards	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1b	Understand $p + q$ as the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	
Essential Understandings	• Addition and subtraction of rational numbers can be represented by movement on a number line, because the sum (or difference) is another rational number whose location is determined by its magnitude and sign. • The sum of a number and its opposite, $p + -p$, is equal to zero because p and $-p$ are the same distance from 0 in opposite directions.	
Materials Needed	• Task sheet. • Additional paper. • Calculator (optional).	

LESSON GUIDE 1

SET-UP PHASE

Can somebody please read this task aloud? Please begin working on this individually for five minutes before working with your partners. You may find it helpful to represent the Positives and Negatives in two colors. For example, you may want to make the Positives yellow and the Negatives red. I have passed out markers so you can draw arrows in these colors. You will also find additional number lines at your tables.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	In what direction does each team move? Where does the ball start?	Can you combine these movements to see where the ball will land at the end of all of the moves?
Counts the distance on the number line. For example: In 3a when drawing arrows for 40 + -30, they count each group of 10 to end the second arrow at +10.	How did you know where to stop each arrow? How did you know the direction of the movement?	Have you noticed a pattern that could help you determine where to stop the arrow without counting?
Uses a numeric strategy. For example: in 3c when figuring out where the ball is after -20 + 60, they break down 60 into 20 + 40 and recognize that the -20 + 20 = 0, so the ball must be at 40.	How did you determine the missing value(s) so that the ball ends at 0?	Is there more than one way for the ball to get back to 0? Why or why not?

SHARE, DISCUSS, AND ANALYZE PHASE

LESSON GUIDE 1

EU: Addition and subtraction of rational numbers can be represented by movement on a number line because the sum (or difference) is another rational number whose location is determined by its magnitude and sign.

- Let's take a look at several strategies that different groups used.
- How did your group know how long to draw an arrow that doesn't start at zero?
- How did you know where -18 + (-7) would end on the football field?
- Can somebody say in his or her own words what this group said about movement on the number line and then explain a different way of thinking about it?
- In #3, how can we determine where movements are going to end? For example, when drawing 40 + (-30), how did you determine where your second arrow would stop?
- What strategy did different groups use to determine the missing value(s) in question 3?
- I am hearing people say that addition and subtraction can be modeled as movement along a number line. **(Marking)** As we continue to study these concepts, we will get more specific about what this modeling looks like.

EU: The sum of a number and its opposite, *p + -p*, is equal to zero because *p* and *-p* are the same distance from 0 in opposite directions.

- In our discussion about movement on the number line, I heard several of you say that opposite values sum to zero. **(Marking)** What is the opposite of a number?
- So if I place any point on the number line, you can determine its opposite? How can you do that? Come up and show us. **(Challenging)** *(It's across zero, but the same distance.)*
- How does this help determine missing values in part 3? Let's look at 40 + (-30) + _____ = 0. Tell us about how we can use opposites to determine the missing value. *(Student 1: They have to be opposites because they equal to 0. Student 2: 40 is the positive, so -30 + something has to be the opposite of 40. Student 3: I marked 40 and -30 on the number line. Then I saw -30 was 10 away from -40 and that is the opposite, so I knew I needed 10 more negatives to make the opposite.)*
- Many groups have different missing values for #3. Are all the values correct? Why do some of the questions have only one correct answer while others have several?
- Is there a way you can check without drawing them on a number line? Let's take a look at the work of a group that used equations and compare this to the number line. What patterns do you notice? *(It was getting hard to make number lines for numbers that were big, so we thought about just using the numbers. Like, -65 is -30 and -35 so in 30 + (-30) + (-35) there are already some opposites and then we just need positive numbers that add up to the opposite of -35.)*
- This group decomposed numbers to look for opposites. **(Revoicing)** We will look at this strategy again as we keep learning about rational numbers.
- Can you explain your strategy for us and show the movement on the number line as you explain?
- So we saw in several examples, both looking at movement up and down the field and by calculating arithmetically, that the sum of a number and its opposite is always zero. **(Recapping)**

LESSON GUIDE 1

Application	You are on a game show where you win money for answering questions correctly and lose money for answering questions incorrectly. The amount of money for each question varies. You have answered two questions incorrectly and have -$45. But then you answer two questions correctly to get back to 0. What could the two questions have been worth?
Summary	Why do opposite numbers add to zero?
Quick Write	No quick write for students.

Support for students who are English Learners (EL):

1. Students who are English Learners may be unfamiliar with the context. Demonstrate several examples of the movement of the game during the set-up so that the context is more concrete.

Name____

TASK
2

Football Part 2

1. The statistician is calculating total yardage and organizes the plays into two groups according to a pattern.

   A. Determine the total yardage (sum) for each pair of plays.

Set 1
-40 + 50
30 + (-20)
-30 + 50
40 + (-15)

Set 2
15 + (-50)
-30 + 25
10 + (-40)
-30 + 10

   B. Describe how the plays were organized.

2. If *a* is a positive integer and *b* is a negative integer, make a conjecture about the conditions under which each of the statements true.

   A. a + b > 0

   B. a + b < 0

   C. a + b = 0

   D. a + b = b + a

LESSON GUIDE 2

Football Part 2

Rationale for Lesson: Recognize patterns involving magnitude and direction of positive and negative numbers. In this lesson, students learn methods for determining whether the sum of integers will be positive or negative.

Task: Football Part 2

1. The statistician is calculating total yardage and organizes the plays into two groups according to a pattern.
   A. Determine the total yardage (sum) for each pair of plays.

Set 1
-40 + 50
30 + (-20)
-30 + 50
40 + (-15)

Set 2
15 + (-50)
-30 + 25
10 + (-40)
-30 + 10

2. If a is a positive integer and b is a negative integer, make a conjecture about the conditions under which each of the statements true.
   A. $a + b > 0$
   B. $a + b < 0$
   C. $a + b = 0$
   D. $a + b = b + a$

Tennessee State Standards	7.NS.A.1b	Understand $p + q$ as the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
Standards for Mathematical Practice	MP1	Make sense of problems and persevere in solving them.
	MP4	Model with mathematics.
	MP5	Use appropriate tools strategically.
	MP6	Attend to precision.
	MP7	Look for and make use of structure.
	MP8	Look for and express regularity in repeated reasoning.

Essential Understandings	• The sum of two numbers *p* and *q* is located $\|q\|$ units from *p* on the number line, because addition can be modeled by movement along the number line. When *q* is a positive number, $p + q$ is to the right of *p*. When *q* is a negative number, $p + q$ is to the left of *p*.
Materials Needed	• Task sheet. • Additional number lines. • Calculator (optional).

SET-UP PHASE

Can somebody please read this task aloud? In #3, it says to make a conjecture about the conditions that make these statements true; can someone explain what this means without giving an answer to the problem? Please begin working on this individually for five minutes before working with your partners. Additional number lines and markers are at your tables.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	Can you tell me what is happening in this problem? What movement is occurring? In what direction?	How can you represent this on the number line? Continue using a number line method and look for patterns.
Uses a number line to count.	Tell me about your strategy. How did you solve 40 + (-15)?	What patterns did you see in Set 1? How can you use this pattern to predict whether a sum will be positive or negative?
Decomposes and regroups numbers.	How did decomposing 40 into 25 + 15 help you find the sum?	What will have to be true for your answer to be positive? What will have to be true for your answer to be negative?
Uses an algorithm/ equation.	Can you explain your algorithm?	Can you generalize your strategy using a number line? How can you use your number line to prove that this algorithm will always "work?"

LESSON GUIDE 2

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The sum of two numbers *p* and *q* is located |*q*| units from *p* on the number line, because addition can be modeled by movement along the number line. When *q* is a positive number, *p* + *q* is to the right of *p*. When *q* is a negative number, *p* + *q* is to the left of *p*.

- What patterns did everybody notice while working on question 1?
- I heard one group saying that all the answers in Set 1 are positive and all the answers in Set 2 are negative, but they weren't sure why. Who can help explain this pattern?
- Who can add on to this and explain why the one group of sums is positive while the other is negative?
- What would have to be true about $a + b$ to be greater than 0? How does this relate to the questions in problem 1 where we summed pairs of positive and negative numbers? How can you test this conjecture?
- What would have to be true about a and b for $a + b$ to be less than 0? How can you test this conjecture? *(The negative number has to be more.)*
- Are you saying that the absolute value of the negative number has to be greater than the absolute value of the positive number if the sum is negative? **(Revoicing)**
- How do these relationships appear in the number line? *(The negative is further away from zero than the positive.)*
- What has to be true if the sum is equal to zero? *(They are the same number, but one is negative and one is positive. Could be -3 + 3 or -50 + 50.)*
- I heard (student name) say a and b would have to be opposites in order for $a + b$ to equal zero. **(Revoicing)** Remind us what opposites are and explain why the sum of opposites is zero. Who can show this on the number line and relate this to the game?
- During the Explore phase, I noticed some confusion about $a + b = b + a$. Now that we've discussed the other problems in context with examples shown on the number line, I want you to talk to your partners and come to an agreement about whether this is always true or not when you consider negative numbers. You must have evidence to back up your claim.
- We considered using a number line to find sums and differences and we remembered that we learned yesterday that we can model addition by moving along the number line. Movement to the right indicates adding a positive value and movement to the left indicates adding a negative value. Using this method and an algorithm introduced by (student name), we noticed that when summing a positive and a negative, if the number with the larger absolute value is negative, the sum is negative and if the number with the larger absolute value is positive, the sum is positive. **(Recapping)** Who can summarize for us one more time why this is true?

Application	Write two additional expressions that fit into Set 1 and Set 2. Determine the sums and explain why these expressions fit the pattern.
Summary	What strategies for adding positive and negative integers did you discover in this task?
Quick Write	Summarize in words why $a + b = b + a$.

Support for students who are English Learners (EL):

1. Since the task is a continuation of the previous context that may have been unfamiliar to English Learners, demonstrate several examples of the movement of the game during the set-up so that the context is more concrete.
2. Have students who are English Learners demonstrate movement along the number line as they determine the sum.

TASK
3

Name____

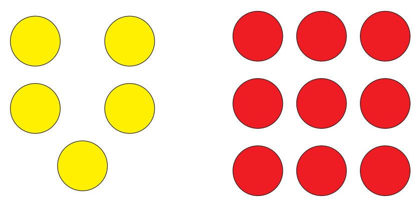
Fundraiser

You may recall that integers can be represented with chips as well as with a number line. Yellow chips represent positive numbers and red chips represent negative numbers.

Renea and her business partner, Alicia, set up a business selling school supplies to raise money for their vacation. They use chips to model their expenses and their income. Expenses are the amount of money spent on supplies and income is the amount of money earned from selling the supplies. Profit is the amount of money a company makes. (Profit may be negative if a company has more expenses than income.)

Renea and Alicia use negative (red) chips for their expenses and positive (yellow) chips for their income.

1. The chip diagram below represents their income and expenses for a given day. Calculate their profit. Use an equation and number line to support your calculation.



2. Model a second situation in which Renea and Alicia end the day having lost or made the same amount of money that they did in question 1. Explain your reasoning.

3. Renea and Alicia had \$15 in expenses and \$7 in income. Determine their profit. Justify your solution with chips as well as a number line.

TASK
3

4. Renea and Alicia have \$57 in expenses and \$32 in income. Alicia ignores the number line and chip method and calculates their profit as shown.

   -57 + 32
   (-25 + -32) + 32
   -25 + (-32 + 32)
   -25 + 0
   -25

   A. In your own words, describe Alicia's method for adding positive and negative numbers.

   B. Renea argues that she can do the same thing with chips and the number line.
   Explain Renea's method.

5. Write two different pairs of numbers, one positive and one negative, whose sum is the given number below. Show your answer with chips and a number line.

   A. 4

   B. -8

6. Do you agree or disagree with the following statement: "Adding a negative number is the same as subtracting that number." Justify your answer mathematically. Use examples in your justification.

Extension
Over the course of a week, Renea made a total of only \$8. She made money during four of the days, but lost money during three of the days. Determine the possible earnings for each day of the week. Use number lines, diagrams, and equations to justify your solution.

LESSON GUIDE 3

Fundraiser

Rationale for Lesson: In this lesson, a different visual model is introduced. Students will continue developing understanding of addition by using the new model to make sense of a problem situation. In this lesson, students will formally encounter the strategy of decomposing and regrouping numbers using zero-sums to add positive and negative integers.

Task: Fundraiser
You may recall that integers can be represented with chips as well as with a number line. Yellow chips represent positive numbers and red chips represent negative numbers.
Renea and her business partner, Alicia, set up a business selling school supplies to raise money for their vacation. They use chips to model their expenses and their income. Expenses are the amount of money spent on supplies and income is the amount of money earned from selling the supplies. **Profit** is the amount of money a company makes. (Profit may be negative if a company has more expenses than income.)
See student paper for complete task.

Common Core Content Standards	7.NS.A.1b	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.

Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.
MP8 Look for and express regularity in repeated reasoning.

Essential Understandings

- The sum of two numbers p and q is located $|q|$ units from p on the number line, because addition can be modeled by movement along the number line. When q is a positive number, $p + q$ is to the right of p. When q is a negative number, $p + q$ is to the left of p.
- The sum of a number and its opposite, $p + -p$, is equal to zero because p and $-p$ are the same distance from 0 in opposite directions.
- Rational numbers can be decomposed and regrouped to efficiently add and subtract positive and negative integers.

LESSON GUIDE 3

Materials Needed	• Task sheet. • Additional paper. • Number lines, chips, or other counter. • Calculator (optional).

SET-UP PHASE

Can somebody please read this task aloud? In 6th grade, you may have been introduced to chips as a method to add integers. Can somebody please demonstrate how chips can be used to determine the sum of 3 and -3? Today, I want us to focus on comparing the chip model to the number line model that we've been working with to explore integer relationships. Number lines and chips are available at each of the tables. Please begin working on this individually for 10 minutes before working with your partners.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 20 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	Can you tell me what you know about this problem? How are positives and negatives represented? What do they mean in this context?	How would you show \$9 in expenses and \$5 in income with chips and on the number line? Then refer to our artifact from the previous task. What are zero-sums?
Uses a number line to count.	How did you determine the answer when they had \$15 in expenses and \$7 in income?	Can you come up with a way to determine the answer without counting on a number line? Do you notice any patterns in this addition problem?
Decomposes and regroups numbers and uses zero-sums. For example, when determining -15 + 7, the student writes -7 + -8 + 7 in order to regroup and use zero-sum.	How did you know how to break down the numbers? How did you know how to regroup?	Can you generalize your method into an algorithm? Can you determine a strategy so that you can calculate the answer without grouping? How will this appear on the number line?
Develops an algorithm.	Can you describe your equations? How do you know the answer is correct?	Can you use your number line to show that this will always be true?

LESSON GUIDE 3

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The sum of two numbers *p* and *q* is located |*q*| units from *p* on the number line because addition can be modeled by movement along the number line. When *q* is a positive number, *p* + *q* is to the right of *p*. When *q* is a negative number, *p* + *q* is to the left of *p*.

- Before we discuss the exact answer or algorithms, I want different groups to explain how you know a sum will be positive or negative.
- How does the chip model support what this group just said? *(If there are more red chips, the sum will be negative and if there are more yellow chips, the sum will be positive.)*
- Who agrees with this statement and can add on? *(A yellow chip plus a red chip is zero. So, they cancel each other out. If there is more of one color than the other, that is the color that will be left over so that is the sign of the sum.)*
- So, we can see before we calculate whether the sum will be negative or positive. When calculating the sum of a positive number and a negative number using chips, we find zero pairs. **(Revoicing)** Is there a way to SEE the sign of the sum on a number line before you calculate the sum? **(Challenging)** *(The number farther from 0 is the sign of the sum, like 8 + (-5) will be positive because 8 is farther from 0 than -5.)*
- So what I hear is that the number represented by the greatest number of chips, or the number farther from zero on the number line, determines the sign of the sum. **(Marking)**
- How can we use this idea of the number with the greater absolute value determining the sign of the sum in examining Alicia's method?

EU: Rational numbers can be decomposed and regrouped to efficiently add and subtract positive and negative integers.

- What did Alicia do when she was adding -57 + 32? Who can summarize how she broke down the number and rewrote it using the regrouping method?
- Why did she decompose and regroup them in this way?
- How is it the same/different as the chip and number line model? How do you know that the sum results in zero? What can you say about their relative position on the number line?
- Let's use this idea to see how different groups approached number 5. In this problem, we know the sum and have to determine the addends.

EU: The sum of a number and its opposite, *p* + -*p*, is equal to zero because *p* and -*p* are the same distance from 0 in opposite directions.

- It looks as though we have many different answers. Is everyone's pair of numbers correct? What is it about this problem that allows so many correct answers?
- When looking at the pairs with different signs, what do they all have in common? How does this support the strategy we developed in Number 4?
- Did any groups pick numbers that were not whole numbers? Can we do this? **(Challenging)**

LESSON GUIDE 3

Application	Renea wrote the following equation to calculate her profit. Solve: 45 + (-32) Alicia wrote the equation as -32 + 45. Will her answer be the same? Why or why not?
Summary	Compare and contrast adding integers on a number line versus adding integers with chips. What do they both show you about integers that will help you add without either of these tools?
Quick Write	Do you agree or disagree with the following statement: "Adding a negative number is the same as subtracting that number." Justify your answer mathematically. Use examples in your justification.

Support for students who are English Learners (EL):

1. Co-create a running list of strategies for adding positive and negative numbers. Make the process concrete for English Learners by referencing the number line and chips during classroom discussions.

TASK
4

Name____

If A then B

1. Determine the sum of each expression.

A. 8 + -8

B. -8 + 18

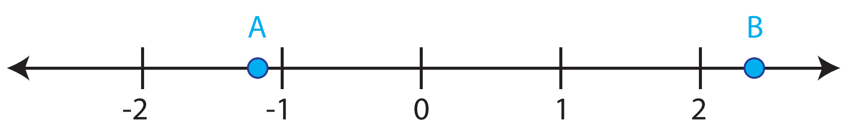
C. 18 + -8

D. -8 + 28

E. -8 + 1028

Summarize your strategy for calculating the sum of a positive and negative integer.

2. Determine whether each of the expressions below has a value that is positive, negative, or equal to 0. Explain your reasoning.



a + 1

b + (-1)

a + (-2)

a+ (-a)

a + b

If A then B

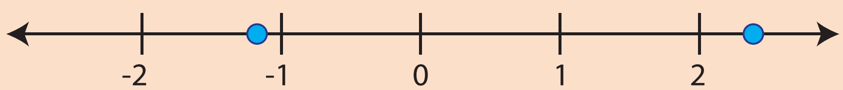
LESSON GUIDE 4

Rationale for Lesson: Solidify a method for adding positive and negative integers. In this lesson, students determine a pattern and determine an algorithm through repeated reasoning.

Task: If A then B

1. Determine the sum of each expression.
   A. 8 + -8 B. -8 + 18
   C. 18 + -8 D. -8 + 28
   E. -8 + 1028
   Summarize your strategy for calculating the sum of a positive and negative integer.

2. Determine whether each of the expressions below has a value that is positive, negative, or equal to 0.



Explain your reasoning.

See student paper for complete task.

Tennessee State Standards	7.NS.A.1b	Understand $p + q$ as the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	
Essential Understandings	• The sum of two numbers p and q is located $\lvert q \rvert$ units from p on the number line, because addition can be modeled by movement along the number line. When q is a positive number, $p + q$ is to the right of p. When q is a negative number, $p + q$ is to the left of p. • Rational numbers can be decomposed and regrouped to efficiently add and subtract positive and negative integers.	
Materials Needed	• Task sheet. • Additional paper. • Number lines, chips, or other counters. • Calculator (optional).	

LESSON GUIDE 4

SET-UP PHASE

Can somebody please read this task aloud? In #2, do we know the exact values of *a* and *b*? What do we know about *a* and *b*? Please begin working on this individually for five minutes before working with your partners. Number lines and chips are available at your tables to use as you need them.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Uses a chip method correctly.	Can you explain your method? What do the chips represent and how do they help you determine the sum?	Can you show this on the number line or with another method? How will you figure out -8 + 1028?
Uses a number line to show the addition. (May include incorrect work.)	Can you explain your method? How will you know what sign the solution has?	Can you determine the sum numerically? For example, how will you figure out -8 + 1028 when our number lines only go up to 10?
Uses a numerical strategy of decomposition. -8 + 1028 = -8 + 1020 + 8 = 1020	Can you explain your strategy?	Can you summarize an algorithm for adding a positive and negative integer?
Chooses a value for a and b based on approximate location and solves each problem.	Can you explain your method? How did you get the values of *a* and *b*?	Can you figure out the answer without using a specific value?
Finishes early.	How do you know addition of positive and negative values is commutative?	Can you think of any operations for which the order does matter?

SHARE, DISCUSS, AND ANALYZE PHASE

LESSON GUIDE 4

EU: The sum of two numbers *p* and *q* is located |*q*| units from *p* on the number line because addition can be modeled by movement along the number line. When *q* is a positive number, *p* + *q* is to the right of *p*. When *q* is a negative number, *p* + *q* is to the left of *p*.

- Walking around, I noticed groups using several different methods. Some groups used chips, others used a number line, while quite a few used a numeric strategy. Let's look first at this group's work. They used a number line. Then we will compare and contrast their strategy with the numeric strategy used by another group.
- Who can tell us how to determine whether the sum is going to be positive or negative for each of the methods?
- How can we determine the exact value using a number line? *(Starting at the first number, I count spaces to the left or right. For -8 + 18, I start at -8 and count 18 spaces to the right. I ended up at 10.)*
- As we compared the chip method, the number line method, or an algorithm, we saw that each method has strengths and weaknesses. Who can identify some of those strengths and weaknesses in their own words?
    - *(The chip method lets us see at a glance whether the sum is negative or positive and I can see the zero pairs easily. It is hard to do with really big numbers though, because we don't have enough chips.)*
    - *(The number line method is nice because I can see why the sum is negative or positive. If I start at a negative and count right, I can see if I move across the zero or not. The number line is hard to use for really big numbers because I would need a big number line and it gets hard to count the spaces.)*
    - *(The rule of decomposing the numbers and looking for zero pairs works for all numbers, but sometimes it can be hard to know how to decompose the number or to know what is left over. If I had to add -8 + 42, I would have to subtract to figure out how to break up 42 into 8 and something else.)*

LESSON GUIDE 4

EU: Rational numbers can be decomposed and regrouped to efficiently add and subtract positive and negative integers.

- Several of you noted that the chip method or number line become difficult when working with very large numbers. Can you explain why?
- Since it was not practical or efficient to use the number line or chip model to solve these problems, what strategies did you use to calculate the sum?
- Say more about zero-sum. Where does this appear in your method? Did others use this idea when thinking through the number line? *(I did when I added 8 + (-18). I drew a line to 8 from 0 and then I moved left 18 with another line and I thought that where there were two lines between 0 and 8, that was like when a red and a yellow are together and make zero.)*
- I hear you saying that for the sum *a* + *b*, for example, we can draw a line from 0 to *a* and then a second line from *a* to the right with the magnitude of *b*. Where the lines overlap, the sum is zero. That is the zero pair(s). **(Revoicing)**
- Who understood this and can say it in their own words? Can you point to the number line when explaining it?
- Can we generalize this into an algorithm that we can use in the future? How do we know it will always work?
- How can we use this algorithm to answer question 2 where we are looking at *a* and *b* on the number line and considering different sums?
- In question d, you are adding *a* and the opposite of *a*. What is always true about adding a number and its opposite? *(They add to zero.)*
- In question e, even though we don't know what *a* and *b* are, can we determine the sign of the answer when we add them together? What have we learned about adding integers that will help us? *(If the numbers have different signs, the sum is the sign of the number that is farther away from zero.)*
- In question f, does it matter what order we add *a* and *b*? Can you show examples with chips or a number line to show that this is true?
- We saw that all of the methods of summing positive and negative can be understood in terms of decomposition and zero-sums. The first group showed us by using chips that the decomposition and zero-sums are visible. On the number line (student name) showed us the overlapping regions or movements representing zero-sums. We saw some examples of numeric decomposition and zero-sums as well when (students' names) rewrote -8 + 28 as -8 + 20 + 8. **(Recapping)**

Application	If *a* is positive and *b* is negative, what will have to be true about *a* and *b* for the sum to be negative?
Summary	Describe the different methods for adding integers. Why does our algorithm work?
Quick Write	No quick write for students.

Support for students who are English Learners (EL):

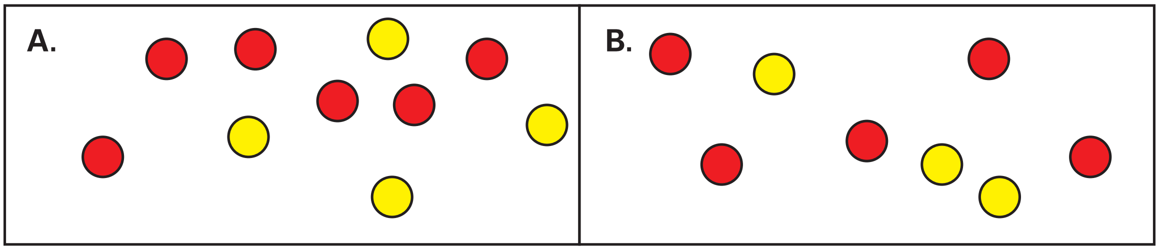
1. Co-create a running list of strategies for adding positive and negative numbers. Make the process concrete for English Learners by referencing the number line and chips during classroom discussions.

Name____

TASK 5

Choose Your Chips Wisely

1. Red and yellow chips represent the sum of positive and negative numbers for two different equations shown below.



A. Write the equation and the sum represented in each chip diagram.

B. Explain why the expressions have the same sum.

C. Write another equation that has the same sum. Justify your reasoning using chips.

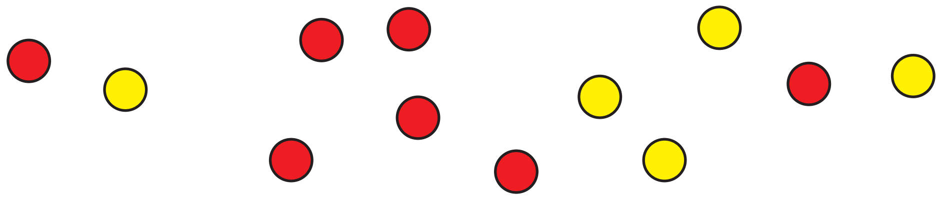
2. Represent each number in two different ways using chips. Each representation must include positive and negative chips.

Number	Chip Representation 1	Chip Representation 2
-3		
2		

TASK
5

3. Red and yellow chips represent the sum of positive and negative numbers.

   A. Explain why the diagram represents -2.



   B. Use the chip diagram above to answer the following subtraction problems.

      I. -2 - 3

      II. -2 - 2

      III. -2 - 1

      IV. -2 - 0

      V. -2 - (-1)

      IV. -2 - (-2)

      Describe any patterns that you notice.

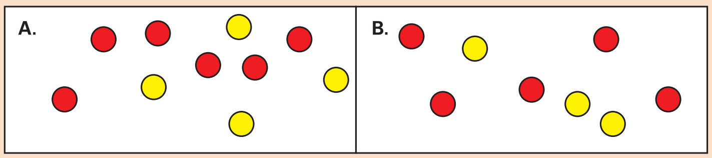
Choose Your Chips Wisely

LESSON GUIDE 5

Rationale for Lesson: Now we turn our attention to subtraction of positive and negative numbers. Students will build on their understanding of adding positive and negative numbers.

Task: Choose Your Chips Wisely

1. Red and yellow chips represent the sum of positive and negative numbers for two different equations shown below.



   A. Write the equation and the sum represented in each chip diagram.

See student paper for complete task.

Tennessee State Standards	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1a	Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
	7.NS.A.1b	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.

LESSON GUIDE 5

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP8 Look for and express regularity in repeated reasoning.
Essential Understandings	• The difference of two numbers $p - q$ is equal to $p + (-q)$ because both can be modeled by the same movement on the number line. That is, if q is positive, $p - q$ and $p + (-q)$ are both located at the point q units to the left of p and if q is negative, both are located at a point q units to the right of p.
Materials Needed	• Task sheet. • Additional paper. • Number lines, chips, or other counter. • Calculator (optional).

LESSON GUIDE 5

SET-UP PHASE

Can somebody please read the task aloud? Without giving away the answer, what do the red and yellow dots represent? Which ones are positive? Which ones are negative? Take a few minutes to work on the task individually before we start working with partners. Extra number lines and chips are available at your tables.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	What numerical value does a red chip represent? A yellow chip?	How can you combine two chips to create a value of zero? What is "left over" and what does this mean mathematically?
Students are having difficulty representing a sum in more than one way.	Can you think of two different pairs of numbers that will add to 7? What about a negative number such as -3?	Can you write the sum so that one value is positive and one is negative? How can you use the chips to represent each of these addends?
Students are having difficulty making a generalization.	How did you use your chips to determine the sums/differences?	In question 3, what patterns do you notice in your answers? Why is that?
Students have difficulty connecting the manipulatives to the equation.	How can you tell that the red and yellow dots shown represent -2?	What happens when we subtract 3? 2? 1? What pattern do you notice? Why is this occurring?

LESSON GUIDE 5

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The difference of two numbers $p - q$ is equal to $p + (-q)$ because both can be modeled by the same movement on the number line. That is, if q is positive, $p - q$ and $p + (-q)$ are both located at the point q units to the left of p and if q is negative, both are located at a point q units to the right of p.

- How did you use chips to help you think about this task?
- Who heard what this group said about looking for zeros and can explain what they mean?
- How does this strategy help us to determine the sum for all of the chips?
- Why do each of the two diagrams in question 1 represent the same sum?
- How does this help you determine the differences? In other words, why may you want to represent the same number in different ways in order to represent different addition and subtraction problems?
- Who can summarize this idea?
- I noticed a couple groups used number lines. How does the chip method relate to a number line? *(The chip method showed 4 + (-6). If we do that on the number line, we start at 4. The space from 0 to 4 is like 4 yellow chips. To add -6, we move 6 spaces left. So, the first 4 spaces we move left are like 4 red chips that are balancing out the 4 yellow chips. Then the next two spaces left are like the two extra red chips.)*
- I noticed that a group modeled the subtraction problem -2 – 1 by taking two red chips and then putting in another red chip. Help us understand how you thought about the problem by using the chips.
- In the subtraction problem -2 – (-1), would someone please explain to us how can you use a yellow chip to help find the answer? *(I noticed that if I have 2 red chips and I take one away, that is the same as if I have two red chips and I add a yellow, because the yellow will cancel out the red. So, -2 – (-1) is the same as -2 + 1.)*
- Many of us are saying that subtracting a negative is the same as adding a positive. **(Revoicing and Marking)** Why is this? Show me using our models.
- Can anyone summarize how you might perform subtraction without the manipulatives?
- Why does this work? (*I can use the number line. Like, -2 – 2 is starting at -2 and going left 2. But this is the same as -2 + -2.*)
- Someone noticed that when we use a number line to model subtraction, the movement we use to subtract a positive value is represented by the same movement that we use for adding a negative. This helps us understand why subtraction is the same as adding the inverse. **(Revoicing and Recapping)** Let's hear a few people's thoughts about this. What do you understand now about subtraction and what are you still wondering about?

Application	Use both red and yellow chips to make a drawing that represents the value -5. Then determine the differences -5 – 2 and -5 – (-2).
Summary	Would someone please use his or her own words to summarize for us a strategy for determining the difference of integers?
Quick Write	Why is subtraction the same as adding a negative?

Support for students who are English Learners (EL):

1. Have a section on the wall that shows different ways to model subtraction and addition of integers. Add the chip idea to the wall as another modeling strategy. Remind the class what the red and yellow colors represent.
2. Clarify vocabulary, particularly the difference between an expression and an equation.

TASK 6

Name____

Traveling on the Number Line

1. Model the following subtraction problems with chips and a number line to determine the difference. Explain your reasoning.

a. 2 – 6	b. 2 – (-6)
c. -2 – 6	d. -2 – (-6)

2. Calculate the following differences:

   A. 2 – 6.3

   B. 2 – (-6.3)

   Describe your strategy for calculating differences in problems involving decimals.

3. Louise claims that subtracting a negative number is equivalent to adding that number. For example, 2.3 – (-4) = 2.3 + 4. Do you agree with Louise? Justify your response mathematically using chips or a number line.

4. Decide whether the following expressions are equal. Support your answer by representing subtraction on the number line.

   A. -4 – 12 = 12 – (-4)

   B. 3 – 10 = 10 – 3

   C. 5 – (-2) = -2 – 5

   What patterns do you notice?

   Is it possible for $a - b$ to equal $b - a$? Explain your reasoning.

Traveling on the Number Line

LESSON GUIDE 6

Rationale for Lesson: Extend previous understanding of subtraction to the number line, but now students are expected to work with decimal values and explore the commutative property of subtraction.

Task: Traveling on the Number Line

1. Model the following subtraction problems with chips and a number line to determine the difference. Explain your reasoning.

a. 2 – 6	b. 2 – (-6)
c. -2 – 6	d. -2 – (-6)

See student paper for complete task.

Tennessee State Standards	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. . MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.

LESSON GUIDE 6

Essential Understandings	• The difference of two numbers p and q is located q units from p on the number line because p and q represent linear distances with direction from zero. When q is positive, $p - q$ is to the left of p. When q is negative, $p - q$ is to the right of p. • The differences $b - a$ and $a - b$ are opposites because they represent the same distance between two points on the number line. Subtraction of a lesser number minus a greater number will result in a negative difference, while subtraction of a greater number minus a lesser number will result in a positive difference. • The order of the values being added does not affect the sum, but the order of the values being subtracted does affect the difference because $a + b$ models the same movement on the number line as $b + a$, while $a - b$ and $b - a$ model movement in opposite directions on the number line.
Materials Needed	• Task sheet. • Additional paper. • Number lines, chips, or other counter. • Calculator (optional).

SET-UP PHASE

Can somebody please read the task aloud? Try to complete the problems on your own for the next five minutes and then I will give you an opportunity to discuss the problems with your partners. You'll find chips and extra number lines at your tables to use as needed.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Students model with chips.	Can you tell me how you used chips to represent these problems? What is the difference between your starting pile of chips and your ending pile of chips?	How can you represent this problem on a number line? How will decimals change your strategy?
Students are working with the number line.	Can you tell me how you represented these problems with a number line? How will you use a number line to show subtraction?	How will you subtract a negative number? How can you show that on the number line? How will decimals change your strategy?
Students have difficulty with decimals. For example, can subtract 2 – 6, but struggles with 2 – 6.3	How would you represent and solve 2 – 6 (using chips, number lines, etc.)?	Can you use distance on a number line to help you answer the question? In what direction are you moving? After the whole number, how much more?
Finishes early.	How did you decide whether $b - a$ is equal to $a - b$?	Can you explain or prove why $a - b$ is not equal to $b - a$ (except for when $a = b$)?

LESSON GUIDE 6

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The difference of two numbers *p* and *q* is located *q* units from *p* on the number line because *p* and *q* represent linear distances with direction from zero. When *q* is positive, *p* – *q* is to the left of *p*. When *q* is negative, *p* – *q* is to the right of *p*.

- This group used chips to model the first set of problems. Tell us about how you used the chips to think about the subtraction.
- Who understood their model and can use it to subtract 7 – 4 and 7 – (-4)?
- This other group used the number line. Listen as they explain their process and think about how it relates to the chip method.
- How does this relate to the first group's method? How are these representations the same/different? *(Using chips, if the signs of the numbers being subtracted are the same, I just took away the second number of chips. For 7 – 4, there are 7 yellow chips and I just take away 4 of them. The number line model is kind of the same. I just start at 7 and go back 4. When the signs are different, I had to add in some other chips. So, for 7 – (-4), I had to add in 4 red chips and 4 yellow chips so that I could take away the red ones. The number line model is kind of like that, too. I had to go up to subtract -4.)*
- We just found the answers to the problems 2 – 6 and 2 – (-6). Would someone please talk to us about the relationship between each of the answers and their direction from 2?
- Someone said that 2 – (-6) is the same as 2 + 6. **(Marking)** Who agrees and can explain why this is true? *(We saw this, the chips and the number line, how you have to go up using both models. Like, with the chips, there is no -6 to take away, so in order to take it away, you have to put them in there, but we always add zero pairs, so we put in 6 reds and 6 yellows. Then we can take away the reds, but the yellows stay, so the total went up even though we subtracted.)*
- When you look at the differences on the number line, how can you determine the distance from 2 to -6?
- Both values are positive when you add 2 and 6. This distance is referred to as **absolute value**. Can we say that the distance from -6 to 2 is also 8? What if we subtract -6 – 2? How is this different? **(Challenging)** *(The distance from -6 to 2 is 8. I counted on the number line. -6 – 2 is -8, so the difference is negative instead of positive, but the absolute value is still the same.)*
- Interesting. So, the 2 – (-6) is the opposite of -6 – 2? We will think more about this and consider whether this is unique to these numbers or if there is a generalization we can make about this. **(Marking)**
- Who can represent why this is true on the number line as well as with the manipulatives?
- Do the chip and number line models both work for the difference of non-integers? In number 2, we considered 2 – 6.3 and 2 – (-6.3). Were we still able to use both of these models? *(No, the chips only work for integers.)*

EU: The order of the values being added does not affect the sum, but the order of the values being subtracted does affect the difference because *a* + *b* models the same movement on the number line as *b* + *a*, while *a* – *b* and *b* – *a* model movement in opposite directions on the number line.

- We talked before about why addition is commutative. What patterns did you notice for subtraction?
- I heard a few people mention specific examples. Who can demonstrate the movement along the number line with these examples?
- Is the distance between 2 and 5 and 5 and 2 the same? So then why is 2 – 5 not the same as 5 – 2? **(Challenging)**
- Who can summarize why addition is commutative but subtraction is not?

EU: The differences *b* – *a* and *a* – *b* are opposites because they represent the same distance between two points on the number line. Subtraction of a lesser number minus a greater number will result in a negative difference, while subtraction of a greater number minus a lesser number will result in a positive difference.

- Let's talk about number 4. Here we were considering whether or not it matters what order we subtract in.
- How can we use the number line to show the answers to -2 – 5 and 5 – (-2)? What do you notice about the differences? Why is that? *(One is negative and one is positive, but they have the same absolute value.)*
- So, we see that subtraction is not commutative, but more interestingly, that changing the order of the numbers being subtracted results in the opposite difference. **(Revoicing)** Why do you think this happens? **(Challenging)**
- *(It is the same number of spaces between the numbers, so that is why the absolute value is the same.)*
- *(You have to go in opposite directions. -2 – 5 is moving to the left. 5 – (-2) is moving to the right.)*
- *(5 is greater than -2, so taking away a larger number from a smaller one is going to be negative, but taking away a smaller number from a larger is positive.)*
- We noticed that the difference of two numbers is different depending on the order of subtraction and then we noticed that they are actually opposites. Together we figured out that the absolute value of the distance is always the same because that is the distance between the two numbers. The sign is different when the values being subtracted are unequal. A lesser number minus a greater number will be a negative difference and a greater number minus a lesser number will be a positive difference. **(Recapping)**

Application	Determine whether 6 – (-2) is equal to 6 + 2. Explain your reasoning.
Summary	Subtracting a negative number is the same as adding a positive number. Why is that? Explain using as many representations as possible.
Quick Write	Why is addition commutative while subtraction is not?

Support for students who are English Learners (EL):

1. When writing equations, reference the physical models and number line to make this representation more concrete for students who are English Learners.
2. Slow down class discussions and mark key ideas both verbally and in writing.

TASK
7

Name____

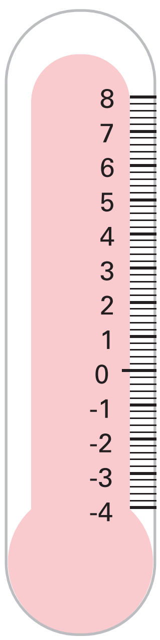
Cold Weather

1. Keisha loves to keep track of the temperature outside.

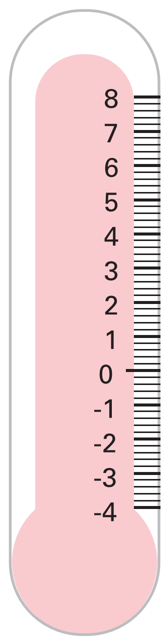
   She looks at the thermometer when she wakes up, at noon, and before she goes to bed and records the temperatures in the table below.

Time of Day	Temperature (in degrees Fahrenheit)
Morning	-2.4
Noon	7.3
Evening	-2.4

   A. Use the thermometers below to show change in temperature from morning to noon and from noon to evening.



Morning to Noon



Noon to Evening

   B. Write equations that can be used to model the change in the temperature from morning to noon and from noon to evening.

TASK 7

2. Evaluate the differences and model your solutions using a number line.

   -6 – 14 =

   14 – (-6) =

3. Why does changing the order of the numbers being subtracted change the sign of the difference?

LESSON GUIDE 7

Cold Weather

Rationale for Lesson: Students have calculated differences using manipulatives and a number line. In this task, students continue analyzing operations and integer properties but move to a vertical number line.

Task: Cold Weather

1. Keisha loves to keep track of the temperature outside.
   She looks at the thermometer when she wakes up, at noon, and before she goes to bed and records the temperatures in the table below.

Time of Day	Temperature (in degrees Fahrenheit)
Morning	-2.4
Noon	7.3
Evening	-2.4

   a. Use the thermometers below to show change in temperature from morning to noon and from noon to evening.

See student paper for complete task.

Tennessee State Standards	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure

Essential Understandings	• The difference of two numbers $p - q$ is equal to $p + (-q)$ because both can be modeled by the same movement on the number line. That is, if q is positive, $p - q$ and $p + (-q)$ are both located at the point q units to the left of p and if q is negative, both are located at a point q units to the right of p. • The distance between a positive number p and negative number q is the sum of their absolute values because this represents each of their distances from zero and therefore their total distance from each other.
Materials Needed	• Task sheet. • Additional paper. • Number lines, chips, or other counter. • Calculator (optional).

LESSON GUIDE 7

SET-UP PHASE

Can somebody please read the problem aloud? What do the values in the table represent? What does it mean for a temperature to be negative?

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	What does the information in the problem represent? What numbers are larger? How is the thermometer labeled?	How can you use what you know to place the number on the vertical number line? How will you calculate the difference? (How is this the same/different from horizontal number lines that we've worked with before?)
Models change using the thermometer/vertical number line.	How did you determine the location on the number line? How is this the same/different from the horizontal number line?	How can you use this model to determine distance? What operation can you use to represent the difference in temperatures with an equation?
Uses an equation.	Can you describe your equation? How does each value and the operation relate to the temperature context?	Can you represent this change on a number line? What do you notice about the difference of a negative number?
Finishes early.	Can you explain your strategy to me?	Why is subtraction of a negative the same as adding? How does this appear in the different representations?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The distance between a positive number *p* and negative number *q* is the sum of their absolute values because this represents each of their distances from zero and therefore their total distance from each other.

- Let's look at how groups represented this problem on the vertical number line and how others represented it with an equation; what are the similarities and differences?
- We can see that some groups represented the change in the temperatures from morning to noon with the equation -2.4 + 9.7 = 7.3, and others represented it as 7.3 – (-2.4) = 9.7. How are these equations related? *(They both show that there is a change of 9.7 degrees between the morning and noon.)*
- Who can summarize what was just said, noting the changes on the vertical number line? *(-2.4 + 9.7 = 7.3 shows that in the morning the temperature was here at -2.4, then it went up 9.7 degrees to end here at 7.3. 7.3 – (-2.4) shows that the difference between morning and noon temperatures here and here is 9.7 and we can count between them to see that there are 9.7 units.) (If we just ignore the negatives, 2.4 + 7.3 = 9.7. And 2.4 is how far -2.4 is from zero and the 7.3 is how far 7.3 is from zero.)*
- I hear you saying that for these two numbers, since one is negative and one is positive, the distance between them is the sum of their distances from zero. **(Revoicing)** Who remembers what the term is for distance from zero?

EU: The difference of two numbers *p* – *q* is equal to *p* + (-*q*) because both can be modeled by the same movement on the number line. That is, if *q* is positive, *p* – *q* and *p* + (-*q*) are both located at the point *q* units to the left of *p* and if *q* is negative, both are located at a point *q* units to the right of *p*.

- When we move to the equation, how can you tell where the group "started" and then where the difference occurred? Explain.
- So -6 – 14 is the same as -6 + -14. Explain why, using the number line.
- What about -6 – 14 and -14 – (-6)? Are they the same? Why or why not?
- So what you're saying is that when subtracting, the order matters. **(Marking)** How is this the same/different from what we studied in previous tasks?
- Why does order matter? Let's look at this group's work that used a number line and then a group that used equations to consider why subtraction and adding the opposite yield the same value.
- So why is the magnitude of the number the same? Why is the sign different?

Application	It is -12 degrees in Quebec and 78 degrees in Phoenix. What is the difference in their temperatures? Represent your answer with a number line and an equation.
Summary	How can we calculate differences on a vertical number line? How is this the same/different than our work with a horizontal number line?
Quick Write	Why is the difference between a positive and negative number the sum of their absolute values?

Support for students who are English Learners (EL):

1. When writing equations, reference the physical model to make this representation more concrete for students who are English Learners.

TASK
8

Name____

Keeping it Real

1. Determine the difference for each expression below.

   A. -8 – 8 B. -8 – 18

   C. -8 – 28 D. -8 – 128

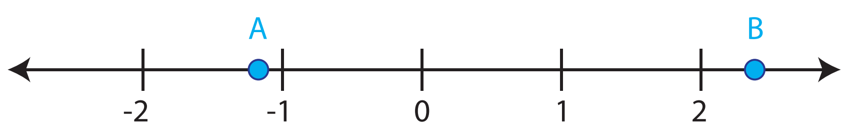
   Describe how to subtract x from -8 for any value x.

2. Determine the difference for each expression below.

   A. -3 – (-4.2) B. -8 – (-18.5)

   C. -8 – (-28) D. -8 – (-128.6)

3. Determine whether each of the expressions has a value that is positive, negative, or equal to 0. Explain your reasoning.



   A. a – 2

   B. a – b

   C. b – a

   D. a – a

   E. a – (-2)

Keeping it Real

LESSON GUIDE 8

Rationale for Lesson: Solidify understanding of how to subtract positive and negative rational numbers by using the strategy, "Look for and express regularity in repeated reasoning." Students do this by applying the rules they have generated based on specific cases to a general case in which variables stand in for unknown numbers.

Task: Keeping it Real

1. Determine the difference for each expression below.

A. -8 – 8   B. -8 – 18

C. -8 – 28   D. -8 – 128

Describe how to subtract x from -8 for any value x.

See student paper for complete task.

Tennessee State Standards	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. . MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	

LESSON GUIDE 8

Essential Understandings	• The difference of two numbers *p* and *q* is located *q* units from *p* on the number line because *p* and *q* represent linear distances with direction from zero. When *q* is positive, $p - q$ is to the left of *p*. When *q* is negative, $p - q$ is to the right of *p*. • The differences $b - a$ and $a - b$ are opposites because they represent the same distance between two points on the number line. Subtraction of a lesser number minus a greater number will result in a negative difference, while subtraction of a greater number minus a lesser number will result in a positive difference.
Materials Needed	• Task sheet. • Additional paper. • Number lines, chips, or other counter. • Calculator (optional).

LESSON GUIDE 8

SET-UP PHASE

Somebody please read the task aloud. I would like you to attempt the problems individually. You will have time to discuss with partners before we share out, though.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Uses a manipulative to calculate distance.	What strategy are you using to determine the differences?	How can you calculate differences for decimals or numbers that are larger (such as -8 – 128)? Do you see any patterns that can help you develop another strategy?
Uses a number line.	Can you explain how you're using a number line to model the movement?	How can you calculate differences for decimals or numbers that are larger (such as -8 – 128)? Can you come up with an algebraic strategy?
Substitutes values on the number line for #3.	Tell me your thinking. How did you decide what values to choose? How did this help your thinking?	How are you deciding in which direction you are moving as you find your differences? Does it matter what number you choose? What values are significant?
Group finishes early.	How would you use the number line to model subtracting x from -8 for any value of x?	How can you generalize your approach for calculating the difference between two real numbers?

LESSON GUIDE 8

SHARE, DISCUSS, AND ANALYZE PHASE

EU: The difference of two numbers *p* and *q* is located *q* units from *p* on the number line because *p* and *q* represent linear distances with direction from zero. When *q* is positive, *p* – *q* is to the left of *p*. When *q* is negative, *p* – *q* is to the right of *p*.

- I noticed many different strategies for subtracting. Let's have a group go over the number line and manipulative methods before moving on to generalizing an algebraic method.
- What patterns do you notice? Who can summarize how these methods show subtraction?
- In order to calculate the difference for the expression -8 – 128, which direction would you move on the number line? Why?
- How can you represent this movement algebraically?
- Can someone state a rule for calculating the difference between two real numbers?
- Who can add on to that idea?
- Does the rule we just came up with change if one of the numbers is positive and the other is negative? Why or why not?
- Any two numbers can be added together in any order **(Marking)**, regardless of whether the numbers are positive or negative. Why is this the case?
- Then why is subtraction not commutative? **(Challenging)**

EU: The differences *b* – *a* and *a* – *b* are opposites because they represent the same distance between two points on the number line. Subtraction of a lesser number minus a greater number will result in a negative difference, while subtraction of a greater number minus a lesser number will result in a positive difference.

- When we subtract two pairs of numbers in different orders, for example 5 – 2 and 2 – 5, what do you notice about the differences? Explain. *(They are opposites, like 3 and -3.)*
- Who can add on to what this group said? *(The distance between the two numbers doesn't change, but the sign changes because a bigger number minus a smaller number is a positive difference, but a smaller number minus a bigger number is a negative difference.)*
- Somebody please summarize, using the number line, the difference between subtracting *a* – *b* and *b* – *a*. *(These differences will always be opposites because the distance between the numbers is the same and part of what the difference tells us is how far apart the numbers are.) (A bigger number minus a smaller number is positive, but a smaller number minus a bigger number is negative and if the numbers are not the same, then one of them has to be bigger than the other.)*
- Over the course of several lessons, we have looked at different methods for calculating sums and differences. We explored and compared methods using chips, a number line, and we developed some general rules and algorithms. In this task, we made sure that we understood the connection between all of these methods and can apply them. When considering differences of *a*, *b*, and known numbers, some groups used strategies involving the number line that included estimating distances and moving that distance based on the operation in the expression. Other groups used the general rules that we have developed that tell us that the sign and magnitude of the values being subtracted give us information about the difference. **(Recapping)** Let's capture some of those rules on chart paper so that we can keep them as a classroom resource moving forward.

Application	Calculate the difference for the expression 12 – (-8.6).
Summary	The expressions ($b – a$) and ($a – b$) represent real numbers. What do you know about each of the numbers? How does the number line help you make sense of these expressions?
Quick Write	Write two sentences to explain how to find the difference between a positive and a negative number.

Support for students who are English Learners (EL):

1. Have a section on the wall that shows different ways to model mathematics. Add the number line to the wall as another modeling strategy. Remind the class what directions represent positive and negative numbers.
2. Clarify vocabulary, particularly when talking about numbers when it relates to the size of negative numbers; for example, represent in multiple ways using the number line and other visual models why the number -2 is larger than the number -15.

Progressions

Appendix O
6-8, The Number System

Progressions for the Tennesee State Standards in Mathematics (draft)

4 July 2013

The Number System, 6–8

Overview

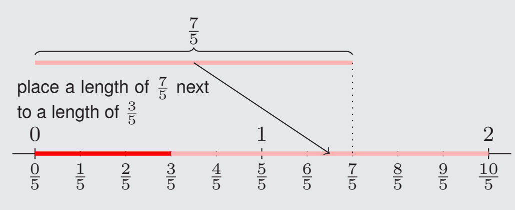
In Grades 6–8, students build on two important conceptions which have developed throughout K–5, in order to understand the rational numbers as a number system. The first is the representation of whole numbers and fractions as points on the number line, and the second is a firm understanding of the properties of operations on whole numbers and fractions.

Representing numbers on the number line In early grades, students see whole numbers as counting numbers, Later, they also understand whole numbers as corresponding to points on the number line. Just as the 6 on a ruler measures 6 inches from the 0 mark, so the number 6 on the number line measures 6 units from the origin. Interpreting numbers as points on the number line brings fractions into the family as well; fractions are seen as measurements with new units, creating by partitioning the whole number unit into equal pieces. Just as on a ruler we might measure in tenths of an inch, on the number line we have halves, thirds, fifths, sevenths; the number line is a sort of ruler with every denominator. The denominators 10, 100, etc. play a special role, partioning the number line into tenths, hundredths, etc., just as a metric ruler is partioned into centimeters and millimeters.

Starting in Grade 2 students see addition as concatenation of lengths on the number line.[2.MD.6] By Grade 4 they are using the same model to represent the sum of fractions with the same denominator: $\frac{3}{5} + \frac{7}{5}$ is seen as putting together a length that is 3 units of one fifth long with a length that is 7 units of one fifth long, making 10 units of one fifths in all. Since there are five fifths in 1 (that's what it means to be a fifth), and 10 is 2 fives, we get $\frac{3}{5} + \frac{7}{5} = 2$. Two fractions with different denominators are added by representing them in terms of a common unit.

[2.MD.6] Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . , and represent whole-number sums and differences within 100 on a number line diagram.

Representing $\frac{3}{5}$ and $\frac{7}{5}$ on the number line



Representing sums as concatenated lengths on the number line is important because it gives students a way to think about addition that makes sense independently of how numbers are represented symbolically. Although addition calculations may look different for numbers represented in base ten and as fractions, addition is the same operation in each case. Furthermore, the concatenation model of addition extends naturally to negative numbers in Grade 7.

Properties of operations The number line provides a representation that can be used to building understanding of sums and differences of rational numbers. However, building understanding of multiplication and division of rational numbers relies on a firm understanding of properties of operations. Although students have not necessarily been taught formal names for these properties, they have used them repeatedly in elementary school and have been with reasoning with them. The commutative and associative properties of addition and mutiplication have, in particular, been their constant friends in working with strategies for addition and multiplication.[1.OA.3, 3.OA.5]

The existence of the multiplicative identity (1) and multiplicative inverses start to play important roles as students learn about fractions. They might see fraction equivalence as confirming the identity rule for fractions. In Grade 4 they learn about fraction equivalence

$$\frac{n \times a}{n \times b} = \frac{a}{b}$$

and in Grade 5 they relate this to multiplication by 1

$$\frac{n \times a}{n \times b} = \frac{n}{n} \times \frac{a}{b} = 1 \times \frac{a}{b},$$

thus confirming that the identity rule

$$1 \times \frac{a}{b} = \frac{a}{b}$$

works for fractions.[5.NF.5]

As another example, the commutative property for multiplication plays an important role in understanding multiplication with fractions. For example, although

$$5 \times \frac{1}{2} = \frac{5}{2}$$

can be made sense of using previous understandings of whole number multiplication as repeated addition, the other way around,

$$\frac{1}{2} \times 5 = \frac{5}{2},$$

seems to come from a different source, from the meaning of phrases such as "half of" and a mysterious acceptance that "of" must mean multiplication. A more reasoned approach would be to observe that if we want the commutative property to continue to hold, then we must have

$$\frac{1}{2} \times 5 = 5 \times \frac{1}{2} = \frac{5}{2},$$

Properties of Operations on Rational Numbers

Properties of Addition

1. Commutative Property. For any two rational numbers a and b, $a + b = b + a$.
2. Associative Property. For any three rational numbers a, b and c, $(a + b) + c = a + (b + c)$.
3. Existence of Identity. The number 0 satisfies $0 + a = a = a + 0$.
4. Existence of Additive Inverse. For any rational number a, there is a number $-a$ such that $a + (-a) = 0$.

Properties of Multiplication

1. Commutative Property. For any two rational numbers a and b, $a \times b = b \times a$.
2. Associative Property. For any three rational numbers a, b and c, $(a \times b) \times c = a \times (b \times c)$.
3. Existence of Identity. The number 1 satisfies $1 \times a = a = a \times 1$.
4. Existence of Multiplicative Inverse. For every non-zero rational number a, there is a rational number $\frac{1}{a}$ such that $a \times \frac{1}{a} = 1$.

The Distributive Property

For rational numbers a, b and c, one has $a \times (b + c) = a \times b + a \times c$.

1.OA.3 Apply properties of operations as strategies to add and subtract.[1]

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2]

5.NF.5 Interpret multiplication as scaling (resizing), by:

a ...

b ...and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

and that $\frac{5}{2}$ is indeed "half of five," as we have understood in Grade 5.[5.NF.3]

When students extend their conception of multiplication to include negative rational numbers, the properties of operations become crucial. The rule that the product of negative numbers is positive, often seen as mysterious, is the result of extending the properties of operations (particularly the distributive property) to rational numbers.

[5.NF.3] Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Grade 6

As Grade 6 dawns, students have a firm understanding of place value and the properties of operations. On this foundation they are ready to start using the properties of operations as tools of exploration, deploying them confidently to build new understandings of operations with fractions and negative numbers. They are also ready to complete their growing fluency with algorithms for the four operations.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions In Grade 6 students conclude the work with operations on fractions, started in Grade 4, by computing quotients of fractions.[6.NS.1] In Grade 5 students divided unit fractions by whole numbers and whole numbers by unit fractions, two special cases of fraction division that are relatively easy to conceptualize and visualize.[5.NF.7ab] Dividing a whole number by a unit fraction can be conceptualized in terms of the measurement interpretation of division, which conceptualizes $a \div b$ as the the measure of of a by units of length b on the number line, that is, the solution to the multiplication equation $a = ? \times b$. Dividing a unit fraction by a whole number can be interpreted in terms of the sharing interpretation of division, which conceptualizes $a \div b$ as the size of a share when a is divided into b equal shares, that is, the solution to the multiplication equation $a = b \times ?$.

Now in Grade 6 students develop a general understanding of fraction division. They can use story contexts and visual models to develop this understanding, but also begin to move towards using the relation between division and multiplication.

For example, they might use the measurement interpretation of division to see that $\frac{8}{3} \div \frac{2}{3} = 4$, because 4 is 4 is how many $\frac{2}{3}$ there are in $\frac{8}{3}$. At the same time they can see that this latter statement also says that $4 \times \frac{2}{3} = \frac{8}{3}$. This multiplication equation can be used to obtain the division equation directly, using the relation between multiplication and division.

Quotients of fractions that are whole number answers are particularly suited to the measurement interpretation of division. When this interpretation is used for quotients of fractions that are not whole numbers, it can be rephrased from "how many times does this go into that?" to "how much of this is in that?" For example,

$$\frac{2}{3} \div \frac{3}{4}$$

can be interpreted as how many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt, or as how much of a $\frac{3}{4}$-cup serving is in $\frac{2}{3}$ of a cup of yogurt. Although linguistically different the two questions are mathematically the same. Both can be visualized as in the margin and expressed using a multiplication equation with an unknown for the

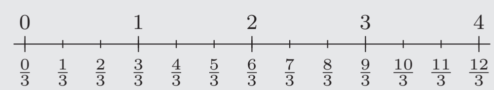
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

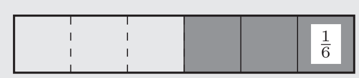
a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

b Interpret division of a whole number by a unit fraction, and compute such quotients.

Visual models for division of whole numbers by unit fractions and unit fractions by whole numbers

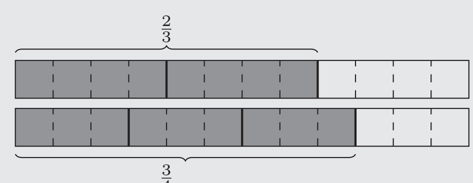


Reasoning on a number line using the measurement interpretation of division: there are 3 parts of length $\frac{1}{3}$ in the unit interval, therefore there are 4×3 parts of length $\frac{1}{3}$ in the interval from 0 to 4, so the number of times $\frac{1}{3}$ goes into 4 is 12, that is $4 \div \frac{1}{3} = 4 \times 3 = 12$.



Reasoning with a fraction strip using the sharing interpretation of division: the strip is the whole and the shaded area is $\frac{1}{2}$ of the whole. If the shaded area is divided into 3 equal parts, then 2×3 of those parts make up the whole, so $\frac{1}{2} \div 3 = \frac{1}{2\times 3} = \frac{1}{6}$.

Visual model for $\frac{2}{3} \div \frac{3}{4}$ and $\frac{2}{3} = ? \times \frac{3}{4}$



We find a common unit for comparing $\frac{2}{3}$ and $\frac{3}{4}$ by dividing each $\frac{1}{3}$ into 4 parts and each $\frac{1}{4}$ into 3 parts. Then $\frac{2}{3}$ is 8 parts when $\frac{3}{4}$ is divided into 9 equal parts, so $\frac{2}{3} = \frac{8}{9} \times \frac{3}{4}$, which is the same as saying that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

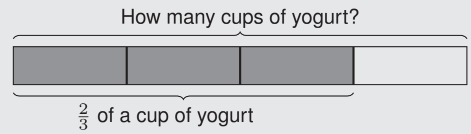
first factor:

$$\frac{2}{3} = ? \times \frac{3}{4}.$$

The same division problem can be interpreted using the sharing interpretation of division: how many cups are in a full container of yogurt when $\frac{2}{3}$ of a cup fills $\frac{3}{4}$ of the container. In other words, $\frac{3}{4}$ of what amount is equal to $\frac{2}{3}$ cups? In this case, $\frac{2}{3} \div \frac{3}{4}$ is seen as the solution to a multiplication equation with an unknown as the second factor:

$$\frac{3}{4} \times ? = \frac{2}{3}.$$

Visual model for $\frac{2}{3} \div \frac{3}{4}$ and $\frac{3}{4} \times ? = \frac{2}{3}$



The shaded area is $\frac{3}{4}$ of the entire strip. So it is 3 parts of a division of the strip into 4 equal parts. Another way of seeing this is that the strip is 4 parts of a division of the shaded area into 3 equal parts. That is, the strip is $\frac{4}{3}$ times the shaded part. So $? = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$.

There is a close connection between the reasoning shown in the margin and reasoning about ratios; if two quantities are in the ratio $3:4$, then there is a common unit so that the first quantity is 3 units and the second quantity is 4 units. The corresponding unit rate is $\frac{3}{4}$, and the first quantity is $\frac{3}{4}$ times the second quantity. Viewing the situation the other way around, with the roles of the two quantities interchanged, the same reasoning shows that the second quantity is $\frac{4}{3}$ times the first quantity. Notice that this leads us directly to the invert-and-multiply for fraction division: we have just reasoned that the ? in the equation above must be equal to $\frac{4}{3} \times \frac{2}{3}$, which is exactly what the rules gives us for $\frac{2}{3} \div \frac{4}{3}$.[6.NS.1]

[6.NS.1] Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

The invert-and-multiply rule can also be understood algebraically as a consequence of the general rule for multiplication of fractions. If $\frac{a}{b} \div \frac{c}{d}$ is is defined to be the missing factor in the multiplication equation

$$? \times \frac{c}{d} = \frac{a}{b}$$

then the fraction that does the job is

$$? = \frac{ad}{bc},$$

as we can verify by putting it into the multiplication equation and using the already known rules of fraction multiplication and the properties of operations:

$$\frac{ad}{bc} \times \frac{c}{d} = \frac{(ad)c}{(bc)d} = \frac{a(cd)}{b(cd)} = \frac{a}{b} \times \frac{cd}{cd} = \frac{a}{b}.$$

Compute fluently with multi-digit numbers and find common factors and multiples In Grade 6 students consolidate the work of earlier grades on operations with whole numbers and decimals by becoming fluent in the four operations on these numbers.[6.NS.2, 6.NS.3] Much of the foundation for this fluency has been laid in earlier grades. They have known since Grade 3 that whole numbers are fractions[3.NF.3c] and since Grade 4 that decimal notation is a way of writing fractions with denominator equal to a power of 10;[4.NF.6] by Grade 6 they start to see whole numbers, decimals and fractions

[6.NS.2] Fluently divide multi-digit numbers using the standard algorithm.

[6.NS.3] Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

[3.NF.3c] Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

[4.NF.6] Use decimal notation for fractions with denominators 10 or 100.

not as wholly different types of numbers but as as part of the same number system, represented by the number line.

In many traditional treatments of fractions greatest common factors occur in reducing a fraction to lowest terms, and least common multiples occur in adding fractions. As explained in the Fractions Progression, neither of these activities is treated as a separate topic in the standards. Indeed, insisting that finding a least common multiple is an essential part of adding fractions can get in the way of understanding the operation, and the excursion into prime factorization and factor trees that is often entailed in these topics can be time-consuming and distract from the focus of K–5. In Grade 6, however, students experience a modest introduction to the concepts[6.NS.4] and put the idea of greatest common factor to use in a rehearsal for algebra, where they will need to see, for example, that $3x^2 + 6x = 3x(x + 2)$.

[6.NS.4] Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understandings of numbers to the system of rational numbers In Grade 6 the number line is extended to include negative numbers. Students initially encounter negative numbers in contexts where it is natural to describe both the magnitude of the quantity, e.g. vertical distance from sea level in meters, and the direction of the quantity (above or below sea level).[6.NS.5] In some cases 0 has an essential meaning, for example that you are at sea level; in other cases the choice of 0 is merely a convention, for example the temperature designated as 0° in Farenheit or Celsius. Although negative integers might be commonly used as initial examples of negative numbers, the Standards do not introduce the integers separately from the entire system of rational numbers, and examples of negative fractions or decimals can be included from the beginning.

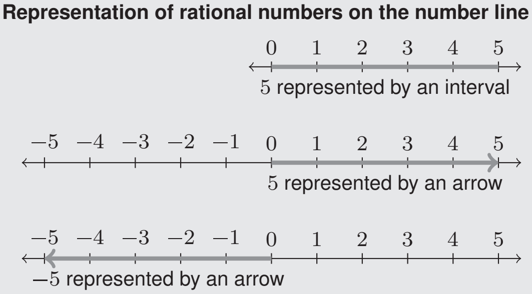
[6.NS.5] Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Directed measurement scales for temperature and elevation provide a basis for understanding positive and negative numbers as having a positive or negative direction on the number line.[6.NS.6a] Previous understanding of numbers on the number line related the position of the number to measurement: the number 5 is located at the endpoint of an line segment 5 units long whose other endpoint is at 0. Now the line segments acquire direction; starting at 0 they can go in either the positive or the negative direction. These directed numbers can be represented by putting arrows at the endpoints of the line segments.

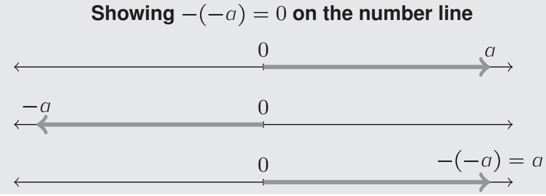
[6.NS.6] Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

Representation of rational numbers on the number line



Showing $-(-a) = 0$ on the number line



Students come to see $-p$ as the opposite of p, located an equal distance from 0 in the opposite direction. In order to avoid the common misconception later in algebra that any symbol with a negative sign in front of it should be a negative number, it is useful for students to see examples where $-p$ is a positive number, for example if $p = -3$ then $-p = -(-3) = 3$. Students come to see the operation of putting a negative sign in front of a number as flipping the direction of the number from positive to negative or negative to positive. Students generalize this understanding of the meaning of the negative sign to the coordinate plane, and can use it in locating numbers on the number line and ordered pairs in the coordinate plane.[6.NS.6bc]

With the introduction of negative numbers, students gain a new sense of ordering on the number line. Whereas statements like $5 < 7$ could be understood in terms of having more of or less of a certain quantity—"I have 5 apples and you have 7, so I have fewer than you"—comparing negative numbers requires closer attention to the relative positions of the numbers on the number line rather than their magnitudes.[6.MS.7a] Comparisons such as $-7 < -5$ can initially be confusing to students, because -7 is further away from 0 than -5, and is therefore larger in magnitude. Referring back to contexts in which negative numbers were introduced can be helpful: 7 meters below sea level is lower than 5 meters below sea level, and $-7°$ F is colder than $-5°$ F. Students are used to thinking of colder temperatures as lower than hotter temperatures, and so the mathematically correct statement also makes sense in terms of the context.[6.NS.7b]

At the same time, the prior notion of distance from 0 as a measure of size is still present in the notion of absolute value. To avoid confusion it can help to present students with contexts where it makes sense both to compare the order of two rational numbers and to compare their absolute value, and where these two comparisons run in different directions. For example, someone with a balance of $\$100$ in their bank account has more money than someone with a balance of $-\$1000$, because $100 > -1000$. But the second person's debt is much larger than the first person's credit $|-1000| > |100|$.[6.NS.7cd]

This understanding is reinforced by extension to the coordinate plane.[6.NS.8]

b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.7 Understand ordering and absolute value of rational numbers.

a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

b Write, interpret, and explain statements of order for rational numbers in real-world contexts.

c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

d Distinguish comparisons of absolute value from statements about order.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Grade 7

Addition and subtraction of rational numbers In Grade 6 students learned to locate rational numbers on the number line; in Grade 7 they extend their understanding of operations with fractions to operations with rational numbers. Whereas previously addition was represented by concatenating the line segments together, now the the line segments have directions, and therefore a beginning and an end. When concatenating these directed line segments, we start the second line segment at the end of the first one. If the second line segment is going in the opposite direction to the first, it can backtrack over the first, effectively cancelling part or all of it out.[7.NS.1b] Later in high school, if students encounter vectors, they will be able to see this as one-dimensional vector addition.

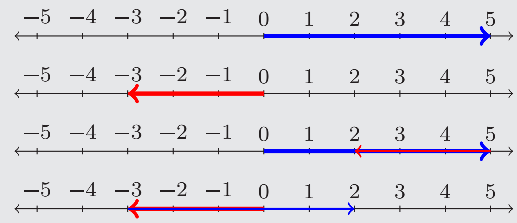
A fundamental fact about addition of rational numbers is that $p + (-p) = 0$ for any rational number p; in fact, this is a new property of operations that comes into play when negative numbers are introduced. This property can be introduced using situations in which the equation makes sense in a context.[7.NS.1a] For example, the operation of adding an integer could be modeled by an elevator moving up or down a certain number of floors. It can also be shown using the directed line segment model of addition on the number, as shown in the margin.[7.NS.1b]

It is common to use colored chips to represent integers, with one color representing positive integers and another representing negative integers, subject to the rule that chips of different colors cancel each other out; thus, a number is not changed if you take away or add such a pair. This is rather a different representation than the number line. On the number line, the equation $p+(-p) = 0$ follows from the definition of addition using directed line segments. With integer chips, the equation $p + (-p) = 0$ is true by definition since it it is encoded in the rules for manipulating the chips. Also implicit in the use of chips is that the commutative and associative properties extend to addition of integers, since combining chips can be done in any order.

However, the integer chips are not suited to representing rational numbers that are not integers. Whether such chips are used or not, the Standards require that students eventually understand location and addition of rational numbers on the number line. With the number line model, showing that the properties of operations extend to rational numbers requires some reasoning. Although it is not appropriate in Grade 6 to insist that all the properties be proved proved to hold in the number line or chips model, experimenting with them in these models is a good venue for reasoning (MP.2).[7.NS.1d]

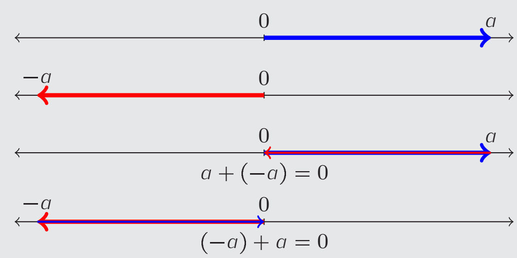
Subtraction of rational numbers is defined the same way as for positive rational numbers: $p-q$ is defined to be the missing addend in $q + ? = p$. For example, in earlier grades, students understand $5 - 3$ as the missing addend in $3 + ? = 5$. On the number line, it

Showing $5 + (-3) = 2$ and $-3 + 5 = 2$ on the number line



The number 5 is represented by the blue arrow pointing right from 0, and the number -3 is represented by the red arrow pointing left from 0. To add $5 + (-3)$ we place the arrow for 5 down first then attach the arrow for -3 to its endpoint. To add $-3 + 5$ we place the arrow for -3 down first then attach the arrow for 5 to its endpoint.

Showing $a + (-a) = 0$, and $(-a) + a = 0$ on the number line



7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a Describe situations in which opposite quantities combine to make 0.

b Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

d Apply properties of operations as strategies to add and subtract rational numbers.

Draft, 9 July 2013, comment at commoncoretools.wordpress.com.

is represented as the distance from 3 to 5. Or, with our newfound emphasis on direction on the number line, we might say that it is how you get from 3 from 5; by going two units to the right (that is, by adding 2).

In Grade 6 students apply the same understanding to $(-5) - (-3)$. It is the missing addend in $(-3) + ? = -5$, or how you get from -3 to -5. Since -5 is two units to the left of -3 on the number line, the missing addend is -2.

With the introduction of direction on the number line, there is a distinction between the distance from a and b and how you get from a to b. The distance from -3 to -5 is 2 units, but the instructions how to get from -3 to -5 are "go two units to the left." The distance is a positive number, 2, wherease "how to get there" is a negative number -2. In Grade 6 we introduce the idea of absolute value to talk about the size of a number, regardless of its sign. It is always a positive number or zero. If p is positive, then its absolute value $|p|$ is just p; if p is negative then $|p| = -p$. With this interpretation we can say that the absolute value of $p - q$ is just the distance from p to q, regardless of direction.[7.NS.1c]

Understanding $p - q$ as a missing addend also helps us see that $p + (-q) = p - q$.[7.NS.1c] Indeed, $p - q$ is the missing number in

$$q + ? = p$$

and $p + (-q)$ satisfies the description of being that missing, number:

$$q + (p + (-q)) = p + (q + (-q)) = p + 0 = p.$$

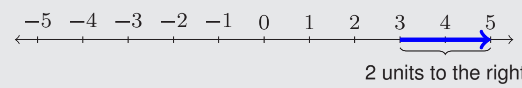
The figure in the margin illustrates this in the case where p and q are positive and $p > q$.

Multiplication and division of rational numbers Hitherto we have been able to come up with visual models to represent rational numbers, and the operations of addition and subtraction on them. This starts to break down with multiplication and division, and students must rely increasingly on the properties of operations to build the necessary bridges from their previous understandings to situations where one or more of the numbers might be negative.

For example, multiplication of a negative number by a positive whole number can still be understood as before; just as 5×2 can be understood as $2 + 2 + 2 + 2 + 2 = 10$, so 5×-2 can be understood as $(-2) + (-2) + (-2) + (-2) + (-2) = -10$. We think of 5×2 as five jumps to the right on the number line, starting at 0, and we think of $5 \times (-2)$ as five jumps to the left.

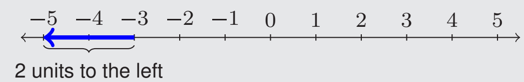
But what about $\frac{3}{4} \times -2$, or -5×-2? Perhaps the former can be undersood as going $\frac{3}{4}$ of the way from 0 to -2, that is $-\frac{3}{2}$. For the latter, teachers sometimes come up with metaphors involving going backwards in time or repaying debts. But in the end these metaphors do not explain why $-5 \times -2 = 10$. In fact, this is a

Showing $5 - 3 = 2$ on the number line.



You get from 3 to 5 by adding 2, so $5 - 3 = 2$.

Showing $(-5) - (-3) = -2$ on the number line.

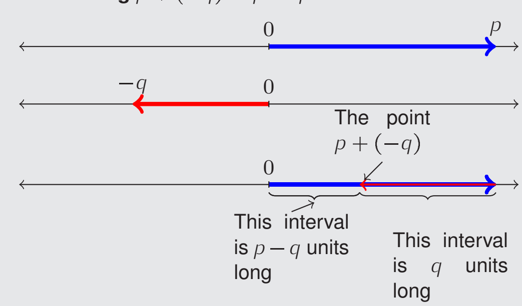


You get from -3 to -5 by adding -2, so $(-5) - (-3) = -2$

7.NS.1c Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Showing $p + (-q) = p - q$ on the number line



The red directed interval representing $-q$ is q units long, so the remaining part of the blue directed interval representing p is $p - q$ units long.

choice we make, not something we can justify by appeals to real world situations.

Why do we make the choice of saying that a negative times a negative is positive? Because we want to extend the operation of multiplication to rational number is such a way that *all* of the properties of operations are satisfied.[7.NS.2a] In particular, the property that really makes a difference here is the distributive property. If you want to be able to say that

$$4 \times (5 + (-2)) = 4 \times 5 + 4 \times (-2),$$

you have to say that $4 \times (-2) = -8$, because the number on the left is 12 and the first addend on the right is 20. This leads to the rules

$$\text{positive} \times \text{negative} = \text{negative} \quad \text{and} \quad \text{negative} \times \text{positive} = \text{negative}.$$

If you want to be able to say that

$$(-4) \times (5 + (-2)) = (-4) \times 5 + (-4) \times (-2),$$

then you have to say that $(-4) \times (-2) = 8$, since now we know that the number on the left is -12 and the first addend on the right is -20. This leads to the rule

$$\text{negative} \times \text{negative} = \text{positive}.$$

Why is it important to maintain the distributive property? Because when students get to algebra, they use it all the time. They must be able to say $-3x - 6y = -3(x + 2y)$ without worrying about whether x and y are positive or negative.

The rules about moving negative signs around in a product result from the rules about multiplying negative and positive numbers. Think about the various possibilities for p and q in

$$p \times (-q) = (-p) \times q = -pq.$$

If p and q are both positive, then this just a restatement of the rules above. But it still works if, for example, p is negative and q is positive. In that case it says

$$\text{negative} \times \text{negative} = \text{positive} \times \text{positive} = \text{positive}.$$

Just as the relationship between addition and subtraction helps students understand subtraction of rational numbers, so the relationship between multiplication and division helps them understand division. To calculate $-8 \div 4$, students recall that $(-2) \times 4 = -8$, and so $-8 \div 4 = -2$. By the same reasoning,

$$-8 \div 5 = -\frac{8}{5} \quad \text{because} \quad -\frac{8}{5} \times 5 = -8.$$

7.NS.2a Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

This means it makes sense to write

$$-8 \div 5 \quad \text{as} \quad \frac{-8}{5}.$$

Until this point students have not seen fractions where the numerator or denominator could be a negative integer. But working with the corresponding multiplication equations allows students to make sense of such fractions. In general, they see that[7.NS.2b]

$$-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}$$

for any integers p and q, with $q \neq 0$.

Again, using multiplication as a guide, students can extend division to rational numbers that are not integers.[7.NS.2c] For example

$$\frac{2}{3} \div (-\frac{1}{2}) = -\frac{4}{3} \quad \text{because} \quad -\frac{4}{3} \times -\frac{1}{2} = \frac{2}{3}.$$

And again it makes sense to write this division as a fraction:

$$\frac{\frac{2}{3}}{-\frac{1}{2}} = -\frac{4}{3} \quad \text{because} \quad -\frac{4}{3} \times -\frac{1}{2} = \frac{2}{3}.$$

Note that this is an extension of the fraction notation to a situation it was not originally designed for. There is no sense in which we can think of

$$\frac{\frac{2}{3}}{-\frac{1}{2}}$$

as $\frac{2}{3}$ parts where one part is obtained by dividing the line segment from 0 to 1 into $-\frac{1}{2}$ equal parts! But the fact that the properties of operations extend to rational numbers means that calculations with fractions extend to these so-called complex fractions $\frac{p}{q}$, where p and q could be rational numbers, not only integers. By the end of Grade 7, students are solving problems involving complex fractions.[7.NS.3]

Decimals are special fractions, those with denominator 10, 100, 1000, etc. But they can also be seen as a special sort of measurement on the number line, namely one that you get by partitioning into 10 equal pieces. In Grade 7 students begin to see this as a possibly infinite process. The number line is marked off into tenths, each of which is marked off into 10 hundreths, each of which is marked off into 10 thousandths, and so on ad infinitum. These finer and finer partitions constitute a sort of address system for numbers on the number line: 0.635 is, first, in the neighborhood betweeen 0.6 and 0.7, then in part of that neighborhood between 0.63 and 0.64, then exactly at 0.635.

The finite decimals are the rational numbers that eventually come to fall exactly on one of the tick marks in this decimal address system. But there are numbers that never come to rest, no

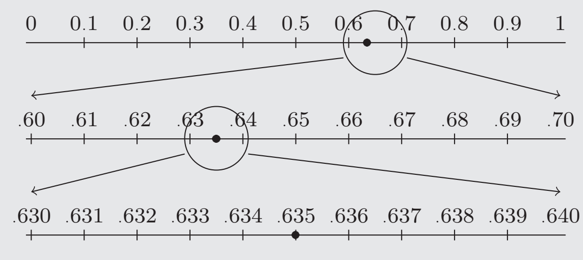
7.NS.2b Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Zooming in on 0.635



The finite decimal 0.635 can eventually be found sitting one of the tick marks at the thousandths level.

matter how far down you go. For example, $\frac{1}{3}$ is always sitting one third of the way along the third subdivision. It is 0.33 plus one-third of a thousandth, and 0.333 plus one-third of a ten thousandth, and so on. The decimals 0.33, 0.333, 0.3333 are successively closer and closer approximations to $\frac{1}{3}$. We summarize this situation by saying that $\frac{1}{3}$ has an infinite decimal expansion consisting entirely of 3s

$$\frac{1}{3} = 0.3333\ldots = 0.\overline{3},$$

where the bar over the 3 indicates that it repeats indefinitely. Although a rigorous treatment of this mysterious infinite expansion is not available in middle school, students in Grade 7 start to develop an intuitive understanding of decimals as a (possibly) infinite address system through simple examples such as this.[7.NS.2d]

For those rational numbers that have finite decimal expansions, students can find those expansions using long division. Saying that a rational number has a finite decimal expansion is the same as saying that it can be expressed as a fraction whose numerator is a base-ten unit (10, 100, 1000, etc.). So if $\frac{a}{b}$ is a fraction with a finite expansion, then

$$\frac{a}{b} = \frac{n}{10} \quad \text{or} \quad \frac{n}{100} \quad \text{or} \quad \frac{n}{1000} \quad \text{or} \quad \ldots,$$

for some whole number n. If this is the case, then

$$\frac{10a}{b} = n \quad \text{or} \quad \frac{100a}{b} = n \quad \text{or} \quad \frac{1000a}{b} = n \quad \text{or} \quad \ldots.$$

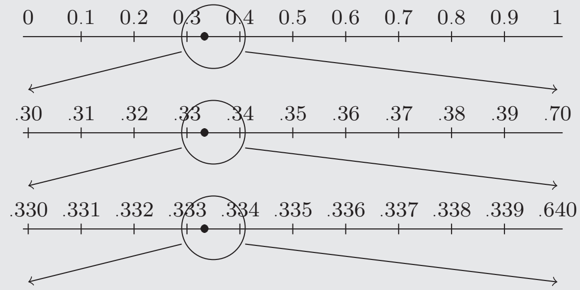
So we can find the whole number n by dividing b successively into $10a$, $100a$, $1000a$, and so on until there is no remainder.[7.NS.2d] The margin illustrates this process for $\frac{3}{8}$, where we find that there is no remainder for the division into 3000, so

$$3000 = 8 \times 375,$$

which means that

$$\frac{3}{8} = \frac{375}{1000} = 0.375.$$

Zooming in on $\frac{1}{3}$



The fraction $\frac{1}{3}$ is never eventually on one of the tick marks. It is always one third the way along the third subdivision.

7.NS.2d Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Division of 8 into 3 times a base-ten unit

$$\begin{array}{r} 3 \\ 8\overline{)30} \\ \underline{24} \\ 6 \end{array} \qquad \begin{array}{r} 37 \\ 8\overline{)300} \\ \underline{240} \\ 60 \\ \underline{56} \\ 4 \end{array} \qquad \begin{array}{r} 375 \\ 8\overline{)3000} \\ \underline{2400} \\ 600 \\ \underline{560} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Notice that it is not really necessary to restart the division for each new base-ten unit, since the steps from the previous calculation carry over to the next one.

Grade 8

Know that there are numbers that are not rational, and approximate them by rational numbers In Grade 7 students encountered infinitely repeating decimals, such as $\frac{1}{3} = 0.\overline{3}$. In Grade 8 they understand why this phenomenon occurs, a good exercise in expressing regularity in repeated reasoning (MP8).[8.NS.1] Taking the case of $\frac{1}{3}$, for example, we can try to express it as a finite decimal using the same process we used for $\frac{3}{8}$ in Grade 7. We successively divide 3 into 10, 100, 1000, hoping to find a point at which the remainder is zero. But this never happens; there is always a remainder of 1. After a few tries it is clear that the long division will always go the same way, because the individual steps always work the same way: the next digit in the quotient is always 3 resulting in a reduction of the dividend from one base-unit to the next smaller one (see margin). Once we have seen this regularity, we see that $\frac{1}{3}$ can never be a whole number of decimal base-ten units, no matter how small they are.

A similar investigation with other fractions leads to the insight that there must always eventually be a repeating pattern, because there are only so many ways a step in the algorithm can go. For example, considering the possibility that $\frac{2}{7}$ might be a finite decimal with, we try dividing 7 into 20, 200, 2000, etc., hoping to find a point where the remainder is zero. But something happens when we get to dividing 7 into 2,000,000, the left-most division in the margin. We find ourselves with a remainder of 2. Since we started with a numerator of 2, the algorithm is going to start repeating the sequence of digits from this point on. So we are never going to get a remainder of 0. All is not in vain, however. Each calculation gives us a decimal approximation of $\frac{2}{7}$. For example, the left-most calculation in the margin tells us that

$$\frac{2}{7} = \frac{1}{1000000}\frac{2000000}{7} = 0.285714 + \frac{2}{7} \times 0.0000001,$$

and the next two show that

$$\begin{aligned} \frac{2}{7} &= 0.2857142 + \frac{6}{7} \times 0.00000001 \\ \frac{2}{7} &= 0.28571428 + \frac{4}{7} \times 0.000000001. \end{aligned}$$

Noticing the emergence of the repeating pattern 285714 in the digits, we say that

$$\frac{2}{7} = 0.\overline{285714}.$$

The possibility of infinite repeating decimals raises the possibility of infinite decimals that do not ever repeat. From the point of view of the decimal address system, there is no reason why such number should not correspond to a point on the number line. For

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Division of 3 into 100, 1000, and $10{,}000$

$$\begin{array}{r} 33 \\ 3\overline{)100} \\ \underline{90} \\ 10 \\ \underline{9} \\ 1 \end{array} \qquad \begin{array}{r} 333 \\ 3\overline{)1000} \\ \underline{900} \\ 100 \\ \underline{90} \\ 10 \\ \underline{9} \\ 1 \end{array} \qquad \begin{array}{r} 3333 \\ 3\overline{)10000} \\ \underline{9000} \\ 1000 \\ \underline{900} \\ 100 \\ \underline{90} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Repeated division of 3 into larger and larger base ten units shows the same pattern.

Division of 7 into multiples of 2 times larger and larger base-ten units

$$\begin{array}{r} 285714 \\ 7\overline{)2000000} \\ \underline{1400000} \\ 600000 \\ \underline{560000} \\ 40000 \\ \underline{35000} \\ 5000 \\ \underline{4900} \\ 100 \\ \underline{70} \\ 30 \\ \underline{28} \\ 2 \end{array} \qquad \begin{array}{r} 2857142 \\ 7\overline{)20000000} \\ \underline{14000000} \\ 6000000 \\ \underline{5600000} \\ 400000 \\ \underline{350000} \\ 50000 \\ \underline{49000} \\ 1000 \\ \underline{700} \\ 300 \\ \underline{280} \\ 20 \\ \underline{14} \\ 6 \end{array} \qquad \begin{array}{r} 28571428 \\ 7\overline{)200000000} \\ \underline{140000000} \\ 60000000 \\ \underline{56000000} \\ 4000000 \\ \underline{3500000} \\ 500000 \\ \underline{490000} \\ 10000 \\ \underline{7000} \\ 3000 \\ \underline{2800} \\ 200 \\ \underline{140} \\ 60 \\ \underline{56} \\ 4 \end{array}$$

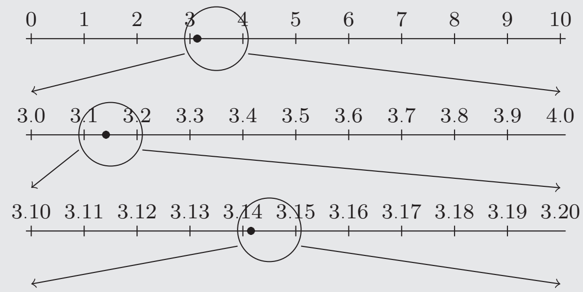
The remainder at each step is always a single digit multiple of a base-ten unit so eventually the algorithm has to cycle back to the same situation as some earlier step. From then on the algorithm produces the same sequence of digits as from the earlier step, ad infinitum.

example, the number π lives between 3 and 4, and between 3.1 and 3.2, and between 3.14 and 3.15, and so on, with each successive decimal digit narrowing its possible location by a factor of 10.

Numbers like π, which do not have a repeating decimal expansion and therefore are not rational numbers, are called *irrational*.[8.NS.1] Although we can calculate the decimal expansion of π to any desired accuracy, we cannot describe the entire expansion because it is infinitely long, and because there is no pattern (as far as we know). However, because of the way in which the decimal address system narrows down the interval in which a number lives, we can use the first few digits of the decimal expansion to come up with good decimal approximations of π, or any other irrational number. For example, the fact that π is between 3 and 4 tells us that π^2 is between 9 and 16; the fact that π is between 3.1 and 3.2 tells us that π^2 is between 9.6 and 10.3, and so on.[8.NS.2]

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Zooming in on π



The number π has an infinite non-repeating decimal expansion which determines each successive sub-interval to zoom in on.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

High School, Number*

The Real Number System

Extend the properties of exponents to rational exponents In Grades 6–8 students began to widen the possible types of number they can conceptualize on the number line. In Grade 8 they glimpse the existence of irrational numbers such as $\sqrt{2}$. In high school, they start a systematic study of functions that can take on irrational values, such as exponential, logarithmic, and power functions. The first step in this direction is the understanding of numerical expressions in which the exponent is not a whole number. Functions such as $f(x) = x^2$, or more generally polynomial functions, have the property that if the input x is a rational number, then so is the output. This is because their output values are computed by basic arithmetic operations on their input values. But a function such as $f(x) = \sqrt{x}$ does not necessarily have rational output values for every rational input value. For example, $f(2) = \sqrt{2}$ is irrational even though 2 is rational.

The study of such functions brings with it a need for an extended understanding of the meaning of an exponent. Exponent notation is a remarkable success story in the expansion of mathematical ideas. It is not obvious at first that a number such as $\sqrt{2}$ can be represented as a power of 2. But reflecting that

$$\left(\sqrt{2}\right)^2 = 2$$

and thinking about the properties of exponents, it is natural to define

$$2^{\frac{1}{2}} = \sqrt{2}$$

since if we follow the rule $(a^b)^c = a^{bc}$ then

$$\left(2^{\frac{1}{2}}\right)^2 = 2^{\frac{1}{2}\cdot 2} = 2^1 = 2.$$

Similar reasoning leads to a general definition of the meaning of a^b whenever a and b are rational numbers.[N-RN.1] It should be noted high school mathematics does not develop the mathematical ideas necessary to prove that numbers such as $\sqrt{2}$ and $3^{\frac{1}{5}}$ actually exist;

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

*This progression concerns Number and Quantity standards related to number. The remaining standards are discussed in the Quantity Progression.

in fact all of high school mathematics depends on the fundamental assumption that properties of rational numbers extend to irrational numbers. This is not unreasonable, since the number line is populated densely with rational numbers, and a conception of number as a point on the number line gives reassurance from intuitions about measurement that irrational numbers are not going to behave in a strangely different way from rational numbers.

Because rational exponents have been introduced in such a way as to preserve the laws of exponents, students can now use those laws in a wider variety of situations. For example, they can rewrite the formula for the volume of a sphere of radius r,

$$V = \frac{4}{3}\pi r^3,$$

to express the radius in terms of the volume,[N-RN.2]

$$r = \left(\frac{3}{4}\frac{V}{\pi}\right)^{\frac{1}{3}}.$$

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers An important difference between rational and irrational numbers is that rational numbers form a number system. If you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. For example, if you multiply the irrational number $\sqrt{2}$ by itself, you get the rational number 2. Irrational numbers are defined by not being rational, and this definition can be exploited to generate many examples of irrational numbers from just a few.[N-RN.3] For example, because $\sqrt{2}$ is irrational it follows that $3+\sqrt{2}$ and $5\sqrt{2}$ are also irrational. Indeed, if $3+\sqrt{2}$ were an irrational number, call it x, say, then from $3+\sqrt{2} = x$ we would deduce $\sqrt{2} = x - 3$. This would imply $\sqrt{2}$ is rational, since it is obtained by subtracting the rational number 3 from the rational number x. But it is not rational, so neither is $3+\sqrt{2}$.

N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with finite decimal approximations (and therefore with rational numbers), thinking about the properties of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments and attending to precisions (MP.3, MP.6).

Complex Numbers

That complex numbers have a practical application is surprising to many. But it turns out that many phenomena involving real numbers become simpler when the real numbers are viewed as a subsytem of the complex numbers. For example, complex solutions of differential equations can give a unified picture of the behavior of real solutions. Students get a glimpse of this when they study complex solutions of quadratic equations. When complex numbers are brought into the picture, every quadratic polynomial can be expressed as a product of linear factors:

$$ax^2 + bx + c = a(x - r)(x - s).$$

The roots r and s are given by the quadratic formula:

$$r = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad s = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

When students first apply the quadratic formula to quadratic equations with real coefficients, the square root is a problem if the quantity $b^2 - 4ac$ is negative. Complex numbers solve that problem by introducing a new number, i, with the property that $i^2 = -1$, which enables students to express the solutions of any quadratic equation.[N-CN.7]

N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.

One remarkable fact about introducing the number i is that it works: the set of numbers of the form $a + bi$, where $i^2 = -1$ and a and b are real numbers, forms a number system. That is, you can add, subtract, multiply and divide two numbers of this form and get another number of the same form as the result. We call this the system of complex numbers.[N-CN.1]

N-CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N-CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

All you need to perform operations on complex numbers is the fact that $i^2 = -1$ and the properties of operations.[N-CN.2] For example, to add $3 + 2i$ and $-1 + 4i$ we write

$$(3 + 2i) + (-1 + 4i) = (3 + -1) + (2i + 4i) = 2 + 6i,$$

using the associative and commutative properties of addition, and the distributive property to pull the i out, resulting in another complex number. Multiplication requires using the fact that $i^2 = -1$:

$$(3 + 2i)(-1 + 4i) = -3 + 10i + 8i^2 = -3 + 10i - 8 = -11 + 10i.$$

Division of complex numbers is a little tricker, but with the discovery of the complex conjugate $a - bi$ we find that every non-zero complex number has a multiplicative inverse.[N-CN.3] If at least one of a and b is not zero, then

$$(a + bi)^{-1} = \frac{1}{a^2 + b^2}(a - bi)$$

because

$$(a + bi)(a - bi) = a^2 - (bi)^2 = a^2 + b^2.$$

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Students who continue to study geometric representations of complex numbers in the complex plane use both rectangular and polar coordinates which leads to a useful geometric interpretation of the operations.[N-CN.4, N-CN.5] The restriction of these geometric interpretations to the real numbers yields and interpretation of these operations on the number line.

N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

One of the great theorems of modern mathematics is the Fundamental Theorem of Algebra, which says that every polynomial equation has a solution in the complex numbers. To put this into perspective, recall that we formed the complex numbers by creating a solution, i, to just one special polynomial equation, $x^2 = -1$. With the addition of this one solution, it turns out that every polynomial equation, for example $x^4 + x^2 = -1$, also acquires a solution. Students have already seen this phenomenon for quadratic equations.[N-CN.9]

N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Although much of the study of complex numbers goes beyond the college and career ready threshold, as indicated by the (+) on many of the standards, it is a rewarding area of exploration for advanced students.

kidcentral tn
KIDCENTRALTN.COM

Department of Education May 2015; Publication Authorization No. 331997; 1984 copies.
This public document was promulgated at a cost of $11.85 per copy.